Synchronized SIS process and a possibly largest non-Markovian threshold

Qiang Liu

*in collaboration with Piet Van Mieghem*

Delft University of Technology, the Netherlands

NetSciX18, Hangzhou, China
Jan. 05 - 08, 2018
1. The (Markovian) SIS process on networks

2. The Weibullian SIS process
   2.1 The limiting case \( \alpha \to \infty \)

3. Further discussions about the limiting case \( \alpha \to \infty \): the synchronized SIS process
The (Markovian) SIS process on networks
The Susceptible-Infected-Susceptible (SIS) process on networks: Each node is either *Infected* or *Susceptible* (healthy); The infection and curing process are **Poisson processes** (the time length between two adjacent infections is exponentially distributed) 

Mean-field thresholds — HMF: $\frac{\langle D \rangle}{\langle D^2 \rangle}$ NIMFA: $\frac{1}{\lambda_1}$

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**References:**

The Weibullian SIS process
The Poisson infection process is a special case. The infected nodes can be cured with rate $\delta$ (Poisson).

The Weibull infection process can represent very different infections: from long-tailed to Gaussian-like.

The epidemic threshold varies with $\alpha$ (No longer $1/\lambda_1$).
What is the range of the epidemic threshold for any infection process?

Qiang Liu and P. Van Mieghem, to appear PRE (Extended abstract see Complex Networks 2017)
\( \alpha \to \infty \) under mean-field approximation

Mean-field approximation:

\[
E[X_i(t)X_j(t)] = E[X_i(t)]E[X_j(t)].
\]

The same assumption made as in NIMFA for Markovian process leads to the epidemic threshold,

\[
\frac{1}{\ln(1 + \lambda_1)}
\]
Epidemic threshold for $\alpha \in [0, \infty]$.

The epidemic threshold changes approximately from 0 to $\frac{1}{\ln(1+\lambda_1)}$. 

The graph shows the epidemic threshold for different types of networks and their corresponding shape parameters. The threshold values for ER, rectangular grid, and scale-free networks are given as $\frac{1}{\ln(1+\lambda_1)}$ for each type of network, with values $0.4437$ for ER, $0.6242$ for rectangular grid, and $0.3899$ for scale-free network.
Further discussions about the limiting case $\alpha \rightarrow \infty$: the synchronized SIS process
The synchronized infection ($\alpha \to \infty$) seems to be the hardest situation for the infection persist on the network among all possible infection processes. Multiple infected neighbors is equivalent to one.
Properties when $\alpha \to \infty$

When $\tau > \frac{1}{\ln(\lambda_1+1)}$, \text{Maximum prevalence} < 1 + \lambda_1$.

When $\tau < \frac{1}{\ln(\lambda_1+1)}$, the prevalence is upper bounded by an exponentially decreasing function of time \((e^{-\delta(\lambda_1 + 1)^\beta})^t\).
An example: the spreading of computer virus

Improving the virus

Burst on the network

Each development iteration takes $1/\beta$ time units

Curing with rate $\delta$

The development cycle of the virus and the topology of the network collectively determines whether the infection can persist or not.
Conclusion

We show that non-Markovian infection process can lead to a non-constant infection probability in the steady state.

We provide a possible largest epidemic threshold for the SIS process on a network for any infection process $\frac{1}{\ln(\lambda_1+1)}$.

We discussed the properties of the synchronized SIS process (the limiting case $\alpha \to \infty$).

Markovian SIS process $\tau_c^{(1)} = \frac{1}{\lambda_1}$

Heavy-tailed interaction time

Gaussian-like interaction time

Susceptible-Infected process $\tau_c = 0$

Synchronized SIS process $\tau_c^{(1)} = \frac{1}{\ln(\lambda_1+1)}$

Email: Q.L.Liu@TuDelft.nl