NUMERICAL LINEAR ALGEBRA
ACADEMIC YEAR 2007-2008

MATLAB assignment day 4

In this assignment we will implement the SOR method and investigate its convergence.

Download the file `sor.m` from the course webpage. This file contains code to generate a 2D-Poisson matrix on a square equidistant $n \times n$ grid. The right-hand-side vector is such that the solution x equals one. The code splits the matrix A into a strictly lower triangular part L, a main diagonal D, and a strictly upper triangular part U.

Assignment 1: Implement the SOR method to solve this system. Use the most efficient variant of the algorithm. Use the relative residual to test for convergence. Test your algorithm with $n = 10$, maximum number of iterations 1000, tolerance 10^{-6}, and omega = 1.

Assignment 2: Determine numerically a near-optimal value for omega (hint: omega should be between 1 and 2).

Assignment 3: Investigate how the number of iterations depends on the gridsize n. Do this by determining the number of iterations for $n = 4$, $n = 8$, $n = 16$ and $n = 32$. Take omega=1. Repeat this assignment but now for near optimal values of omega.

The calculation of the residual is an expensive operation. We would therefore like to use a cheaper termination criterion, preferably one on basis of the true error instead of on the residual. In the next assignment we derive such a criterion.

- Show that the spectral radius of $G = M^{-1}R$ approximately satisfies

$$\rho(G) \approx \frac{\|x_{k+1} - x_k\|}{\|x_k - x_{k-1}\|} \, .$$

- Show that if $\rho(M^{-1}R)$ is known, an estimate for the error is given by $\|x - x_k\|_2 \leq \frac{\rho(G)}{1-\rho(G)} \|x_k - x_{k-1}\|_2$.

- Implement this criterion in your code, and test it for $n = 10$.