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## Surface Wave Inversion for a P-wave Velocity Profile via Estimation of the Squared Slowness Gradient

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### SUMMARY

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Surface waves can be used to obtain a near-surface shear wave profile. The inverse problem is usually solved for the locally 1-D problem of a set of homogeneous horizontal elastic layers. The output is a set of shear velocity values

for each layer in the profile. P-wave velocity profile can be estimated if higher modes and P-guided waves are used in the inversion scheme.

Here, we use an exact acoustic solution to invert for the P-velocity profile in an elastic model with a decreasing constant vertical gradient of the squared P-wave slowness, bounded by a free surface on the top and a homogeneous halfspace at the bottom. The exact acoustic solution can be expressed in Airy functions and leads to a dispersion equation.

We can invert several modes of the dispersion equation for the single gradient parameter of the squared P-wave slowness from elastic data.

As a first test case, we invert for the P-wave velocity profile of synthetic 2-D isotropic elastic data with a small  $V_s/V_p$ -ratio, using the first two dispersive P-wave modes.

The method does not require any picking and should be able to provide an initial model for full waveform inversion when applied to real data.

## Introduction

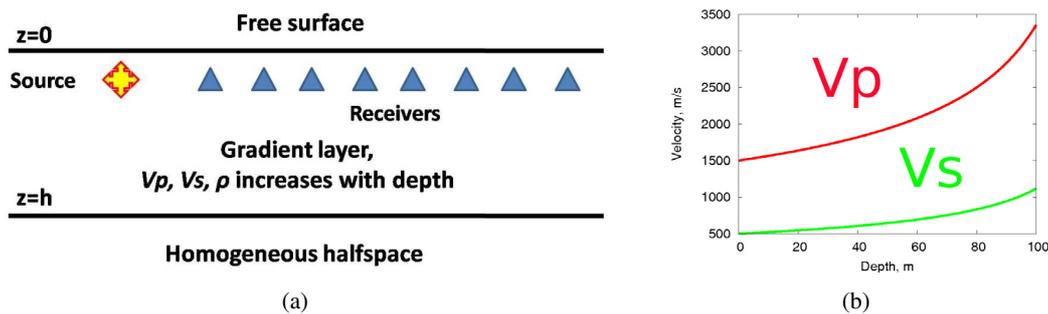
Surface waves can be processed and inverted for a near-surface shear velocity profile. The inverse problem is usually solved for a 1-D model, where the medium is approximated as a sequence of the homogeneous horizontal elastic layers (Socco et al., 2010). The roots of the dispersion equation, the Haskell-Thomson determinant, represent the various modes of the surface waves. Usually, the output of the inversion is a set of shear velocity values, one for each layer in the profile (Socco et al., 2010; Maraschini et al., 2008). A P-wave velocity profile can also be estimated if higher modes as well as P-guided waves are used in the inversion scheme (Ernst, 2008; Boiero et al., 2009).

If one wants to invert for a smoothly varying near-surface velocity model, a large number of homogeneous layers with small velocity jumps between them could in principle be considered. In practice, however, a large number of layers increases the calculation time of the 1-D inversion and may cause convergence problems due to an increased number of local minima. Therefore, we adopted an alternative approach. We consider a smooth velocity model that allows for an exact analytical solution. We obtain a dispersion relation similar to the multi-layered case and then invert it by minimization of the dispersion equation (Maraschini et al., 2010). The goal is to come up with a starting model for full waveform inversion.

In this paper, we concentrate on the P-waves first. If the  $V_s/V_p$ -ratio in the near surface is small, the P-waves can be distinguished from the Rayleigh waves. To model the P-waves, we use a vertical velocity profile that has a squared slowness linear with depth together with a free-surface boundary condition and approximate the elastic equations by an acoustic one. In that case, the exact solution can be expressed in terms of Airy functions (Brekhovskikh, 1980) and describes a dispersive P-wave. The method is illustrated by inverting 2-D synthetic elastic data for a P-wave velocity profile of the same form.

## Theory

We obtain the dispersion relation for the surface waves in the acoustic case and then try to apply it to an elastic model. Consider a 2-D acoustic model with a layer that has a linear vertical decrease of the squared slowness on top of a homogeneous halfspace. A free-surface boundary condition is imposed. The velocity in the layer is  $v_1(z) = v_0[1 - az]^{-1/2}$ . The halfspace below begins at a depth  $h$  and has a constant velocity  $v_2 = v_1(h)$ , so there is no velocity jump. Also, there is no density jump. The elastic version of the model is sketched in Figure 1. In such an acoustic velocity model that depends only on



**Figure 1** (a) Geometry of the model and (b) velocity profiles.

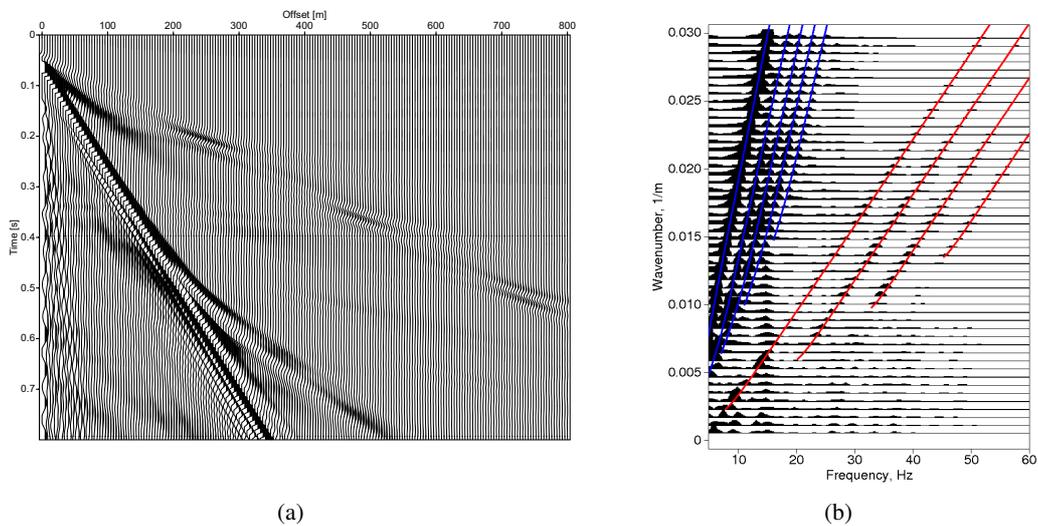
depth, the Fourier transform in time and in wavenumber of the pressure can be expressed in the Airy functions  $U(\tau)$  and  $V(\tau)$  with real argument  $\tau$ . Here,  $\tau = \tau_0 + z/H$ ,  $\tau_0 = H^2(k^2 - k_0^2)$ ,  $H = (ak_0^2)^{-1/3}$ ,  $k_0 = \omega/v_0$  and  $\omega = 2\pi f$  is the angular frequency. Using the free-surface boundary condition of zero pressure and the continuity of pressure and velocity at the boundary between the layer and the halfspace, we obtain the dispersion relation:

$$D = U(\tau)|_{z=0} \left\{ V'(\tau)|_{z=h} + H\alpha_2 V(\tau)|_{z=h} \right\} - V(\tau)|_{z=0} \left\{ U'(\tau)|_{z=h} + H\alpha_2 U(\tau)|_{z=h} \right\} = 0, \quad (1)$$

where the primes denote the derivatives w.r.t.  $\tau$  and  $\alpha_2 = \sqrt{k^2 - (\omega/v_2)^2}$ . The roots of this equation are the functions  $k(f)$  and  $v = \omega/k$  is the dispersive velocity of the guided wave. As in the case of Rayleigh waves, these dispersion curves can be inverted to estimate some of the model parameters.

### Synthetic example

The experiment was made using the 2-D isotropic elastic modelling code REM2D from Hamburg University. The model is sketched in Figure 1(a). An explosive (pressure) source is placed below the free surface and the receivers are buried at the same depth of 4 m. The P- and S-wave velocities at the surface are  $V_{p,0} = 1500$  m/s and  $V_{s,0} = 500$  m/s, hence  $V_{s,0}/V_{p,0} = 0.3$ . The density at the surface is  $2200$  kg/m<sup>3</sup>. The gradient parameter in the layer is  $a = 0.008$  1/m and is taken the same for  $V_p$ ,  $V_s$  and  $\rho$ . The homogeneous halfspace starts at a depth  $h = 100$  m. Figure 1(b) shows the vertical velocity profiles. The source has a central frequency of 20 Hz. The vertical displacement seismograms in the  $x, t$ - and  $f, k$ -domains are shown in Figures 2(a) and 2(b), respectively. Figure 3 displays the recorded pressure for a vertical force source.



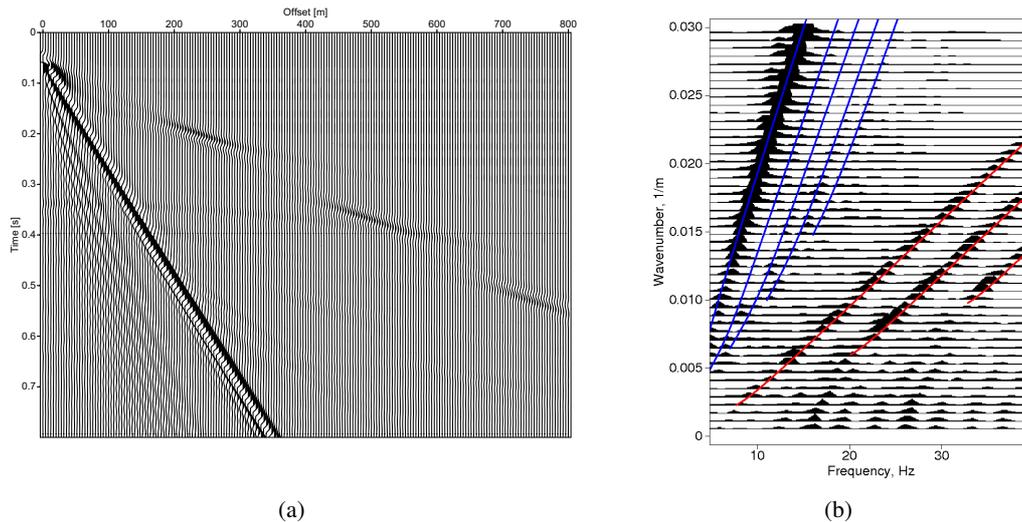
**Figure 2** Synthetic seismogram in the  $x, t$ - (a) or  $f, k$ -domain (b) for an explosive source and vertical-displacement data.

Because of the small  $V_s/V_p$  ratio, the P-waves can be distinguished from the Rayleigh waves. In the  $f, k$ -domain, in Figures 2(b) and 3(b), there are two groups of the spectral maxima. One consists of the Rayleigh dispersion curves, which are marked by blue lines. The other set of dispersion curves can be described via the roots of the dispersion equation (1) and are drawn in red. We will focus on the red curves, as they contain information about the P-wave velocity in the model. The curves can be picked from Figure 2(b) for frequencies above about 20 Hz. The low frequencies are boosted if the explosive source is replaced by a vertical force and the pressure is recorded, as shown in Figure 3. The curves can then be picked starting from a lowest frequency of 7.6 Hz up to 20 Hz and beyond to still higher frequencies. In this paper, we do not pick the dispersion curves but use the real roots of equation (1) instead. The frequency value of 7.6 Hz is the lowest frequency for which the roots of dispersion equation lie on the real axis of the complex plane ( $k, \omega$ ).

### Inverting the dispersion curves

The estimation of the gradient parameter,  $a$ , is the easiest way to obtain the P-wave velocity profile. We applied Newton's method to find the value of  $a$  that minimizes a misfit functional of the form

$$F(a) = \sqrt{\sum_i \sum_j D_{ij}^2(f_{ij}, V_{ij}, a)}. \quad (2)$$



**Figure 3** As Figure 2, but for a vertical force source an pressure data.

Here,  $D_{ij}$  is the value of the dispersion equation for the points on the “observed” dispersion curve, computed for a frequency  $f_{ij}$  and dispersion velocity  $V_{ij}$ ;  $i$  is the index of a point on the one dispersion curve and  $j$  is the index of the dispersion curve itself. The true gradient parameter has  $D_{ij} = 0$ .

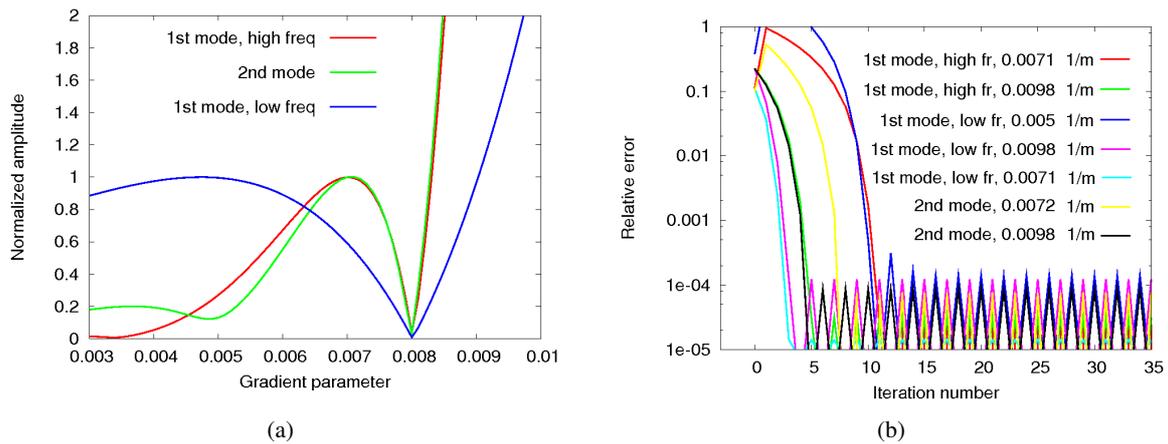
Three normalized misfit functionals are shown in Figure 4(a). One corresponds to the first mode from Figure 2(b). The frequency ranges from 20 to 50 Hz. Another is for the second mode from the same figure and frequency range. The last is for the first mode from Figure 3(b) and the frequency ranges from 7.6 to 20 Hz. Each functional has a local maximum, situated to the left of the true minimum. The first two have local maxima near a value of  $0.007 \text{ m}^{-1}$  for the gradient parameter, the last near  $0.005 \text{ m}^{-1}$ . The initial value for the inversion should therefore be on the correct side of the local maxima for each misfit functional.

In Figure 4(b), the relative error in the estimated gradient parameter  $a$  as a function of the iteration count is displayed for various initial values. The legend of Figure 4(b) lists the corresponding mode number and the initial gradient parameter used for the inversion. Even with a starting values that is quite far from the true  $a = 0.008 \text{ m}^{-1}$ , only 10 iterations are required to obtain an error less that 0.1%, more than sufficient for an accurate result. The convergence stagnates when the numerical errors of the minimization algorithm start to dominate.

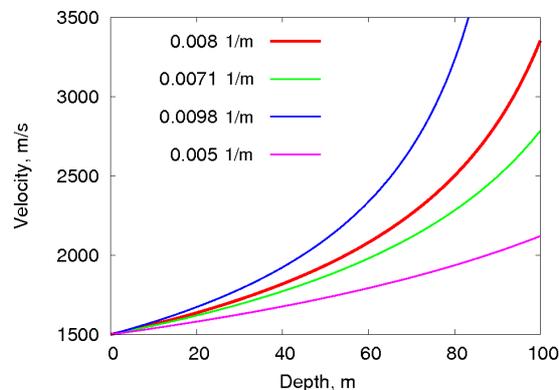
Figure 5 displays the  $V_p$  velocity profiles for different values of the gradient parameter. The red curve marks the true  $V_p$  profile. The other curves correspond to different initial values of the gradient parameter used in the inversion. Even if we start the inversion procedure from a value of the parameter close to the true one, the velocity profile for the initial parameter (green curve) differs a lot from the true velocity profile (red curve). In other cases, the differences between the initial and true velocity profiles are still larger. This implies that the method should be able to find the proper gradient over a wide range of velocity variations.

## Conclusions

We have analysed the possibility to obtain a P-wave velocity profile in an isotropic elastic medium with a gradient in the squared slowness by 1-D inversion of the P-wave dispersion curves, which does not require any picking. We used the first and second mode in the inversion. The misfit functional has a local maximum, which restricts the range in which the initial parameter value for the inversion should be chosen. If we use low frequencies and invert the first mode, the range for the initial guess becomes larger. The large variation in the velocity values for the initial and true values of the gradient parameter gives



**Figure 4** (a) Normalized misfit functional and (b) convergence history for various choices of the initial guess for  $a$  in  $1/m$  and the first or second mode, without or with low frequencies for the first.



**Figure 5**  $V_p$  velocity profiles for different values of the gradient parameter.

we hope that the method will provide a useful initial model for full waveform inversion when applied to real data.

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