D036

A Robust Solver for CSEM Modelling on Stretched Grids

W.A. Mulder* (Shell International Exploration & Production BV)

SUMMARY

Modelling and inversion of marine Controlled-Source Electro-Magnetic data for prospect evaluation requires a fast and robust solver. A multigrid method presented earlier provides excellent performance on equidistant and mildly stretched grids. The grid stretching takes care of the singular behaviour of the electric and magnetic field near the current source. In addition, it reduces the errors due to the artificial boundaries by placing them sufficiently far away from the source and receivers. Unfortunately, the performance of the multigrid method breaks down with stronger grid stretching, as may happen when modelling shallow seawater.

Here, a new and robust multigrid variant based on line relaxation and semi-coarsening is presented. The computation of the electric field for a realistic marine example required only three bicgstab2 iterations when using the new method as a preconditioner. Because its computational cost is an order of magnitude larger than the earlier multigrid method, it only becomes more efficient if the amount of grid stretching between neighbouring cells is larger than about 4%.

Introduction

Marine Controlled-Source Electro-Magnetic (CSEM) data enable the construction of a subsurface resistivity model that may reveal the presence of hydrocarbon reservoirs. The low source frequencies generate diffusive electro-magnetic signals in the water and the earth. Direct processing and interpretation of diffusion data can be difficult. Scenario studies based on modelling as well as full inversion of the data will lead to better prospect evaluation. A fast and robust solution method is essential for reasonable turn-around time.

An efficient multigrid solver for CSEM modelling and inversion was presented earlier (Mulder, 2006). Convergence on uniform grids required about 8 multigrid cycles, independent of the number of grid points. Unfortunately, the performance decreased dramatically on stretched grids. The stretching is needed to resolve the singular behaviour of the electric and magnetic field near the current source and to move the artificial, perfect electric conducting boundaries sufficiently far away from the source and receivers. With mild grid stretching, the use of multigrid as a preconditioner for bicgstab2 (Gutknecht, 1993) – a conjugate-gradient type iterative method – still provides acceptable results. Stronger stretching may be necessary if the seawater is shallow, leading to unacceptably large iteration counts.

Here, a more robust solver based on semi-coarsening and line relaxation is presented that is a further improvement of initial attempts reported by Jönsthövel *et al.* (2006).. The scheme outperforms the earlier, standard multigrid preconditioner when the grid stretching is more severe.

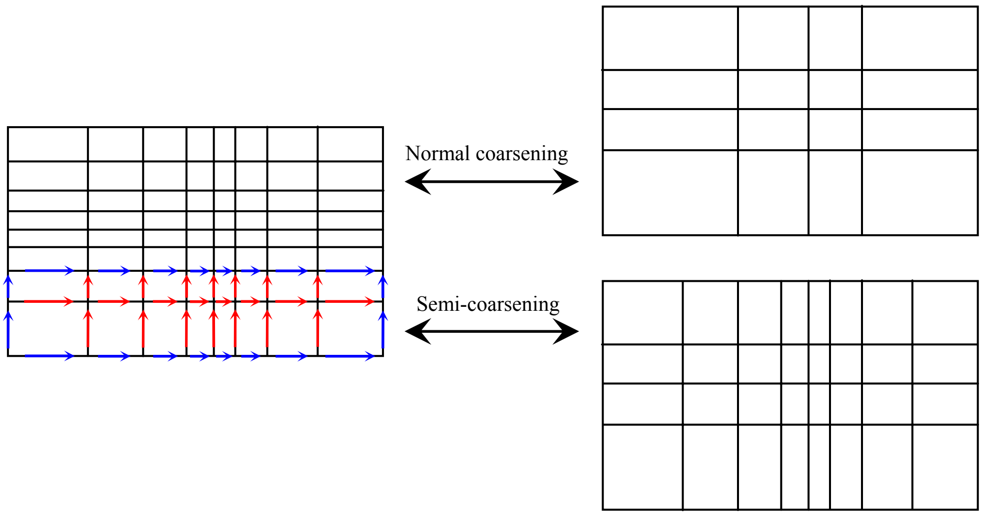
Method

The multigrid technique is an iterative method for solving partial differential equations that are discretised on a grid. For linear equations, the problem is to find a solution $\mathbf{u}$ such that the residual $\mathbf{r} = \mathbf{f} - A\,\mathbf{u}$ is zero. Here A is a large and sparse matrix that describes the discretised equations. In the multigrid method, the solution $\mathbf{u}$ is represented on several grids simultaneously. The one the solution is required on is called the fine grid. Coarser grids are constructed by combining cells (in the present case) into larger ones. The operation that projects the residual from a finer to a coarser grid is called restriction. The interpolation of the coarse-grid correction to the finer grid is called prolongation. On each grid, a relaxation scheme computes an approximate solution. Typical relaxation schemes are built on an approximation to A that is easy to invert. Often, these schemes will find the short-range, oscillatory, or rough components of the solution, but will have difficulty with the long-range, smooth components. Projection of the smooth components to a coarser grid makes them oscillatory, so that a relaxation scheme or smoother can easily remove them on that coarser grid. Applying multigrid recursively on subsequently coarser grids effectively reduces the norm of the residual for both rough and smooth components of the solution, making the method very efficient for a class of problems (see, for instance, Wesseling, 1992).

The multigrid method presented by Mulder (2006) provides solutions in a number of iterations independent of the number of grid points on equidistant grids. With stronger grid stretching, however, the number of iterations increases dramatically. The stretching introduces an effective anisotropy in the discrete equations. Multigrid methods cannot handle that very well. Still, bicgstab2 with multigrid as a preconditioner can provide acceptable convergence rates. With strong grid stretching, however, the resulting number of iterations was still very large in some examples.

Here, a more robust solver, based on semi-coarsening and line relaxation is presented. Figure 1 illustrates normal coarsening and semi-coarsening for the 2D case. On the left, 8×8 cells have the electric field components living on their edges. Normal coarsening turns this into 4×4 cells, as shown in the upper right panel. Semi-coarsening means that the cells are not coarsened in all coordinate directions. In the example on the lower right, coarsening was only applied in the vertical direction. This leads to a higher computational cost because the coarser

Figure 1. Illustration of normal and semi-coarsening and line relaxation in 2D. The horizontal and vertical electric field components live on the edges of cells.

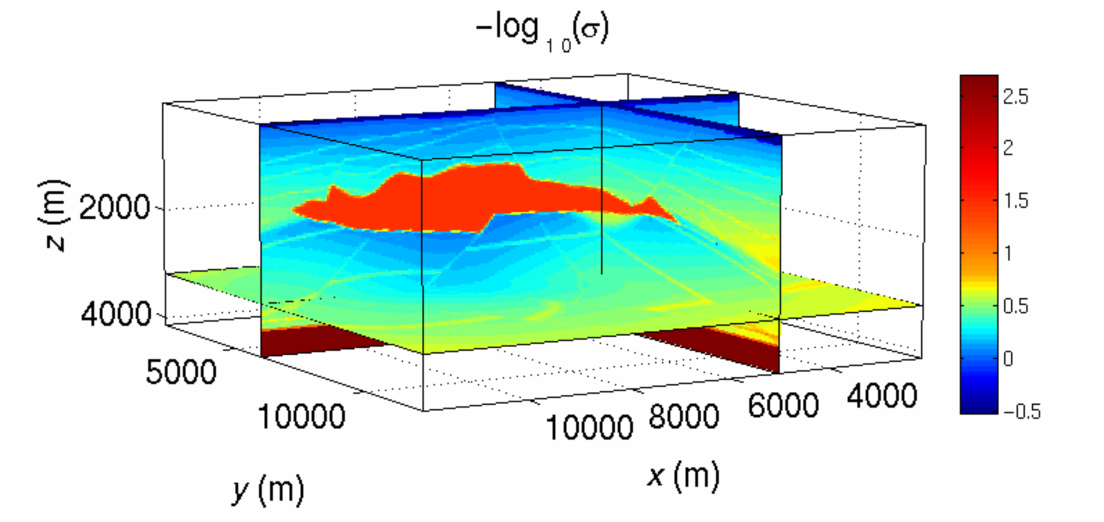


grids now contain more cells, but can also compensate the effects of anisotropy in the discretised differential operators. Line relaxation offers another way of dealing with anisotropy. In the standard scheme (Mulder, 2006), the relaxation scheme simultaneously solved the six electric field components on the edges connected to a single node, before moving to the next node. The tangential electric field components at the boundaries of the domain are excluded because the perfect electric conducting boundary conditions require them to be zero. Line relaxation simultaneously solves all components that live on edges connected to nodes on a single line, before moving to the next line. In 2D, this involves all the red arrows sketched in Figure 1. The red and blue arrows contribute to the residuals involved in that line. In 3D, additional components that stick out of the paper are also involved, two for each node. In Line Gauss-Seidel relaxation, subsequent lines are treated in the natural, lexicographical order using the solution updates of previous lines as soon as these become available. Its symmetric variant repeats this in the opposite order.

The following multigrid variant led to a good performance. First, Symmetric Line Gauss-Seidel (SLGS) relaxation was carried out as pre-smoothing with the line in the x- and in the y-direction, followed by semi-coarsening in the same directions. After the recursive application of multigrid to increasingly coarser grids, still using semi-coarsening in x and y, we return back to the fine grid, after which SLGS is repeated in the same coordinates as post-smoothing. For the next multigrid cycle, we repeat this in the y- and z-direction, and for the third, in the z- and x-direction. These 3 multigrid cycles serve as a single preconditioning operation for bicgstab2. The latter requires two of these expensive preconditioning steps, involving six multigrid cycles in total.

A non-standard Cholesky decomposition was used for the line solve,. The standard decomposition factors a hermitian matrix A into LL^H, where L is a lower triangular matrix and L^H its complex conjugate transpose. In our case, the discretisation is based on the Finite Integration Technique (Weiland, 1977) and provides a matrix A that is complex-valued and symmetric: $A = A^T$, where the superscript ^{T} denotes the transpose. The line relaxation scheme takes a matrix B that is a subset of A along the line. B is a complex symmetric band matrix with eleven diagonals. The non-standard Cholesky decomposition factors the matrix B into LL^T. Because of the symmetry, only the main diagonal and five lower diagonal elements of B need to be computed. The Cholesky decomposition replaces this matrix by L, containing six diagonals, after which the line relaxation can be carried out by simple back-substitution.

Figure 2. Logarithm of the resistivity (1/σ) in Ohm m for a part of the SEG/EAGE salt model.



A marine example

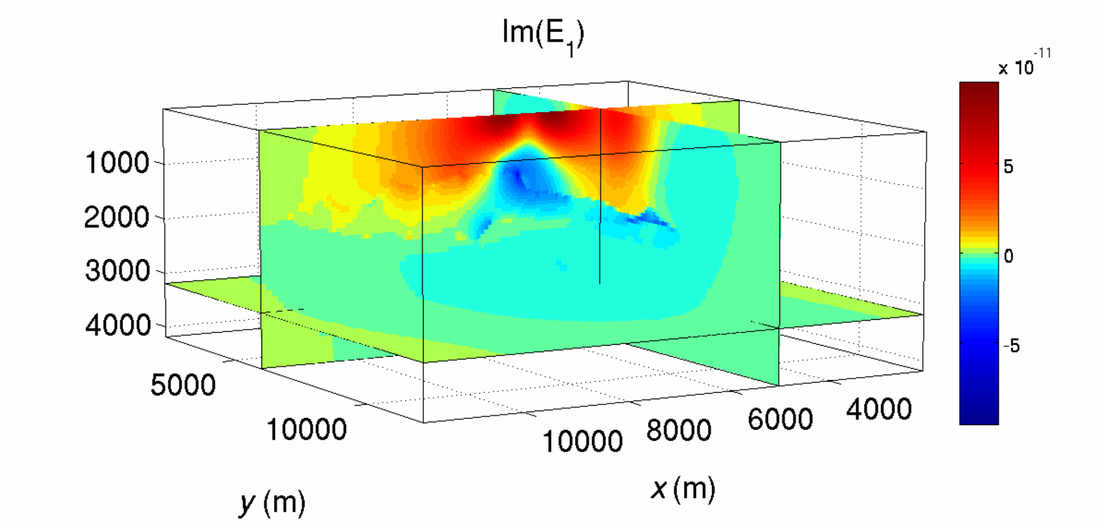
The SEG/EAGE salt model (Aminzadeh et al. 1997), originally designed for seismic simulations, served as a template for a realistic subsurface model. Its dimensions are 13500 by 13480 by 4680 m. The seismic velocities of the model were replaced by resistivity values. The water velocity of 1.5 km/s was replaced by a resistivity of 0.3 Ohm m. Velocities above 4 km/s, indicative of salt, were replaced by 30 Ohm m. Basement, beyond 3660 m depth, was set to 0.002 Ohm m. The resistivity of the sediments was determined by $(v/1700)^{3.88}$ Ohm m, with the velocity v in m/s (Meju et al. 2003). For air, the resistivity was set to 10^8 Ohm m. Figure 2 displays the resistivity on a logarithmic scale for the central part of the model. A finite-length current source was positioned between (6400,6500,50) and (6600,6500,50) m. Receivers were placed at the sea bottom on the line $y = 6500$ m, at depths between 50 and 230 m. We computed the electric field components at 0.25 Hz. The grid was stretched away from the centre of the source in the horizontal directions, using power-law stretching. In the vertical direction, the grid had a constant spacing the water layer and power-law stretching beyond. On a grid of 128×128×128 cells, the spacing in the horizontal coordinates ranged from 25 to 285 m, and in depth from 7.5 to 1667 m. In the example with 256^3 cells, the cell widths were about half those values.

The tolerance for convergence was set to 10^{-6}, meaning that the iterations were stopped as soon as the norm of the residual had decreased by this factor, relative to the residual norm obtained for a zero electric field. Table 1 lists the number of bicgstab2 iterations, the number of multigrid cycles in the preconditioner or as a stand-alone method, and the cpu times measured on a AMD Opteron 248 for a problem discretised on 128^3 and 256^3 cells. The amount of stretching is measured by α: the maximum ratio between cells widths of neighbouring cells is 1+ α. The table lists the maximum over the three spatial coordinates. In

Table 1. Number of bicgstab2 iterations for the standard multigrid method and the one based on semi-coarsening and line relaxation. The number of cells is N^3, the maximum stretching factor between neighbouring cells is given by 1+α.

N	max. α	method	iterations	MG cycles	cpu time (s)
128	0.084	Standard coarsening	52	104	$4.5\ 10^3$
		Semi-coarsening + line	2	12	$1.9\ 10^3$
		Idem without bicgstab2	–	14	$2.3\ 10^3$
256	0.041	Standard coarsening	50	100	$3.6\ 10^4$
		Semi-coarsening + line	3	18	$2.5\ 10^4$
		Idem without bicgstab2	–	24	$3.2\ 10^4$

Figure 3. Imaginary part of E_1 in V/m.



general, the larger α, the larger the number of iterations for the standard scheme. The code was partly written in matlab and partly in C, so the cpu-times provide only a crude indication of the performance. The relaxation scheme and evaluation of the residual are the most costly parts, and these were coded in C. The solver based on semi-coarsening and line relaxation costs 11 times more cpu time than the standard solver per iteration, but requires just a few bicgstab2 iterations.

Figure 3 shows the imaginary part of the *x*-component of the electric field.

Conclusions

A new preconditioner based on multigrid, semi-coarsening, and line relaxation provides solutions for 3D EM diffusion in about three full bicgstab2 iterations, even in the presence of strong grid stretching. As its cost is 11 times that of a standard multigrid preconditioner, it only wins if the amount of stretching from cell to cell exceeds about 4%.

References

Gutknecht, H.H. [1993] Variants of BiCGStab for matrices with complex spectrum. SIAM Journal on Scientific Computing 14, 1020–1033.

Aminzadeh, F., Brac, J., and Kunz, T. [1997] 3-D Salt and overthrust models, Society of Exploration Geophysicists, Tulsa, Oklahoma.

Meju, M.A., Gallardo, L.A., and Mohamed, A.K. [2003] Evidence for correlation of electrical resistivity and seismic velocity in heterogeneous near-surface materials. Geophysical Research Letters 30 (7), 1373–1376.

Mulder, W.A. [2006] A multigrid solver for 3D electromagnetic diffusion. Geophysical Prospecting 54, 633–649.

Jönsthövel, T. B., Oosterlee, C.W., and Mulder, W.A. [2006] Improving multigrid for 3-D electro-magnetic diffusion on stretched grids, in: Proceedings, ECCOMAS European Conference on Computational Fluid Dynamics (P. Wesseling, E. Oñate, J. Périaux, eds.), Egmond aan Zee, The Netherlands, Sept 5–8, 2006.

Weiland , T. [1977] A discretization method for the solution of Maxwell's equations for six-components fields. Electronics and Communications 31, 116–120.

Wesseling, P. [1992] An Introduction to Multigrid Methods, John Wiley & Sons Ltd.