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Waveform Tomography by Correlation Optimisation

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SUMMARY

An important ingredient of any tomography-based velocity inversion method is the determination of traveltime differences. When a ray-based method is used, the modelled traveltimes are directly available and the traveltimes in the observed data need only to be picked once. When using wave-equation-based methods, however, the traveltime difference between two complex waveforms needs to be determined at each iteration. A straight-forward approach automatically picks the onset of relevant arrivals, either directly or via a correlation of the two waveforms. If the waveforms are not very similar, however, this approach is problematic. We propose to measure the traveltime difference via a weighted norm of the correlation of the two waveforms. The weighted norm can be used directly as an optimisation criterion for waveform tomography. We illustrate this with a synthetic and real data example.

Introduction

Wave-equation-based tomography tries to find a velocity model that best explains the data while taking into account the finite-frequency effects of wave propagation. For a given velocity model, the data are modelled by solving the wave equation using some numerical method. The residual between the observed and modelled data is then used as input for an adjoint calculation, which projects the residual back onto the velocity-model space and serves as an update of the velocity. This procedure is repeated in an iterative manner until some stopping criteria is met. Formally, the back-projected residual is the gradient of the misfit function corresponding to the data residual. A key step in such a procedure is the construction of an appropriate misfit function. An intuitively pleasing choice is the least-squares misfit function, resulting in the classical waveform inversion scheme pioneered by Tarantola and Valette (1982). However, it has been noted that the least-squares misfit functional exhibits local minima. As a result of these local minima, the iterative procedure may halt at a sub-optimal solution. From a classical tomography point of view one would like to use the traveltime difference between the observed and modelled waveforms as a misfit criterion. However, unlike ray-based methods, the traveltime difference is not explicitly calculated in wave-equation-based methods. Therefore, several procedures have been proposed to measure the traveltime difference from the waveforms (Cara and Leveque, 1987; Luo and Schuster, 1989; Gee and Jordan, 1992). These procedures rely on the fact that the correlation of the waveforms is maximal when their relative phase shift is zero. This is only valid, however, if the waveforms are identical. Therefore, the two waveforms are usually deconvolved first to normalise the amplitude spectra. We propose another method, also based on the correlation, to extract the phase-shift information. This approach borrows ideas from velocity analysis techniques that are used in reflection seismology. The idea is that the correlation of the two waveforms should be optimally 'focused' when the waveforms overlap. A misfit function can then be constructed that attains a minimum (or maximum) when the phase shift is zero. A simple example suggest that this approach is robust with respect to differences in the amplitude spectra of the waveforms. An inversion of synthetic and real cross-well data illustrates the application of the method.

Waveform correlation

Consider the measurement of the wavefield for a single source-receiver pair. In the frequency domain we may represent this measurement as

$$\hat{d}(\omega) = a(\omega)\exp\left[\imath\omega T\right]. \tag{1}$$

The correlation of the observed data , d_{obs}, and the predicted data, d, is then given by

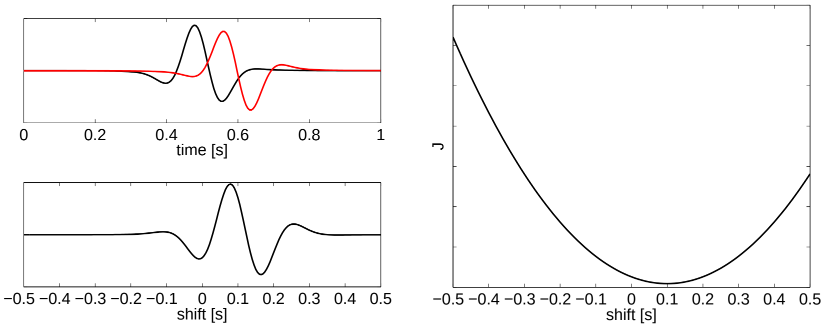
$$C[d, d_{\mathrm{obs}}](\Delta t) = \int \mathrm{d}t\, d_{\mathrm{obs}}(t+\Delta t)d(t) = \int \mathrm{d}\omega\, a^*_{\mathrm{obs}}(\omega)a(\omega)\exp\left[\imath\omega(\Delta t + T - T_{\mathrm{obs}})\right], \tag{2}$$

where we only consider shifts $|\Delta t| \leq T_{\max}$. When the amplitude spectra of the modelled and observed data are the same, the correlation will clearly reveal the phase shift by having a maximum at the corresponding time shift. Based on this observation, Luo and Schuster (1989) propose to use the shift at which the correlation attains a maximum to measure the misfit. When the amplitude spectra are *not* identical, however, the correlation will in general not have a maximum at zero shift. This is illustrated in Figure 1 (a) with a correlation of two waveforms that are phase shifted by 0.1 s and phase rotated over 45° degrees. As an alternative misfit function we propose a weighted norm to measure the misfit:

$$J[d, d_{\mathrm{obs}}] = ||W \cdot C[d, d_{\mathrm{obs}}]||_2^2, \quad W(\Delta t) = \Delta t. \tag{3}$$

The weighted norm measures how well the energy in the correlation is focused around zero shift and should attain a minimum when there is no phase shift between the waveforms. This may be used as an optimisation criterion for waveform tomography but is can also be adapted to determine the phase shift explicitly by considering a correlation of the observed data with a shifted version of the modelled data. The corresponding misfit functional as a function of the shift, τ, is then given by

$$J[d, d_{\mathrm{obs}}](\tau) = ||W \cdot C[d(\cdot + \tau), d_{\mathrm{obs}}]||_2^2, \tag{4}$$



(a) (b)

Figure 1 *(a) Correlation of two waveforms with a relative shift of 0.1 s whose amplitude spectra are phase rotated over 45° w.r.t. each other. The waveforms are displayed in the top panel, their correlation is displayed in the bottom panel. The weighted norm of the correlation as a function of the shift, τ, (cf. equation 4) is displayed in (b). The weighted norm attains a minimum at $\tau = 0.1$ s, revealing the phase shift between the waveforms.*

which should attain a minimum at $\tau = T_{\text{obs}} - T$. The phase shift is now easily determined by scanning over τ. This is illustrated in Figure 1 (b). Note that picking the maximum of the correlation here would have yielded a time shift slightly smaller than the true time shift.

Synthetic Example

We apply the above outlined method to cross-well waveform tomography, using a time-domain finite-difference code (Mulder and Plessix, 2002) to model the data. The correlation is easily calculated in the frequency domain by multiplication. Application of the weight is done by convolution with the Fourier transform of the weighting function. We use a different weighting function:

$$W_\gamma(t) = \exp[-(t/\gamma)^2]. \tag{5}$$

The principle is the same as described above, but now we have to *maximise* the misfit functional instead of minimising it. There are two reasons for choosing this weighting function: 1) it allows for easier implementation in the frequency domain and 2) the weight automatically blanks any crosstalk in the correlation at larger shifts, depending on γ. We normalise the functional by dividing by the energy of the unweighted correlation. We modelled the synthetic data for a constant-density acoustic medium for 101 shots/receivers at 5 m spacing and 200 m inter-well spacing and frequencies from 2 to 200 Hz with a 2 Hz interval. We used a Ricker wavelet with a peak-frequency of 100 Hz. For the inversion we represented the velocity model with linear splines on a grid with a 20 m spacing. The optimisation was carried out using a limited-memory BFGS (Broyden-Fletcher-Shanno-Goldfarb) method, starting from a constant model of 3 km/s. The gradient of the misfit functional can be calculated in a straightforward manner via the adjoint-state method (Plessix, 2006). The results are depicted in Figure 2. The velocity model was not retrieved exactly, but this cannot be expected since we are dealing with an ill-posed inverse problem. The data match nicely, however, and the correlation of the observed and measured data is focused around zero shift and this indicates that there is no phase shift between the observed and modelled data.

Real-data example

The cross-well data set, which was also use for a virtual source study (Mehta et al., 2008), contains 147 shot positions in the well at x = 205.74 m between 7.19 m and 567.9 m depth at a 3.84 m interval and 150 receiver positions at x = 26.82 m between 6.63 m and 578.86 m depth at the same depth interval. The horizontal source and 3C geophones were clamped to the casing. We only used the x-components of the source and the receivers and assumed that the data were not too different from acoustic pressure data from an explosive source. The data where damped to emphasise the first arrival by multiplication

with $e^{-\alpha(t-t_0)}$, $\alpha = 8\,\mathrm{s}^{-1}$, $t_0 = 0.05\,\mathrm{s}$. This translates into a complex angular frequency ω with α as its imaginary part. We inverted frequencies from 70 to 160 Hz with an interval of 2 Hz. Attenuation is taken into account: $\tilde{c} = c/\sqrt{1 + (\mathrm{i} - a)/Q}$, $a = (2/\pi)\log(f/f_r)$, using a reference frequency of 100 Hz. The free surface was also modelled. As a wavelet we used a band-limited delta function.
We use the same misfit functional as for the synthetic example. The inversion was done in several stages. First, an optimal linear velocity $c = c_0 + \alpha z$ was sought for $c_0 = 1551$ m/s at a fixed $Q = 80$. Then, we inverted for a layered model, represented by linear splines at a 15.240 m interval, again at a fixed $Q = 80$. Finally, we allowed Q to vary and use finer spline grid to represent the velocity. The final result is shown in figure 3. We compare the final model with a model that was obtained by standard ray-based tomography constrained by the well-log. The match is quite good. Care must be taken in interpreting the correlation panel here because the wavelets of the modelled and observed data do not match. The correlation does not necessarily peak at zero shift in this case.

Conclusion and discussion

We proposed a method to measure the phase shift between complex waveforms based on the correlation of the waveforms. We showed that a weighted norm of the correlation actually measures the phase shift and can possibly overcome the local minima that plague the least-squares approach. Such alternative misfit functionals for transmission tomography have also been studied by Luo and Schuster (1989) and Plessix et al. (2000) for crosswell tomography and by de Hoop and van der Hilst (2005) for global Earth tomography. The method proposed here has the advantage over previous correlation-based approaches that it is less sensitive to differences in the amplitude spectra of the modelled and observed data. In particular this makes the method less sensitive to errors in the wavelet. This is illustrated by the real-data example where no attempt was made to estimate a wavelet. From this optimisation criterion we may also derive a method to extract the phase shift directly from the correlation.
However, care must be taken to properly regularise the inversion procedure. Unlike the least-squares approach, there is no direct constraint on the amplitudes. This may drive the optimisation towards unphysical velocities to maximise or minimise the amplitudes of the modelled data. Normalisation of the misfit functional and additional smoothing penalties helped to mitigate some of these problems.

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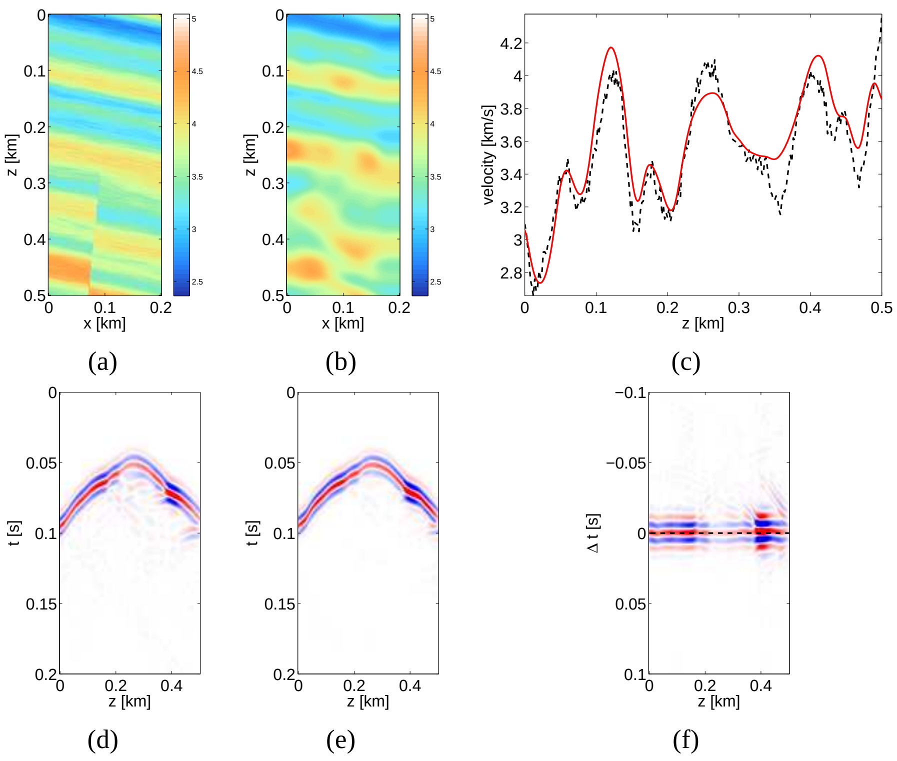

Figure 2 *Synthetic example. The true and final velocity models are depicted in (a,b) respectively. (c) depicts a slice through the velocity models at* $x = 100\,\text{m}$*, the final model in red. The shotgathers at* $z = 150\,\text{m}$ *of the corresponding data are depicted in (d,e). (f) shows the correlation of these shotgathers.*

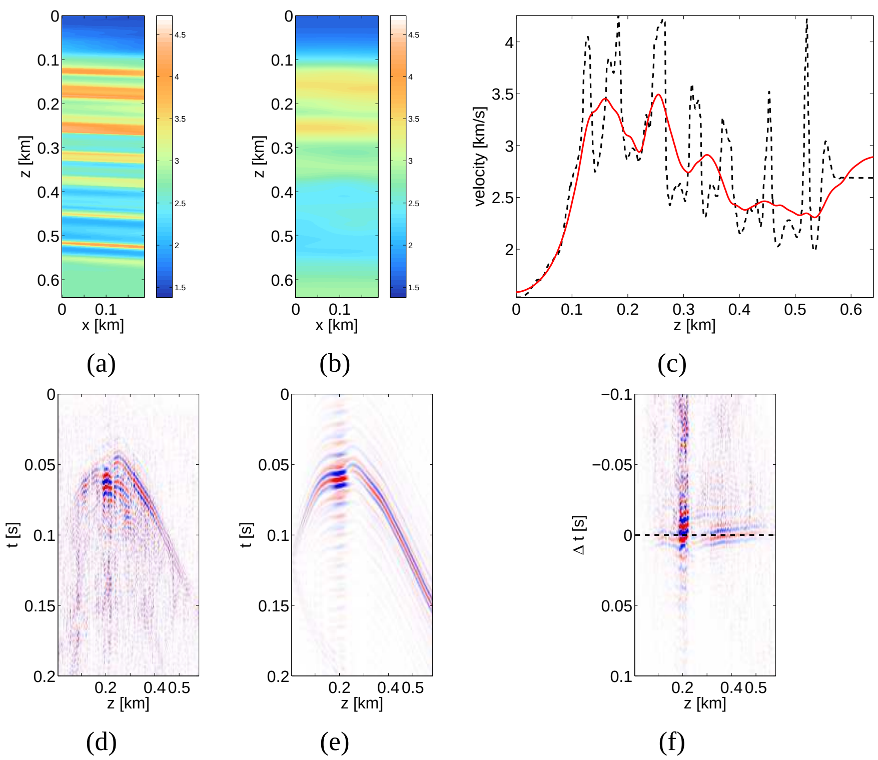

Figure 3 *Real-data example. The ray-based and correlation-based velocity models are depicted in (a) and (b), respectively. (c) depicts a slice through the velocity models at* $x = 92.5066\,\text{m}$*, the correlation-based model in red. The shotgathers at* $z = 233.78\,\text{m}$ *of the corresponding data are depicted in (d) and (e). (f) shows the correlation of these shotgathers.*