E-10 SCATTERING BY SMALL-SCALE INCLUSIONS AND CRACKS

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Abstract

Wavefield modeling techniques have to account for the effect of small-scale inclusions and cracks. Due to the fact that these are small compared to the wavelelength and are present in very large numbers, special methods are required. In the present paper, an overview is given of methods based on integral equations, analytical methods and finite differences.

Introduction

Variations in the subsurface of the earth are present in many scales: from scales much larger than the typical seismic wavelength down to scales that are much smaller. Modeling wave propagation in media containing inclusions smaller than the wavelength of the probing wave field is of considerable interest in seismology. Inclusions or cracks that are much smaller than the seismic wavelength cannot be distinguished individually using seismic waves, but nevertheless can have a significant effect on the amplitude and phase of the transmitted wave field. Wavefield modeling methods addressing this type of geometries have to take into account that these variations are much smaller than the seismic wavelength and, at the same time, that the number of these variations is very large. In this paper, we discuss our work in this area considering both scalar acoustic and elastic wave scattering problems.

Integral-equation methods

Integral-equation methods are very accurate for modeling wavefield scattering by many small cracks or inclusions. If the impulse response, or Green's function, of the embedding medium is known, the unknown quantities are only situated on the cracks or in the inclusions. With the aid of perturbation techniques, efficient expansion functions can be found for the unknowns (using the fact that they are small). For a homogeneous embedding, models containing many thousands of cracks or inclusions can be considered without excessive computational effort. Integral-equation methods take all multiple scattering effects correctly into account (Muijres et al., 1998). In Figure 1, a typical (two-dimensional) geometry is shown containing 4,000 cracks in a homogeneous embedding. In Figure 2, the transmitted pressure is shown for two different choices of the crack boundary condition. For large numbers of cracks, the total transmitted wavefield is strongly influenced by the properties of each crack. For more complex, and therefore also more realistic embeddings, the Green's function is usually not known analytically and has to be computed as well. In that case, integral-equation methods become less efficient.

Analytic methods

Apart from obtaining accurate solutions, integral-equation methods can also be used for the derivation of analytical expressions for the transmitted wavefield if one is willing to make some limiting

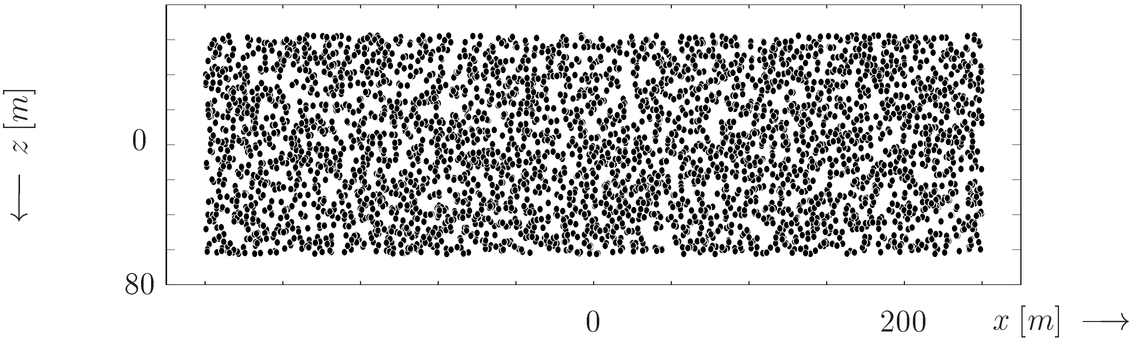


Figure 1: Model containing 4,000 cracks in a homogeneous embedding. Each crack has zero thickness and a half-width of 1 m.

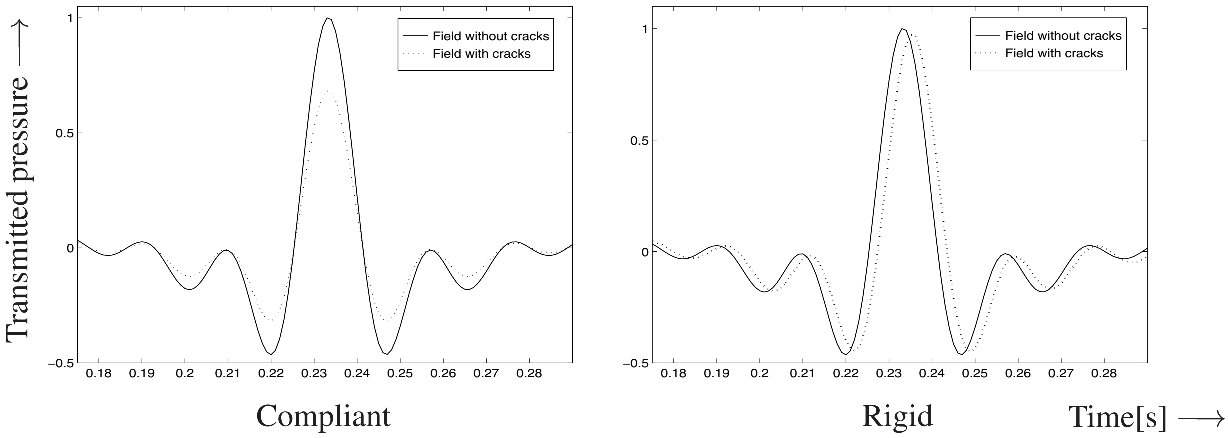
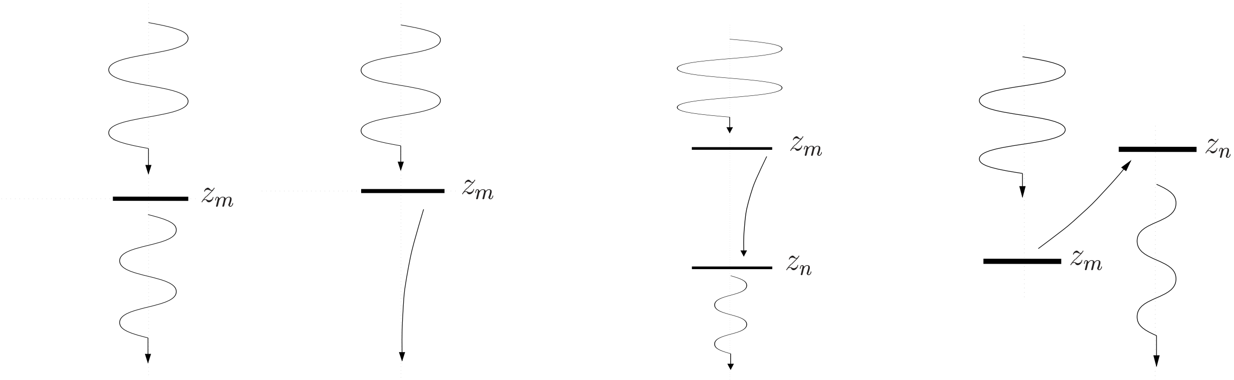


Figure 2: Transmitted pressure for two different choices of the crack boundary condition. The compliant crack is characterized by zero pressure, the rigid crack by zero normal particle velocity. The velocity of the embedding equals 3,000 m/s, the crack half width 1 m and the number of cracks 4,000.

assumptions. First, the incident wave is assumed to be a monochromatic plane wave. Furthermore, the cracks or inclusions are assumed to be organized in horizontal layers and are periodic in each layer. (Periodicities in different layers can be different.) It can be shown that the solution for this geometry can be constructed explicitly in terms of first-, second-, and higher, scattering orders. The first-order transmitted field is shown schematically in Figure 3(a) when the horizontal periodicity of the cracks is **larger** than the seismic wavelength and in Figure 3(b) when the horizontal periodicity of the cracks is **smaller** than the seismic wavelength. We observe that, in the latter case, the first-order transmitted field decays exponentially in the vertical direction and therefore can be neglected at distances larger than a few wavelengths. Therefore, the first-order transmitted field "sees" no horizontal variations smaller than its own wavelength. The second-order transmitted field, however, can be sensitive to these small-scale variations since it can "tunnel" over a certain distance provided this distance between two consecutive scattering processes is not much larger than the seismic wavelength. This is illustrated in Figures 3(c) and 3(d). In Figure 3(c), for instance, the monochromatic plane wave is scattered at the upper layer of cracks and has become evanescent (exponentially decaying). After a second-order scattering proces, part of the wave can become propagating again at a lower layer. With the same methods, it can also be shown that we can replace a medium with cracks with a "slower" medium, resulting in the same transmitted field. This is not an equivalent medium since it is only valid for that particular frequency and angle of incidence. Despite these limitations, it can take higher-order scattering effects into account and



(a) Propagating (b) Evanescent (c) Forward tunneling (d) Pegleg tunneling

Figure 3: Different scattering processes present in the transmitted wave: (a) first-order propagating, (b) first-order evanescent (exponentially decaying), (c) second-order forward tunneling, (d) second-order pegleg tunneling.

Contrast model (”slower” medium) Crack model

Figure 4: Periodic crack model and the corresponding ”slower” medium, resulting in the same transmitted field.

quantify the effect of spatial correlations of crack distributions on the transmitted field (Muijres & Herman, 2000).

Finite-difference methods

When the embedding is heterogeneous, integral-equation methods are computationally expensive due to the complexity of the Green’s function. Finite-difference methods are well suited for this type of problems, but require that the small-scale cracks or inclusions are represented by the finite-difference grid thus giving rise to a prohibitive large number of grid points. An alternative approach is to distribute the cracks over the grid points in such a way, that the scattered field of the ”distributed” cracks is the same as the actual scattered field (van Baren et al., 2001; van Antwerpen et al., 2002). This is illustrated in Figure 5. With this approach, the wavefield scattered by large numbers of small cracks can be computed efficiently, without using small grid spacings. In Figure 6, the horizontal displacement component resulting from a 2D elastic simulation is shown for the case of 2,000 small cracks situated in a low velocity layer. The field, scattered by the cracks, is clearly trapped within the low velocity layer.

Concluding remarks and acknowledgment

Wavefield modeling techniques have to account for the effect of small-scale inclusions and cracks. The extensions of the above methods to the 3D case and to the case of porous media represent a major challenge. This paper is an overview of work jointly carried out with Guus Muijres, Gerben van Baren and Vincent van Antwerpen.

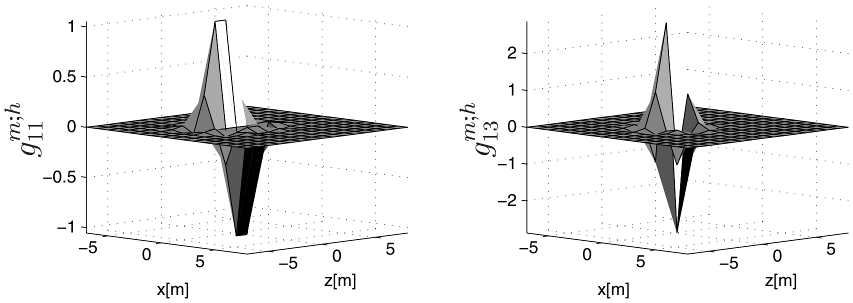
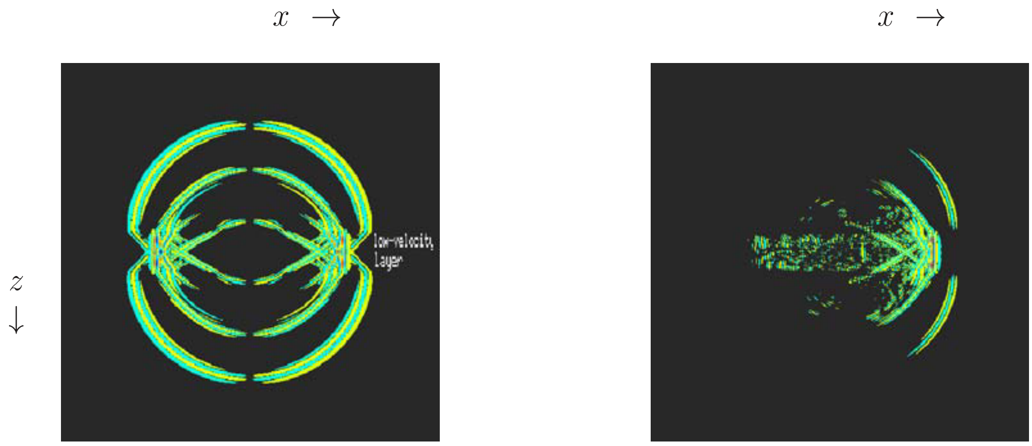


Figure 5: Discrete point-source tensor $g_{\alpha\nu}^{m;h}$ for one crack, computed using the staggered-grid finite-difference operator. The crack is situated in the origin and is oriented along the x-axis. The half width of the crack, a_m, is given by $a_m = 0.5$ m. The grid spacing is 2 m. The area around the crack, necessary to represent it as a distributed point source, is typically chosen as a square containing 7 × 7 grid cells.



(a) Horizontal component incident field (b) Horizontal component scattered field

Figure 6: Snapshots of the horizontal component of the incident and scattered displacement fields. The incident field is the field that would be present in the absence of the cracks, whereas the scattered field represents the effect due to the presence of the cracks.

References

Antwerpen, V.V. van, Mulder, W.A., and Herman, G.C., 2002. *Finite-difference modeling of two-dimensional elastic wave propagation in cracked media*, Accepted for publication in Geophys. J. Int.

Baren, G.B. van , Mulder, W.A., and Herman, G.C., 2001. *Finite-difference modeling of scalar wave propagation in cracked media*, Geophysics 66, pp. 267-276.

Muijres, A.J.H., Herman, G.C., and Bussink, P.G.J., 1998, *Acoustic wave propagation in two-dimensional media containing small-scale heterogeneities*, Wave Motion 27, pp. 137-154.

Muijres, A.J.H., and Herman, G.C., 2000. *Derivation of smooth apparent models for media with small-scale cracks (scalar theory)*, Geophys. J. Int. 140, pp. 587-597.