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Waveform Inversion with Data Correlation

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SUMMARY

A common way to obtain a suitable velocity model for migration is migration velocity analysis. These methods somehow try to focus the migrated image in the depth-domain. Recently, a method was proposed to perform the velocity analysis in the data-domain. And first tests using the convolutional model and NMO velocities proved successful.
In this paper we want to incorporate this method in a waveform inversion scheme. Classical waveform inversion tries to fit the data in a least-squares sence, and it has been shown that it is difficult to get an update of the background velocity this way. By combining classical waveform inversion with the data-correlation method we hope to be able to invert for the background model as well.

Introduction

Waveform inversion tries to find a velocity model from seismic data by minimizing a data-misfit function. A common way of measuring the data-misfit is the least-squares (LS) difference between observed and predicted data (Tarantola, 1984). Although this approach is intuitively pleasing, it has been noted that it is very difficult to obtain information about the slowly varying components of the velocity background model this way. This is caused by the absence of low frequencies in the data. One way to get information about the background model is by migration velocity analysis (MVA). Well-known MVA methods are Differential Semblance Optimization (Symes, 1999; Mulder and ten Kroode, 2002), stack-power and depth-focusing (Shen *et al.*, 2003). All these methods take advantage of the redundancy in the data to create several migrated images and compare these images according to an appropriate criterion, which we will discuss briefly in subsequent sections.

van Leeuwen and Mulder (2007a) proposed a method to perform the velocity analysis in the data domain. This method relies on measuring the amount of focusing of a correlation of the measured and predicted data. First tests using the convolutional model and NMO traveltimes proved successful. In the current paper we use a time-domain finite difference code to model the data for 1D velocity models instead of the much simpler convolutional model.

Modeling

The data $p(t,x)$ are modeled using a time-domain finite difference scheme to solve the wave equation, here denoted by

$$\begin{aligned} \mathcal{L}[m]u(t,x,z) &= f(t,x,z), && (1)\\ p(t,x) &= u(t,x,0). && (2) \end{aligned}$$

where $\mathcal{L}[m]$ is the appropriate operator, which depends on the squared slowness $m(z) = 1/v(z)^2$. We introduce an operator $\mathcal{F}$ that acts on m and produces the data according to Eq. (2). Symbolically, we express this as

$$p(t,x) = \mathcal{F}m. \tag{3}$$

The measured data are denoted with a hat:

$$\hat{p}(t,x) = \mathcal{F}\hat{m}. \tag{4}$$

Waveform inversion tries to find a velocity model m^* that minimizes the LS difference between measured and predicted data

$$m^* = \underset{m}{\operatorname{argmin}}\, ||\mathcal{F}m - \hat{p}||_2^2, \tag{5}$$

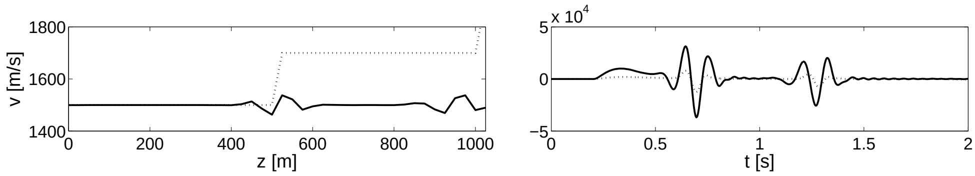
by using a gradient-based optimization method, e.g., Conjugate Gradients or BFGS. It has been noted that, when starting from a smooth initial model, the first iteration is equivalent to a migration. To clarify this, note that the gradient g of the misfit function can be written as a correlation at zero shift of a source and receiver wavefield

$$g[m](z) = \int \mathrm{d}t \int \mathrm{d}x\, u_s(t,x,z)u_r(t,x,z), \tag{6}$$

where

$$\begin{aligned} \mathcal{L}[m]u_s &= f, && (7)\\ \mathcal{L}^H[m]u_r &= \hat{p}. && (8) \end{aligned}$$

In traditional wave equation migration, the gradient is used as the migrated image. However, the image can be improved by iterating.



(a) (b)

Figure 1: *(a) shows the 'true' velocity model (dots) and an initial velocity model (solid) that is produced as described in equation 13. The corresponding near-offset trace is depicted in (b). This shows that the traveltimes of the near offset events are indeed identical.*

Stack-power

To judge the quality of the background model a 'generalization' of the migrated image is used

$$I[m](z,x) = \int \mathrm{d}t\, u_s(t,x,z)u_r(t,x,z). \tag{9}$$

Note that $\int \mathrm{d}x\, I[m](z,x)$ gives the traditional migrated image. If the background model is correct the image should not depend on x and a stack of the image should be maximal because of the constructive interference. In this approach we seek the velocity model that obeys

$$m^* = \underset{m}{\operatorname{argmax}}\, ||I[m](z,x)||_2^2. \tag{10}$$

It is also possible to asses the quality of the migrated image in the data domain. This requires generating new data for the image and comparing this to the observed data

$$\begin{aligned} m^* &= \underset{m}{\operatorname{argmin}}\, ||p[m] - \hat{p}||_2^2, & (11)\\ p[m] &= \mathcal{F}(m + \alpha g[m](z)), & (12)\\ & & (13) \end{aligned}$$

where $g[m]$ is defined in equation 6 and α is chosen such that the amplitudes of $p[m]$ and $\hat{p}$ match. This approach is known as Migration Based Traveltime Tomography (MBTT). By calculating the reflectivity for each background model the reflectors move around in depth but the corresponding events in the data will have the same zero-offset traveltimes for each background model. Figure 1 shows an example. Shown are the 'true' velocity model and the migrated image superimposed on the initial background model. The corresponding near-offset traces ($x = 100$ m) are also displayed. It has been shown that this method is more successful in updating the background velocity model than classical LS inversion.

Depth-focusing

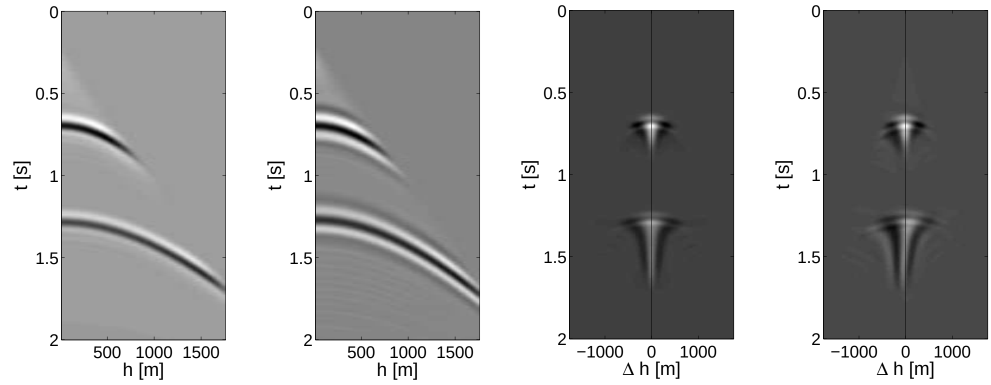
Another generalization of the image can be considered, which uses offset at depth as redundant variable

$$I[m](z,\Delta x) = \int \mathrm{d}t \int \mathrm{d}x\, u_s(t,x,z)u_r(t,x+\Delta x,z). \tag{14}$$

Note that $I[m](z,0)$ gives the traditional migrated image. For the correct velocity model the image should focus at zero shift. Hence, the criterion to find the correct velocity model is

$$m^* = \underset{m}{\operatorname{argmin}}\, ||\Delta x I[m]||_2^2. \tag{15}$$

The weight Δx in the norm will annihilate energy at zero shift, causing a minimum of the misfit function for the correct velocity model.
We have proposed a similar approach, defined in the data-domain. Instead of correlating the



(a) (b) (c) (d)

Figure 2: *In (a) and (b) the true and initial data are shown. This is the data that corresponds to the true and initial velocity models, shown in figure 1. The data-correlation for the true velocity model (i.e., the spatial correlation of the true data with itself) is shown in (c). The data-correlation for the initial velocity model is depicted in (d). There is a clear distinction in the focusing between both panels. The goal of the optimization of the functional is now to focus panel (d).*

wavefields at depth, data is generated for the migrated image and this is correlated with the observed data. It has been shown that this correlation also exhibits a similar focusing behavior as the image-domain correlation. Just as MBTT can be seen as 'natural' extension of the stack-power misfit function to the data-domain, so the data-correlation can be seen as a natural extension of the depth-focusing method. The details of this methods are discussed next.

Data correlation

The velocity model is explicitly divided in a reflectivity and a background model, denoted with r and m respectively. The background model is represented with splines on a coarse grid and the reflectivity part is represented on a fine grid. For a given background model the predicted data $p[m]$ are generated according to equation 13. To measure the misfit, the predicted data is correlated with the measured data

$$C_h[p[m], \hat{p}](t, \Delta h) = \int \mathrm{d}h\, p[m](t, h)\hat{p}(t, h + \Delta h). \tag{16}$$

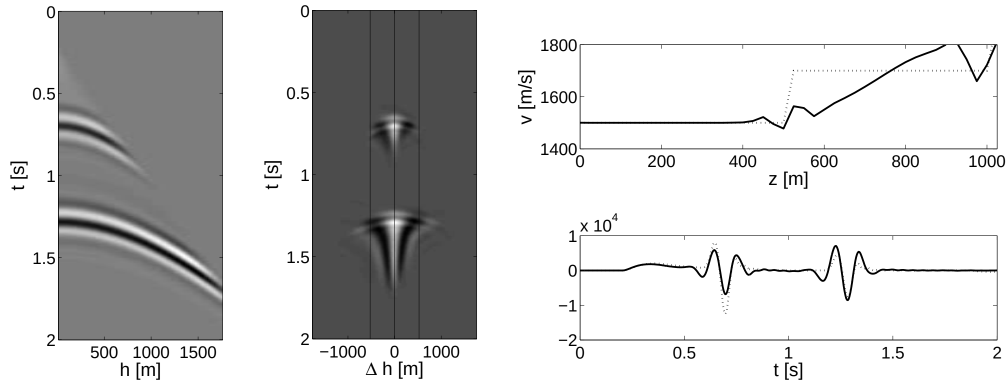
In figure 2, an example of the data and data-correlation is shown. The amount of focusing of this correlation is then measured by a weighted norm. The weight is chosen such that the weighted norm has a maximum when most of the energy is concentrated in the centre. To achieve this we use a gaussian weight. Here, the criterium to find the correct velocity model is

$$m^* = \underset{m}{\operatorname{argmin}} \, ||e^{-\gamma \Delta x^2} C_h[p[m], \hat{p}]||_2^2, \tag{17}$$

Note that the amount focusing is measured in a slightly different way as in the depth-focusing approach. Instead of *minimizing* the amount of energy away from the centre, we are *maximizing* the amount of energy around the center. We refer to (van Leeuwen and Mulder, 2007a,b) for more details on this particular choice of weighting function.

Results

To test the method outlined above we seek to invert synthetic, multiple-free data for a 1D velocity model, depicted in figure 1 (a) with the dotted line. The background velocity model was represented on an equidistant grid of 6 spline nodes. To maximize the misfit function we used the BFGS algorithm. The gradient was calculated using the adjoint-state method. The direct



(a) (b) (c)

Figure 3: *Shown here are the data (a) and data-correlation (b) corresponding to the final velocity model, depicted in (c, top, solid). The near-offset traces of the data are shown in (c, bottom, solid). Although the final velocity model is not identical to the true velocity model, the data-correlation panel is now focused. This indicates that the moveout –and hence the velocity background model– of the true and final data are identical.*

wave was removed from the data by blanking and some time weighting was applied.
Shown are the results after 4 iterations. Figure 3(c) depicts the true and reconstructed velocity model and the corresponding near offset traces. There is not a perfect match of the velocity models, but this is not to be expected; the blocky model cannot be properly represented on splines. However, as seen from figure 3(a,b), the moveout of the events are identical and the data-correlation panel is focused.

Discussion

We have illustrated how the data-correlation method can be incorporated in a full-waveform inversion setting. First results look promising and encourage us to develop the technique further. Using the full wave equation to model the data would enable us to incorporate multiples in the velocity analysis. The idea is that a simulated multiple will correlate with a multiple in the measured data and focus in the data correlation. However, we expect some difficulties in obtaining a suitable reflectivity. A single LS iteration, as is used here, may not suffice in the presence of multiples.

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