# C003 HARD AND SMOOTH MODELS FOR SEISMIC IMAGING BASED ON RAYTRACING

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Understanding of wave propagation is essential in geophysical imaging techniques. Most of these techniques require an *a priori* model of the subsurface. Conventional imaging techniques usually employ a high-frequency approximation: only the wave front is considered. In the limit of infinite frequency, only discontinuous variations of the velocity produce reflections. It therefore seems an obvious choice to require the velocity model to be smooth: all reflection data are caused by discontinuities that can be considered as perturbations on the smooth background model.

Two aspects of this approach make the construction of a proper background model less obvious. First, frequencies are only measured in a finite band. Secondly, variations of the medium parameters occur at several length scales. Large-scale fluctuations might be defined as variations that have a length scale larger than the dominant wavelengths in the data, where small-scale variations occur on length scales much smaller that the dominant wavelengths. Unfortunately, this distinction depends on the angle of incidence: for one range of angles the medium may appear smooth, whereas for other angles the same medium may appear discontinuous. As a consequence, it is difficult to separate those aspects from the model that may be considered smooth from those that may be viewed as perturbations.

It is an open question how wave propagation in the high-frequency approximation in a smooth medium is related to the propagation in a discontinuous medium, except in one-dimensional media. In this paper we study the relation between smooth models and piecewise constant models (i.e., exhibiting velocity jumps at interfaces) by studying wave propagation in a one-parameter family of media that interpolate between the two extremes. We show evidence that the wave propagation in the limit from a smooth to a discontinuous model is *singular*. The reflection coefficient, however, has a smooth limit. This is in contrast with the situation in one dimension, where recent results show that propagation is non-singular in the limit from a smooth to a hard model.

As an illustration, we consider a medium that consists of two half-spaces, with a constant velocity of 3.5 on the "left" and 1.5 on the "right". Figure 1 shows a two-dimensional finite-difference solution obtained in this medium. The wave front consists of several parts: the direct wave and transmitted wave, a reflected wave (part of a circle) and a refracted wave (a straight line). The refracted wave becomes tangent to the reflected wave at a given point, marked by the black dot in Fig. 2d.

Figure 2a–c show traces of ray-tracing solutions for smoothed media, superimposed on a 3D finite-difference solution in the hard medium. The amount of smoothing is characterised by a smoothing length $H$. Figure 2d shows the exact wave front. The wave front obtained by ray tracing can be seen to approach the wave front in the discontinuous case for decreasing $H$ (Fig. 2a to 2c), except for one feature. The missing part corresponds to the reflected wave for smaller angles. This is the part of the trace data in Fig. 2d beyond the black dot (i.e., for later times). This shows that the limit for vanishing smoothing length of the wave front is singular.

We draw the following conclusions:

- In a smooth medium, ray-tracing produces a *continuous* wave front.
- The various branches of the ray-tracing wave front can be identified with the direct wave, the transmitted wave, the refracted wave, and part of the reflected wave in the limit of vanishing smoothing length. This limit is singular.
- The part of the reflected wave that is captured by the ray-tracing in a smoothed medium is the part that lies between the critical angle and the point where the refraction wave becomes tangent to the reflected wave.

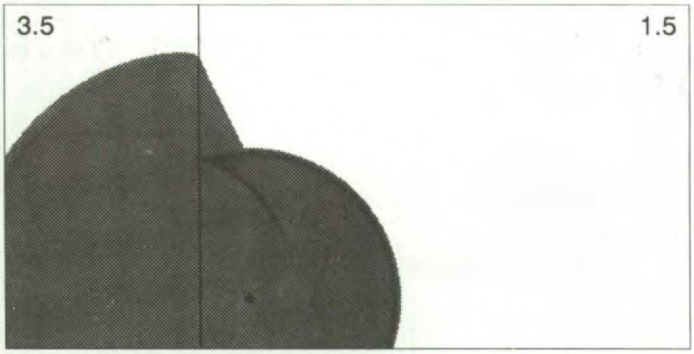


**Fig. 1**. 2D finite-difference solution obtained for a piecewise constant medium with a velocity of 3.5 at the left and 1.5 at the right. The position of the source is marked by a dot. The size of the domain is 14×7.

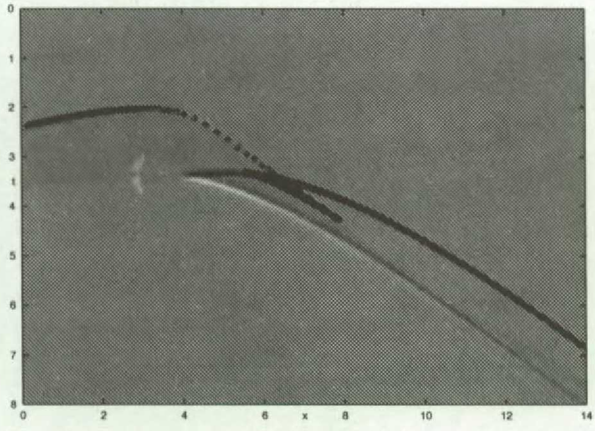

**Fig. 2a**. Ray-tracing result (diamonds) for a smooth model with $H = 0.5$. The gray-scale is obtained from a 3D finite-difference solution in the piecewise constant model ($H = 0$).

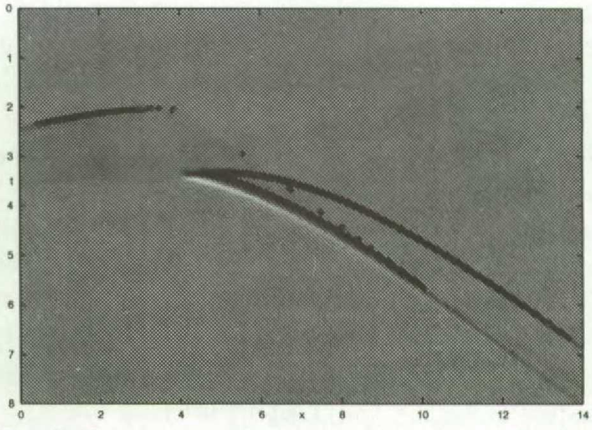

**Fig. 2b**. Ray-tracing result for a smooth model with $H = 0.2$ compared to a finite-difference solution with $H = 0$.

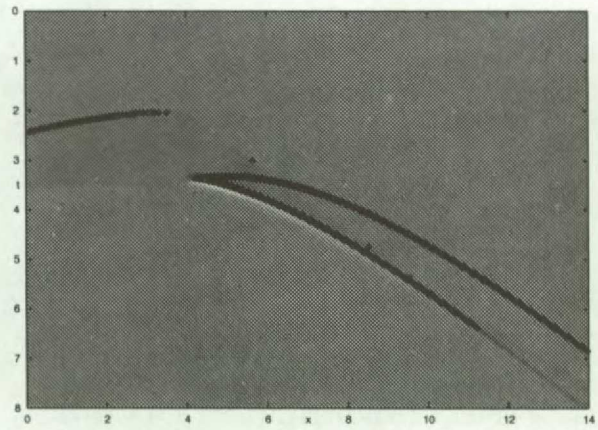

**Fig. 2c**. Ray-tracing result for a smooth model with $H = 0.1$ compared to a finite-difference solution with $H = 0$.

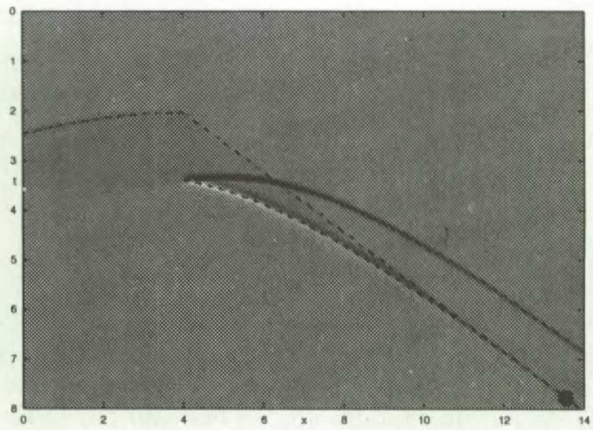

**Fig. 2d**. Exact ray-tracing solution (dashed lines) and finite-difference result.