NUMERICAL LINEAR ALGEBRA
ACADEMIC YEAR 2007-2008

MATLAB assignment day 1

A first step in the calculation of the eigenvalues of a matrix A is to bring the matrix into a simpler shape via a simularity transformation

$$B = S^{-1}AS$$

with S the transformation matrix.

Assigment 1: Prove that the matrices A and B have the same eigenvalues.

We first choose as transformation matrix

$$S = [v; Av; A^2v; \cdots ; A^{n-1}v];$$

in which we take the vector v that has ones as its entries. The space spanned by the vectors $v, Av, A^2v, \cdots , A^{n-1}v$ is called the Krylov subspace generated by the matrix A and vector v. As you will see in the remainder of this course, this subspace plays an important role in most modern iterative methods for solving eigenvalue problems or linear systems.

Assignment 2. Use the test matrix `'lehmer'` that can be obtained with the matlab command `A = gallery('lehmer',n);`, in which `n` is the size of the matrix.

- Investigate by printing A on your screen the properties of A. Take $n = 5$. Also compute the eigenvalues of A using the command `eig`.
- Generate S for $n = 5$, and compute the transformed matrix B. Inspect the properties of B by printing the matrix on your screen. Compute the eigenvaluesof B. Are they the same as of A?
- Repeat the above assignments for $n = 9$ and $n = 10$. Explain your results.

We are now going to use for S the matrix that has an orthonormal basis for the Krylov subspace as its columns. This can be done using the following (modified Gram-Schmidt) algorithm:

$v_1 = v/||v||$

for $j = 1, .., n-1$

$t = Av_j$

for $i = 1, .., j$

$t = t - (v_i, t)v_i$

end

$v_{j+1} = t/\|t\|$

end

$S = [v_1; ...; v_n]$

Assignment 3.

- Generate S for $n = 5$, and compute the transformed matrix B. Inspect the properties of B by printing the matrix on your screen. Compute the eigenvalues of B. Are they the same as of A?
- Repeat the above assignments for $n = 9$ and $n = 10$. Explain your results.