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Electromagnetic and Seismoelectric Sensitivity Analysis Using Resolution Functions

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SUMMARY

For multi-parameter problems, such as the seismoelectric system, sensitivity analysis through resolution functions is a low-cost, fast method of determining whether measured fields are sensitive to certain subsurface parameters. We define a seismoelectric resolution function for the inversion of a bulk density perturbation. The synthetic data and Green's functions required to construct the resolution function are computed using the seismoelectric modelling code ESSEMOD. First, we consider the purely electromagnetic problem with a conductivity perturbation at a single point in an isotropic homogenous half-space. The result is nearly identical to a published result based on analytical Green's functions. It correctly maps the position of the scatterer. Next, we perform an electromagnetic sensitivity analysis for the case of a layered background medium. Again, the resolution function is capable of correctly mapping the scatterer when it is above as well as below a layer of increased conductivity; although in the latter case with less resolution. Finally, we generate multi-component synthetic data with our forward modelling code and compute the seismoelectric resolution function for inversion of a bulk-density perturbation. We find that the seismoelectric system is sensitive to a perturbation in bulk density and that the position of the perturbation can be correctly recovered.

Introduction

The seismoelectric (SE) method might be a promising geophysical tool because it takes advantage of subsurface coupling between the seismic poro-elastic wavefield and the electromagnetic diffusive field (Pride, 1994). Several studies have already shown that the SE-method can provide supplemental information about porosity and permeability, or on pore-fluid properties such as viscosity. This can be used to detect or monitor gas-water or oil-water contacts (Smeulders et al., 2014) or for the detection, characterization, and monitoring of aquifers (Garambois and Dietrich, 2002). Furthermore, SE might provide information on the near-borehole flow properties by the use of well-logging techniques (Zhu and Toksöz, 2005). Since the seismoelectric effect is determined by a combination of many subsurface parameters which are often mutually related, inversion of seismoelectric data for each of these parameters individually is costly. Therefore, performing a sensitivity analysis prior to inversion is crucial. It can, for example, focus on acquisition design or investigate time-lapse perturbations and more importantly, show whether the measured fields are actually sensitive to the parameter of interest. Our aim is two-fold; first we perform an EM parameter sensitivity analysis for a point perturbation in conductivity and compare this with literature results, after which we investigate the EM sensitivity to point scatterers above and below highly conductive layers. Second, we expand the theory to include a parameter sensitivity analysis for the fully coupled seismoelectric system. We illustrate the theory of resolution functions for the specific SE case by deriving the formal resolution function for inversion for a bulk density contrast and compute it as the least-squares solution to the *normal equation*. We use it to demonstrate the use of the seismoelectric forward modelling code, called ESSEMOD, and conclude with a seismoelectric sensitivity analysis for a bulk density perturbation from single-frequency multicomponent line data.

Theory - Seismoelectric Resolution function for bulk density contrast

We define a large domain \mathbb{D} where all quantities are known and a smaller domain $\mathbb{D}^s \subset \mathbb{D}$ where one or more of the medium parameters and the associated fields are unknown. The domain \mathbb{D}^s is referred to as the scattering domain and its associated field as the scattered field. The larger domain \mathbb{D} is known as the background or incident domain and is associated with the incident field, which is the field that would be present in the entire domain if the scattering domain has no contrast with the embedding domain. Using the Born approximation, the total field may be defined in the space-frequency domain as the following integral equation (Maas, 2014)

$$\hat{G}_{js}^{ve}(\mathbf{x}^r, \mathbf{x}^s, \omega) = \hat{G}_{js}^{ve;i}(\mathbf{x}^r, \mathbf{x}^s, \omega) + \int_{\mathbb{D}^s} [\chi^\rho(\mathbf{x}) \hat{D}_{js}(\mathbf{x}^r, \mathbf{x}^s, \mathbf{x}, \omega)] d^3\mathbf{x}. \quad (1)$$

Equation (1) holds for a fixed pair of source- and receiver-locations \mathbf{x}^s and \mathbf{x}^r , respectively. It shows that the total field is the sum of the incident field and scattered field, which consists out of the contribution of all scatterers within the scattering domain \mathbb{D}^s . Here \hat{G}_{js}^{ve} denotes the total Green's field for the j -component of the particle velocity field generated by the s -component of an electric current source. The additional superscript i indicates it describes an incident field. All Green's fields together form a three-by-three Green's matrix. Furthermore, the two-way wavefield operator is given by

$$\hat{D}_{js}(\mathbf{x}^r, \mathbf{x}^s, \mathbf{x}, \omega) = j\omega \hat{G}_{jm}^{vf;i}(\mathbf{x}^r, \mathbf{x}, \omega) \hat{G}_{ms}^{ve;i}(\mathbf{x}, \mathbf{x}^s, \omega), \quad (2)$$

where $\hat{G}_{jm}^{vf;i}$ is the incident Green's particle velocity field generated by a bulk-force source. $\chi^\rho(\mathbf{x})$ denotes the spatially dependent contrast function. Resolving it is the problem to be solved and the resulting expression is the resolution function for a bulk density contrast. This requires *perfect* knowledge about the embedding domain and the associated incident fields such that we can use them in equation (1) and we are left with the scattered field only. We therefore assume enough prior knowledge is available for the sensitivity analysis of the scattering problem, that we may assume that we are already in, or at least sufficiently close to, the global minimum of the objective function of our inverse problem.

In order to come to an expression for the resolution function, it is assumed that the inverse two-way

wavefield operator $\hat{\mathcal{D}}_{kr}(\mathbf{x}^r, \mathbf{x}^s, \mathbf{x}, \omega)$ (note the different use of \mathcal{D}) satisfies Fourier orthogonality and is defined as follows

$$\int_{\omega=-\infty}^{\infty} \int_{\mathbf{x}^r \in \mathbb{R}^3} \int_{\mathbf{x}^s \in \mathbb{R}^3} \hat{\mathcal{D}}_{kr}(\mathbf{x}^r, \mathbf{x}^s, \mathbf{x}^I, \omega) \hat{\mathcal{D}}_{rp}(\mathbf{x}^r, \mathbf{x}^s, \mathbf{x}^p, \omega) d^3 \mathbf{x}^s d^3 \mathbf{x}^r d\omega = \delta_{kp} \delta(\mathbf{x}^p - \mathbf{x}^I). \quad (3)$$

If a possible image location \mathbf{x}^I coincides with a true scattering location \mathbf{x}^p then the resolution function will show an image in that location only, when considering a perfect solution with infinite bandwidth. However, in case of limited bandwidth, smearing around that scattering location will occur and, instead, the resolution function will highlight a volume in which the scatterer is likely to be located. Applying the inverse operator to both the left- and right-hand side of the expression for the scattered field (equation (1) without the incident field), integrating the right-hand side over all frequencies, all source and receiver locations and subsequently integrating it over the scattering domain \mathbb{D}^s , we obtain a *resolution function for inversion* for a bulk density point perturbation;

$$\chi^{\rho:Inv}(\mathbf{x}^I) \delta_{kp} = \int_{\omega=-\infty}^{\infty} \int_{\mathbf{x}^r \in \mathbb{R}^3} \int_{\mathbf{x}^s \in \mathbb{R}^3} \hat{\mathcal{D}}_{kr}(\mathbf{x}^r, \mathbf{x}^s, \mathbf{x}^I, \omega) \hat{G}_{rp}^{ve;s}(\mathbf{x}^r, \mathbf{x}^s, \mathbf{x}^p, \omega) d^3 \mathbf{x}^s d^3 \mathbf{x}^r d\omega. \quad (4)$$

Following Slob and Mulder (2011), we compute the formal resolution function by expressing the scattering problem as the *normal equation* for a linear inverse problem for subsurface scatterers. Its regularized least-squares solution then reads

$$\chi_{rp}^{\rho:Inv} = \left(\hat{\mathbf{D}}_{sj}^* \hat{\mathbf{D}}_{jk} + \varepsilon \mathbf{I} \right)^{-1} \hat{\mathbf{D}}_{sj}^* \hat{\mathbf{S}}_{jk}. \quad (5)$$

The two-way field operator $\hat{\mathbf{D}}_{jk}$ (consisting of submatrices) contains all source-side and receiver-side multicomponent field data for all possible image locations \mathbf{x}^I , where each submatrix is computed according to equation (2). $\hat{\mathbf{D}}_{sj}^*$ is defined as the complex conjugate transpose of $\hat{\mathbf{D}}_{jk}$, where the superscript asterisk denotes complex conjugation and the subscripts take care of transposition. The scattered data matrix $\hat{\mathbf{S}}_{jk}$ contains the *true* scattered data, computed for a known bulk density point perturbation at the *true* scattering location \mathbf{x}^p . Furthermore, \mathbf{I} is the identity matrix and ε the regularization parameter. The resulting resolution function matrix $\chi_{rp}^{\rho:Inv}$ then consists out of nine subcomponents, where each component represents one seismoelectric sensitivity for the entire investigated area.

Numerical Results

Here we present the results of three scenarios: a purely EM scenario with a single scatterer in an isotropic homogeneous background medium, another purely EM scenario with a single scatterer in a layered background medium and finally a SE scenario of a single bulk density contrast scatterer in an isotropic homogeneous background medium. The line acquisition geometry for these experiments consists of 201 sources located at the surface with a horizontal source spacing of 25 m and 41 receivers, located 50 m below the surface with a horizontal receiver spacing of 250 m. For the EM scenarios, these are x -directed electric current sources and electrodes, whereas for the SE scenario these are three-component electric current sources, as well as bulk-force sources, and particle velocity receivers. The investigated area is centrally located in the x, z -plane below the acquisition line and has a total offset of 3000 m and a depth coverage of up to 1600 m; see Figure 2-a. To generate synthetic data on which the resolution functions can be tested, we use the analytically-based numerical forward modelling code ESSEMOD (Grobbe et al., 2014). We do this by modelling an isotropic homogeneous half-space and compute the multi-component response to a unit-impulse source for all source-side and receiver-side fields. In order to test our approach, we first investigate the scalar resolution function for a conductivity point perturbation for a purely EM scenario as derived by Slob and Mulder (2011).

Figure 1-a shows the EM resolution function for inversion where both the two-way field operator as well as the data were computed through the use of explicit analytical homogeneous-space Green's function solutions (Slob and Mulder, 2011). Figure 1-b shows the one where both the two-way field operator

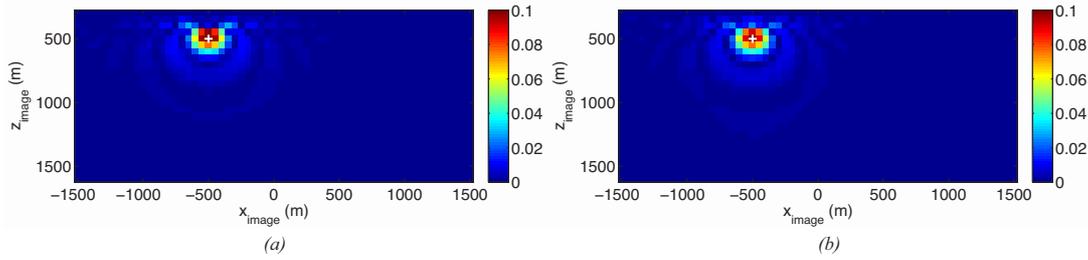


Figure 1 EM resolution functions for inversion, (a) from Green's function solutions and (b) from ESSEMOD data for a single frequency of 0.5 Hz. The white 'plus'- sign indicates the true location of the conductivity point perturbation $(-500, 500) m$.

as well as the data have been computed using the forward modelling code ESSEMOD, where we assume that there is no coupling between mechanical waves and electromagnetic fields. It may be observed that in both cases the resolution function is capable of correctly mapping the conductivity perturbation and that the horizontal location and depth are equally well resolved. We may also observe that both methods deliver nearly identical results. This provides a solid basis for extending the sensitivity analysis to include the fully coupled seismoelectric system. However, it also allows us to explore more complex cases for which deriving explicit analytical solutions is more challenging.

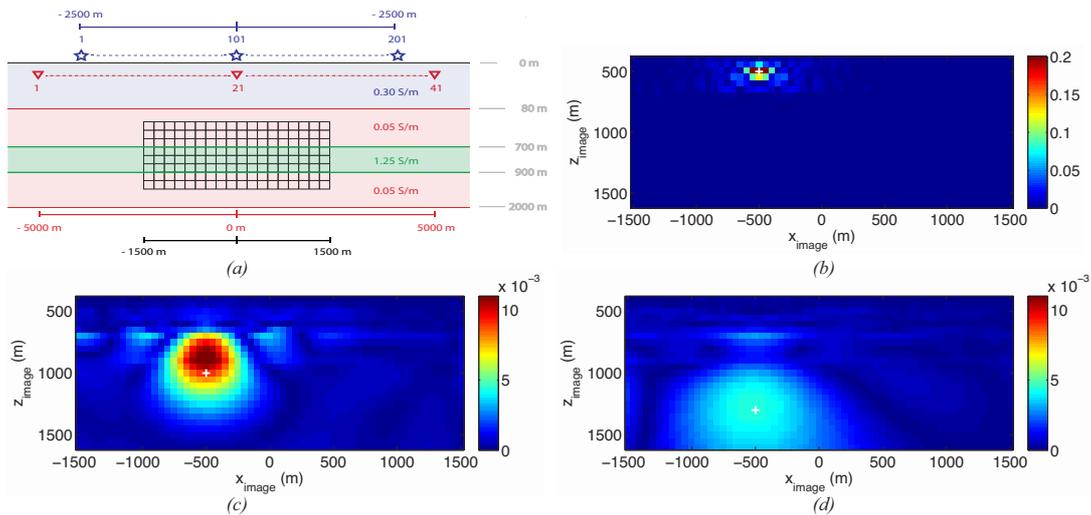


Figure 2 EM resolution functions for a conductivity point perturbation at (a) $(-500, 500) m$, (c) $(-500, 1000) m$ and (d) $(-500, 1300) m$ in a layered background medium for a single frequency of 0.5 Hz.

Figure 2-a shows the case of a layered background medium. Figure 2-b shows the EM sensitivity analysis for a conductivity perturbation above a layer of increased conductivity. It may be observed that the resolution function is still capable of correctly mapping the scatterer. However, figures 2-c and 2-d show the sensitivity analysis for the scatterer below that layer at two different depths. It can be observed that although the EM resolution function correctly maps the perturbation, the resolution has clearly deteriorated for the deeper perturbation. This may be because the layer above the scatterer is likely reflecting the incident field back towards the surface, therefore decreasing the amount of energy available to illuminate the scatterer itself.

Figures 3-a and 3-c show the diagonal element χ_{22}^p , whereas figures 3-b and 3-d show the off-diagonal element χ_{31}^p of the seismoelectric resolution function matrix for inversion for a density perturbation. This result was computed according to equation (5) where the regularization parameter ϵ was determined by finding the maximum of the approximate Hessian, and multiplying it with an arbitrary small number. It may be observed that the diagonal element χ_{22}^p clearly resolves the density perturbation at the correct

location. At the same time, it can be seen that the off-diagonal elements show patterns with amplitudes that are generally one order smaller than those on the diagonal of the resolution functions. Looking at equation (4), this is to be expected: in an ideal case, the diagonal terms show a clear point-response whereas the off-diagonal terms should be zero.

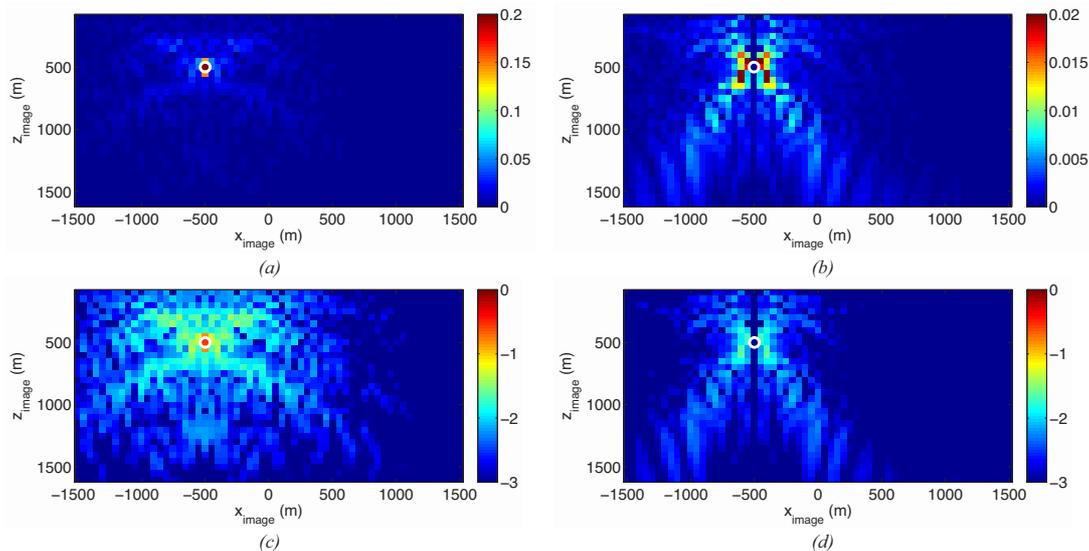


Figure 3 SE resolution function for a density perturbation at $(-500, 500)$ m as indicated by the white circle. (a) & (c) diagonal element χ_{22}^p , (b) & (d) off-diagonal element χ_{31}^p . (a) & (b) plotted according to true amplitude and (c) & (d) plotted on a logarithmic scale.

Conclusions

We performed two electromagnetic and one seismoelectric sensitivity analysis for three examples using a seismoelectric forward modelling code for horizontally layered media (ESSEMOD). We tested the approach on an example with single-component electromagnetic line-data for a point conductivity perturbation in a homogeneous isotropic half-space for which explicit analytical Green's function solutions are available. We then used the forward modelling code for EM sensitivity analysis in the case of a layered background medium. It turns out that single-component electromagnetic line-data have a resolution function that still can resolve a point scatterer but when the scatterer is placed below a layer with increased conductivity, the resolution has severely decreased. Finally, for the case of multi-component data for the coupled seismoelectric system in a homogeneous isotropic half-space, we have successfully computed a resolution function for inversion of a perturbation in bulk density in one point. We conclude that the coupled seismoelectric system is sensitive to this perturbation and that its position can be correctly recovered.

References

- Garambois, S. and Dietrich, M. [2002] Full waveform numerical simulations of seismoelectromagnetic wave conversions in fluid-saturated stratified porous media. *Journal of Geophysical Research*, **107**(B7), 2148.
- Grobbe, N., Hunziker, J. and Slob, E. [2014] Seismoelectric wave propagation modeling for typical laboratory configurations: A numerical validation. *SEG Technical Program Expanded Abstracts 2014*, 1993, Society of Exploration Geophysicists, 2072–2077.
- Maas, P.J. [2014] *Electromagnetic & Seismoelectric sensitivity analysis using resolution functions*. Master's thesis, Delft University of Technology.
- Pride, S. [1994] Governing equations for the coupled electromagnetics and acoustics of porous media. *Physical Review B*, **50**, 15678 – 15696.
- Slob, E. and Mulder, W. [2011] CSEM Sensitivity Analysis Through Resolution Functions for Migration and Inversion. *73rd EAGE Conference & Exhibition*, (May 2011), 23–26.
- Smeulders, D.M.J., Grobbe, N., Heller, H.K.J. and Schakel, M.D. [2014] Seismoelectric conversion for the detection of porous medium interfaces between wetting and nonwetting fluids. *Vadose Zone Journal*, **13**(5).
- Zhu, Z. and Toksöz, M.N. [2005] Seismoelectric and seismomagnetic measurements in fractured borehole models. *GEOPHYSICS*, **70**(4), F45–F51.