S043

Simultaneous Imaging of Velocity and Attenuation Perturbations from Seismic Data Is Nearly Impossible

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SUMMARY

We generated synthetic reflection data for the constant-density visco-acoustic wave equation in the Born approximation. Perturbations in velocity and attenuation that have rapid variations on small length scales create reflections. In the frequency domain, a single complex-valued model parameter that depends on subsurface position describes both. Its real part is related to the classic reflectivity, its imaginary part also involves attenuation variations. When we applied a weighted Hilbert transform in the depth coordinate to our scattering model, we obtained almost the same synthetic data for scatterers with small dip. This means that, in practice, it will be impossible to simultaneously determine the real and imaginary part of the scattering parameters by linearized inversion without imposing additional constraints.

Introduction

Migration maps seismic data recorded at the earth's surface into an image of the subsurface. Although qualitative structural images are often acceptable, quantitative characterization of a scatterer's reflection and attenuation properties may enable a distinction between fluid-bearing and dry geological formations. For migration, the implicit assumption is that the data only contain primaries and that surface or interbed multiples are absent or negligible. We can generate such data with the linearized wave equation or Born approximation. In this approximation, the subsurface model is split into a part that does not produces reflections and a perturbation that generates the reflection data. We consider the constant-density visco-acoustic wave equation with perturbations both in velocity and attenuation. If we apply a weighted Hilbert transform in the depth coordinate to the perturbations of the model, we obtain almost the same reflection data as for the original perturbations. We explain the details, discuss the implication for attenuation scattering imaging, and indicate under which conditions the ambiguity occurs.

Scattering

The constant-density acoustic wave equation in the frequency domain reads

$$-\frac{\omega^2}{v^2}p - \Delta p = s, \tag{1}$$

with $p(\mathbf{x},\omega)$ the pressure, $\omega = 2\pi f$ the angular frequency, f the frequency, $v(\mathbf{x})$ the velocity, and $s(\mathbf{x},\omega)$ a source term. The complex velocity obeys

$$\frac{1}{v^2} = \frac{1}{c^2}\left[1 - \frac{2}{\pi Q}\log(f/f_r) + \frac{\imath}{Q}\right],$$

where $c(\mathbf{x})$ is the real-valued velocity and $Q(\mathbf{x})$ denotes the quality factor. Typically, $Q^{-1} \ll 1$. The logarithmic term is required for causality and is measured relative to some reference frequency f_r, which we set to 1 Hz in the numerical experiment. Our Fourier convention is $\hat{p}(\mathbf{x},t) = (2\pi)^{-1}\int_{-\infty}^{\infty} p(\mathbf{x},\omega)e^{-\imath\omega t}d\omega$. The linearized version of equation (1), known as the Born approximation, is the pair of equations

$$-\omega^2\nu_0 p_0 - \Delta p_0 = s, \quad -\omega^2\nu_0 p_1 - \Delta p_1 = \omega^2\nu_1 p_0. \tag{2}$$

Here $\nu = 1/v^2 = \nu_0 + \nu_1$, with $\nu_0(\mathbf{x})$ a smooth background velocity model that should not produce significant scattering in the seismic frequency band. If we solve equation (2) without a free-surface boundary condition, we will mainly obtain primary reflections generated by the complex-valued reflectivity $\nu_1(\mathbf{x})$ and no free-surface or interbed multiples. Note that ν_1 represents the difference between the original model and its smoothed version and therefore is a spatially rapidly varying or oscillatory function of subsurface position.

We have analysed the scattering response for the simple case of horizontally layered perturbations in an otherwise homogeneous background model (Mulder and Hak, 2009). It turned out that the recorded pressure field at the surface hardly changes if the reflectivity $\nu_1(\mathbf{x})$ is replaced by $\tilde{\nu}_1(z) = -\imath|z|\mathcal{H}_z[\nu_1(z)/z]$. Here, $\mathcal{H}_z[a]$ denotes the Hilbert transform in depth, z, of a function $a(z)$, which amounts to its convolution with $(\pi z)^{-1}$. For a surface seismic experiment, $\nu_1(z) = 0$ for negative z. The transformed model $\tilde{\nu}_1(z)$ may have non-zero values for negative z. Because of symmetry, we can take $\tilde{\nu}_1(z)$ for $z < 0$, mirror it with respect to $z = 0$, and add it to the transformed model at positive z. This can be summarized by defining the depth-weighted, scaled Hilbert transform of a function $g(z)$ as

$$\mathcal{M}[g](z) = \tilde{g}(z) + \tilde{g}(-z), \quad \tilde{g}(z) = -\imath|z|^p\mathcal{H}_z[g/z^p],$$

where $z \geq 0$ and we assume that $g(z) = 0$ for $z \leq 0$. For 3D wave propagation, $p = 1$, whereas in 2D, $p = 1/2$.

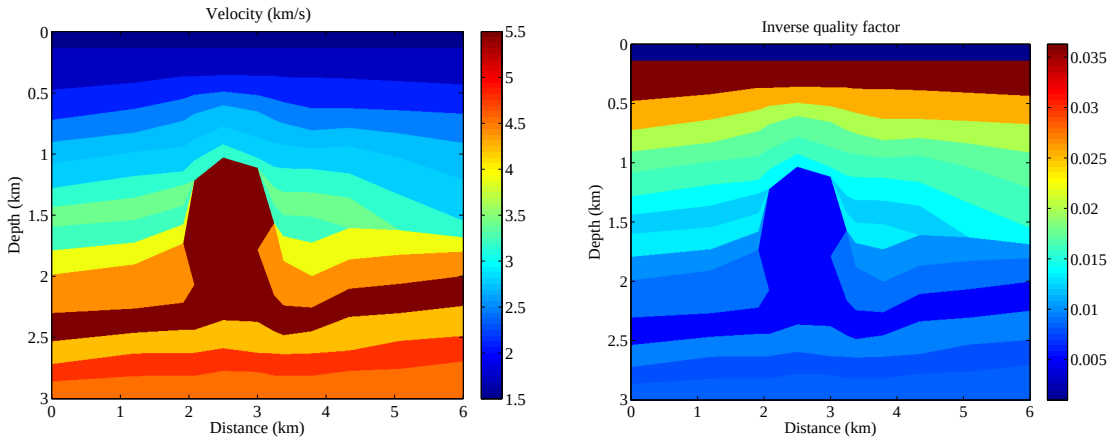

Figure 1: Simple salt-diapir model with velocity c (left) and inverse quality factor $1/Q$ (right).

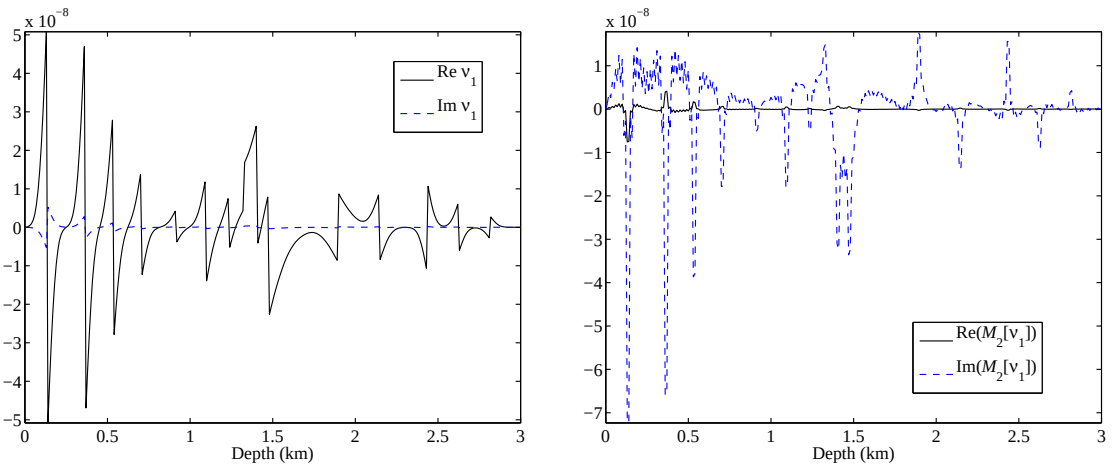

Figure 2: Cross sections at 2 km distance from the origin. The left panel shows the real (drawn) and imaginary (dashed) part of the scatterers, the right shows the scatterers after the depth-weighted Hilbert transform.

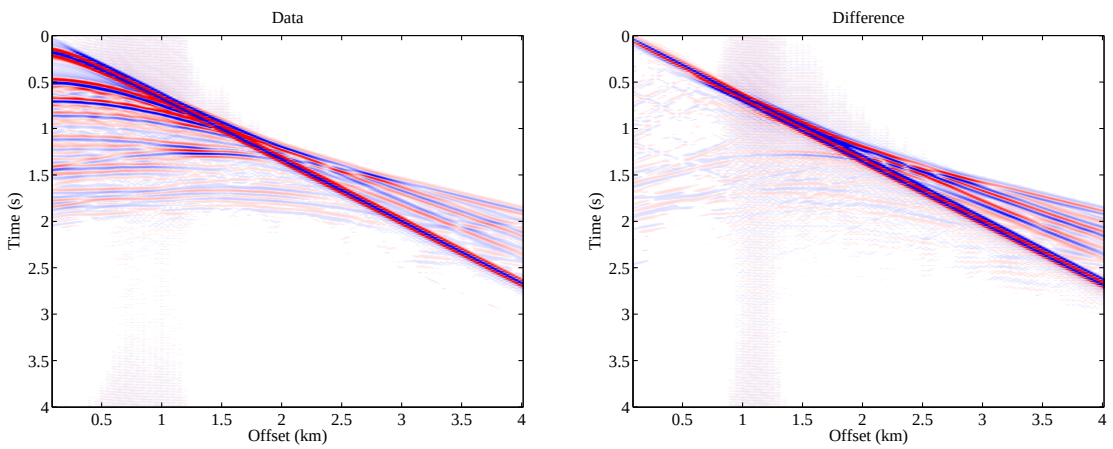

Figure 3: The data (left) and the difference (right) between data generated in the transformed and in the original model, both with the same scale. The shot was located $x_s = 1000$ m and a depth z_s of 5 m. The receivers had the same depth. We applied a clip at 5% of the maximum absolute value of the data in the left panel.

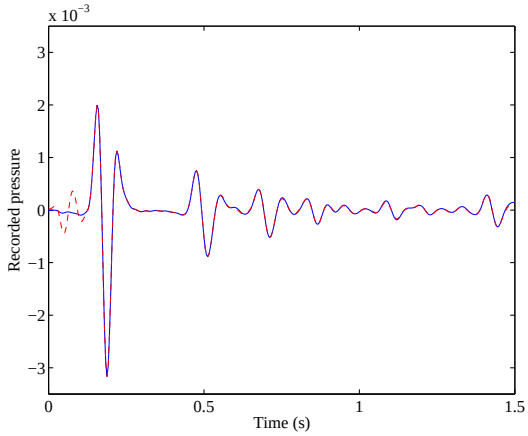

Figure 4: Single trace at 100 m offset for the shot panel shown in Figure 3. The drawn line represents the data for original model, the dashed line for the transformed one. The data are almost identical except for the sea-bottom reflection.

2D numerical experiment

We generated synthetic data with a 2D frequency-domain finite-difference code (Østmo *et al.*, 2002; Mulder and Plessix, 2004). Figure 1 shows the velocity and inverse quality factor for the original model. We heavily smoothed this model and obtained the perturbation ν_1 as the difference between the original and smoothed model. The left panel of Figure 2 displays a vertical cross section of the real and imaginary part of the perturbation. After application of the weighted Hilbert transform $\mathcal{M}[\nu_1]$ in depth at each distance x of the model grid, we obtained the right panel of Figure 2, which does not necessarily have a physical meaning.

We computed synthetic data in both models for a shot at $x_s = 1000$ m and a depth z_s of 5 m. The receiver line had positions x_r between 1100 and 5000 m at an interval of 25 m and z_r at 5 m depth. Absorbing conditions were applied at all boundaries. The frequency-domain data were transformed to the time domain with a suitable wavelet. Figure 3 displays the scattered wavefield for a single shot gather in the original model. The right panel shows the difference between the data in the transformed and original model, using the same plot scale and a strong clip to bring out the weak events. The difference between the data is very small, with the exception of the sea-bottom and shallow reflectors at large offsets. Also, there are differences for reflections off the dipping interfaces, in particular the top of the salt diapir. Figure 4 compares the results for a single trace. Apart from the sea-bottom reflection, the data agree remarkably well.

Among all linear combinations $\alpha\mu + (1-\alpha)\{-\imath\mathcal{H}[\mu]\}$, with $\mu(z)$ a function of depth, the one with $\alpha = 1/2$ has the smallest norm. Note that $\mu_0 = 1/2\,(\mu - \imath\mathcal{H}[\mu])$ obeys $\mu_0 = -\imath\mathcal{H}[\mu_0]$. When we carried out a true-amplitude two-way wave-equation migration (Plessix and Mulder, 2004) of the data in the frequency domain, we found that the real and imaginary parts of the complex-valued 2D migration result $m(x,z)$ were approximately related by $m/z^p \simeq -\imath\mathcal{H}_z[m/z^p]$, for $z > 0$ and with $p = 1/2$. We therefore conclude that this type of migration produces a reconstruction that approximately has the minimum-norm property mentioned above. As a consequence, simultaneous linearized inversion of seismic data for velocity and attenuation perturbations should be nearly impossible.

Discussion and conclusion

We found that two different models provided almost identical data. This implies that, in practice, it will be nearly impossible to uniquely determine simultaneously both the velocity and attenuation perturbations from the data, unless additional constraints are imposed. The transformed model, for instance, when added to the background model, corresponds to unphysical values of the perturbed attenuation, both in size and sign. Suitable constraints on admissible solution will reduce, but possibly not remove, the ambiguity. Requiring the result to be without attenuation will remove the ambiguity, as will the constraint that the scattering is caused by attenuation perturbations only.

Our result seems to contradict conclusions on visco-acoustic inversion by several authors,

including ourselves, so some remarks are in place. Chaderjian (1994) proofs uniqueness of the linearized inverse problem. This author as well as Ribodetti *et al.* (1995) and Hak and Mulder (2008) consider delta-function type perturbations or point scatterers. Its Hilbert transform is proportional to $1/z$ and will produce near-surface scattering even if the scatterer itself is buried at large depth. Clearly, our observed ambiguity will not occur in that case and linearized inversion should be feasible. For scatterers that resemble the first or higher derivative in z of a delta-function or gaussian with small width, the corresponding Hilbert transform will decay more rapidly with distance and be small compared to the transform of other scatterers. This is the case in the example presented here. The sea bottom is an exception, as are shallow refractions at large offsets and reflections off dipping interfaces.

Ribodetti *et al.* (1995) demonstrated that density, velocity, and attenuation perturbations can be recovered by inversion, but they only consider perturbations in one of these parameters at the time. With the constraint that the inverted parameter is either real or imaginary, the ambiguity is removed.

Ribodetti and Virieux (1998) presented linearized inversion results for scattering by blocky models. The Hilbert transform of a square wave has logarithmic singularities at positions corresponding to the ramps of the square wave. Such singularities should be visible as large peaks. Indeed, the reconstructed attenuation profiles in (Ribodetti and Virieux, 1998) sometimes show peaks at the jumps, rather than the under- and overshoots next to these jumps that are typical for least-squares inversion of blocky models in the real-valued case. Also Innanen and Weglein (2007) consider blocky models and can invert for the scattering model parameters.

We should emphasize that our observations do not relate to attenuation estimates from diving waves or direct arrivals in crosswell or VSP transmission data. These provide a characterization of the background model.

Acknowledgments

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