PERIODIC ORBITS AND WARPS

W.A. Mulder
Sterrewacht, Huygens Laboratorium, P.O. Box 9513,
2300 RA Leiden, The Netherlands

Orbit calculations were done in a rotating triaxial system with a
density distribution in accordance with recent observations of spiral
galaxies. A search was made for simple closed orbits which are tilted
with respect to the plane of the galaxy. A family of stable prograde
tilted orbits was found which can explain warps as stationary phenomena.

Tilted disks are common phenomena in many early type and elliptical
galaxies (Bertola et al., 1978; Hawarden et al., 1981). Spiral galaxies
often show warps. Attempts have been made to explain these configurations
in terms of stable closed orbits in rotating triaxial systems. Merritt
(Heisler, Merritt and Schwarzschild, 1982) found a family of retrograde
tilted orbits. Binney (1981) used Mathieu's equation to show that orbits
in the plane of a rotating triaxial potential can be unstable for per-
turbations perpendicular to that plane.

Here the triaxial density distribution is chosen to be \( \rho = \rho_0 \, m^p \)
with \( m^2 = x^2/a^2 + y^2/b^2 + z^2/c^2 \) and \( a > b > c \). For the bulge of our Galaxy
\( p = -1.8, c/a = 0.4 \) and \( M = (9 \pm 2) \times 10^9 \, M_\odot \) within 1 kpc (Sanders and
Lowinger, 1972; Isaacman, 1981). Burstein et al. (1982) found \( p = -1.7 \pm 0.1 \)
from the rotation curves of 21 Sc galaxies. The density distribution
pertsains this value even at large radii. Therefore, bulge and halo are
modelled with a single density distribution.

The axial ratio's were chosen to be \( b/a = 0.8 \) and \( c/a = 0.5 \) with
\( p = -1.8 \) and the rotation frequency of the triaxial system \( \omega = 0.1 \) around
the z-axis. Periodic orbits were calculated numerically with the method
described by Magenmat (1982). There are tilted orbits near every \( (\omega_O - \omega) : \omega_z \)
resonance, where \( \omega_O \) is the rotation frequency in the non-rotating frame
and \( \omega_z \) the oscillation frequency perpendicular to the plane. Most of
these orbits are self intersecting, have sharp turnings or are not sym-
metric with respect to the origin. The most important orbits for gas were
found at the \( \pm 1:1 \) resonance, ocuring for the retrograde orbits within
CR and for the prograde orbits outside the OLR. The first correspond to
those found by Merritt (Heisler et al., 1982). The latter are shown in

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fig. 1. They have an increasing tilt with increasing energy and start to
have loops at a specific value of the energy, implying that gas in this
type of orbit will have a maximum tilt.

Fig.1. Stable prograde
orbits at the -1:1 resonance.
The plotted orbits have
energies E of 2.75, 3.00,
3.25 and 3.50. The orbit
with the loops is at E=4.40.
The tilt increases as E in-
creases. CR is at r = 8.4.

The following conclusions can be drawn:
(1) Under the assumption that the motion of gas in the outer part of a
spiral galaxy can be described in terms of simple stable periodic orbits,
a warp can be explained as a stationary phenomenon in a rotating triaxial
system. As this assumption does not hold strongly, the presented calcu-
lations are only indicative.
(2) A model based upon these periodic orbits provides a length scale for
the galaxy, predicting the locations of the other resonances in the
assumed density distribution. It also defines the direction of the long
axis of the triaxial figure.
(3) It should be noted that highly non-circular motions can occur in a
rotating triaxial system. Interpretations of observations in terms of
circular motion might unnecessarily give rise to multi component models
for the mass distribution and notions as "expanding gas features".

References.
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