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Squared-slowness Gradient Profile from Surface-wave Inversion as a Starting Model for Full-waveform Inversion

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SUMMARY

Full-waveform inversion (FWI) is widely used for the detailed reconstruction of subsurface velocity models from seismic data collected in different environments. The method requires an adequate starting model. Otherwise, an incorrect model may be obtained, due to the presence of local minima in the least-squares objective function. This is in particular true in case of a complex near-surface velocity structure. Classic surface-wave inversion provides a layered representation of the medium, which can subsequently be smoothed to serve as a starting model for FWI.

As an alternative to the layered representation of the medium, we consider a continuous, smoothly varying P-wave velocity profile. The profile has its squared slowness increasing linearly with depth and allows for analytical evaluation of the dispersion curves. We investigate the application of the reconstructed profile as an initial guess for acoustic FWI. A synthetic example shows how we can reconstruct both the background structure as well as a high-velocity inclusion in this way.

Introduction

Full-waveform inversion (FWI) is a modern, not so robust method for the detailed reconstruction of subsurface velocity models. It is widely used among geoscientists on seismic data collected from different environments. For proper subsurface imaging, FWI needs an adequate starting model. Otherwise, the iterative reconstruction prodedure may lead to the wrong result. This is particularly true in the complex near surface (Ernst, 2013). Classic surface-wave inversion (SWI) can provide such a shallow starting model for FWI (Boiero et al., 2013). The horizontally layered representation of the medium leads to a dispersion equation for the surface waves. Then, inversion of the Rayleigh and Scholte dispersion curves produces a S-wave velocity profile, whereas inversion of the P-guided waves leads to a P-wave velocity profile (Boiero et al., 2013; Socco et al., 2010). After spatial smoothing, these locally 1-D profiles can be combined into a 2-D or 3-D starting model for acoustic or elastic FWI (Mulder and Plessix, 2008).

Earlier, we considered an alternative way to the layered representation of the medium. We proposed an analytical approach to invert for a continuous, smoothly varying P-wave velocity profile. The latter has its squared slowness increasing linearly with depth. Using the analytical solution in the acoustic case for an unbounded medium with a constant gradient of the squared slowness (Brekhovskikh, 1980), we derived the analytical dispersion equation for a 2-D acoustic model. This dispersion equation has a multimodal structure. We demonstrated the feasibility of reconstructing a P-wave velocity profile for elastic data with small V_s/V_p-ratio by inverting the dispersion curves (Ponomarenko et al., 2013b). Later, we applied the method to field data and obtained an estimate of the P-wave velocity profile that was comparable to a multi-layered inversion result (Ponomarenko et al., 2013a).

Here, we investigate the use of this squared-slowness gradient model as an initial guess for acoustic FWI with a number of synthetic examples. We reconstruct both the background structure as well as a high-velocity inclusion.

Estimation of the initial model

We considered a 2-D acoustic model, consisting of a layer with a constant squared-slowness gradient bounded by a free surface at zero depth and by a homogeneous halfspace from below, and derived the dispersion equation for the surface P-wave (Ponomarenko et al., 2013b). The velocity in the layer was $v_1(z) = v_0[1 - az]^{-1/2}$. The halfspace below started at a depth h and had a constant velocity $v_2 = v_1(h)$. There were no velocity and density jumps. We used the misfit functional (Maraschini et al., 2008)

$$F_j(v_0, a, h) = \sqrt{\sum_i D_{ij}^2(f_{ij}, V_{ij}, v_0, a, h)}, \qquad (1)$$

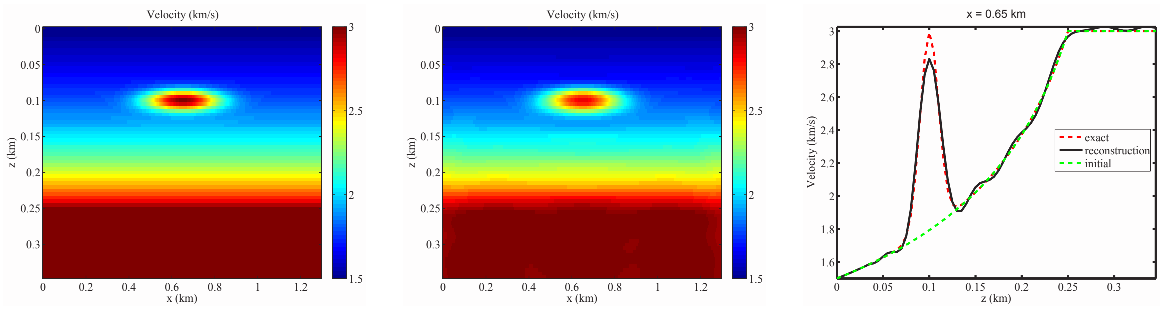
where D_{ij} is the value of the dispersion equation, computed for a frequency f_{ij} and dispersion velocity V_{ij}; i is the index of a point on the dispersion curve and j is the number of the picked dispersion curve or mode. In an earlier synthetic study (Ponomarenko et al., 2013b), we minimized the functional (1) for the gradient parameter a, assuming that the true values of V_0 and h were known. Here, we apply 2-D direct grid search over the a,h-space to find the global minimum of the misfit functional. The resulting velocity profile served as starting model for FWI with a 2-D acoustic frequency-domain code.

Background model with a constant squared slowness gradient

As a first example, we considered the model shown in Figure 1(a). The background velocity model consists of a layer with a constant squared-slowness gradient on top of a homogeneous halfspace, with the free surface at $z = 0$. We added a gaussian high-velocity inclusion. The sources and receivers were put just below the free surface. The central frequency of the source was 10 Hz. Figure 2(a) displays the synthetic seismogram for a source in the left corner of the model. Because the wave equation for this background velocity model has an analytical solution (Ponomarenko et al., 2013b), the velocity profile can be readily estimated from the theoretic dispersion curves – the red curves in Figure 2(b).

As a first test, we tried to reconstruct the inclusion starting from the exact background model, skipping its reconstruction from the theoretical dispersion curves. We used 60 sources with a 10-Hz Ricker wavelet and 105 frequencies from 4 to 25 Hz simultaneously for the inversion.

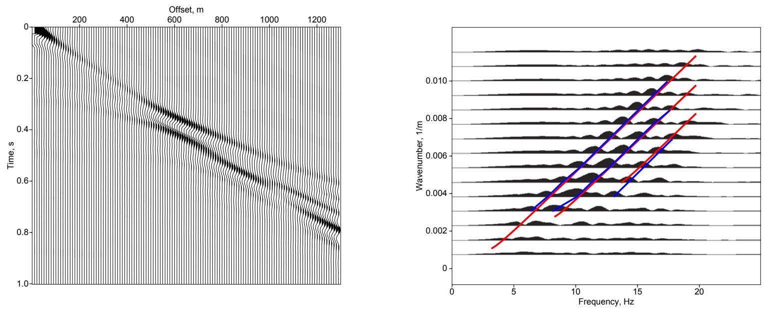
The reconstructed model after 30 iterations is shown in Figure 1(b) and vertical cross sections through the centre of the exact, initial and reconstructed models in Figure 1(c). The inversion result is a bit smoother than the true model because of a smoothness penalty term.



(a) (b) (c)

***Figure 1** Inclusion in the background model: (a) true model; (b) reconstructed model after 231 iterations; (c) vertical cross sections at $x = 650\,m$ of the true (dashed red), initial (dashed green) and reconstructed (solid black) model.*

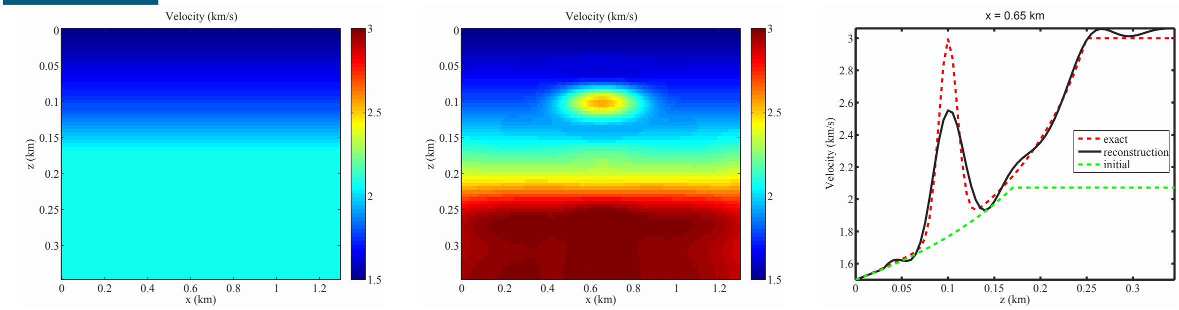
Secondly, we replaced the true background model by an estimate based on the inversion of the dispersion curves of the surface P-waves. We picked three dispersion events from the synthetic seismogram in the f,k-domain, shown as the blue curves at Figure 2(b), and inverted them with a direct grid search in the a,h-domain, fixing the true value of v_0. Because the picked curves do not perfectly fit the theo-



(a) (b)

***Figure 2** (a) Seismogram from exact model in the x,t-domain and (b) f,k-domain. The red curves mark and theoretical dispersion curves, the blue the picked ones.*

retical curves, the depth of the top of the halfspace is underestimated and, as a consequence, so is the halfspace velocity value. In the current example, the true values of the parameters are $a = 0.003\,1/\mathrm{m}$ and $h = 250\,\mathrm{m}$. The values, estimated from the three dispersion curves, are $a = 0.0028\,1/\mathrm{m}$ and $h = 170\,\mathrm{m}$. This underestimation does not strongly affects the quality of the FWI reconstruction, but only increases the required number of iterations. The initial model with its underestimated parameters is shown in Figure 3(a). Figure 3(b), displays the reconstructed model after 140 iteration and Figure 3(c) shows vertical cross sections through the centre of the model. The position and geometry of the inclusion as well as the background velocity were properly reconstructed, but the peak velocity value of the reconstructed inclusion is underestimated. This may be improved by adding higher frequencies and reducing the amount of smoothing in the penalty terms.

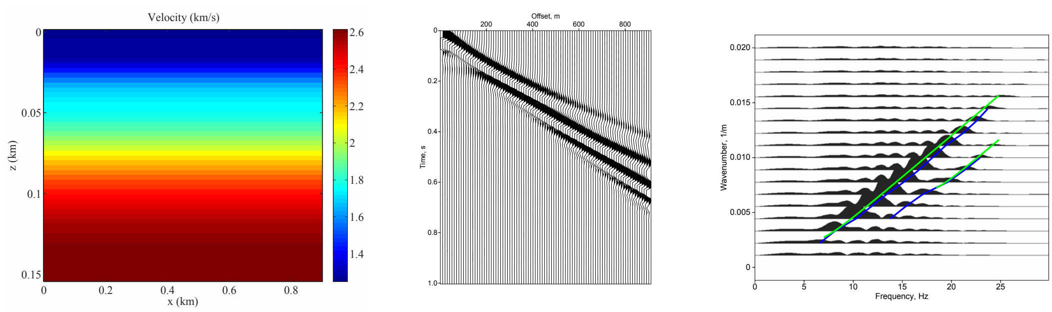


(a) (b) (c)

***Figure 3** (a) Initial model from dispersion curve inversion (b) reconstructed model after 200 iterations; (c) vertical cross section at $x = 650\,m$ of the true (dashed red), initial (dashed green) and reconstructed (solid black) model.*

Smoothed layered model

As a second example, we considered the smoothed layered model from Ponomarenko et al. (2013a). Figure 4(a) shows the model, Figure 4(b) a synthetic x,t seismogram for one source in the left corner of the model, and Figure 4(c) its transformed version in the f,k-domain. The picked dispersion curves, shown as blue curves in Figure 4(c), were inverted with a direct grid search for the minimum of the misfit functional (1) in the the h,a-domain for a fixed v_0. The resulting starting model for FWI is shown in Figure 5(a) and its corresponding dispersion curves are drawn in green in Figure 4(c). For FWI, we



(a) (b) (c)

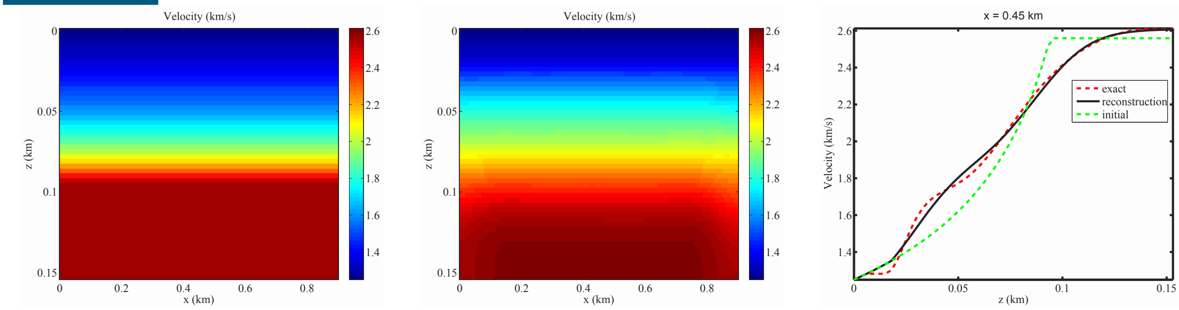
***Figure 4** Smooth 1-D model: (a) true model; (b) seismogram for the true model in the x,t-domain and (c) f,k-domain.*

used 40 sources with a 10-Hz Ricker wavelet and simultaneously inverted 80 frequencies from 4 to 20 Hz. Figure 5(b) shows the inversion result. The vertical cross section through the centre of the model in Figure 5(c) depicts the initial model (dashed green), obtained by inversion of the dispersion curves. The reconstruction (solid black) closely matched the true model (dashed red).

As a slightly more complicated example, we took the previously estimated 1-D velocity model as background and added a high-velocity gaussian inclusion. Figure 6(a) depicts the model. We inverted 60 shots with a 40-Hz Ricker wavelet using 240 frequencies from 6 to 60 Hz. Figures 6(b) and 6(c) show the result of the reconstruction, which is a little bit smoother version of the true model due to the smoothness constraints.

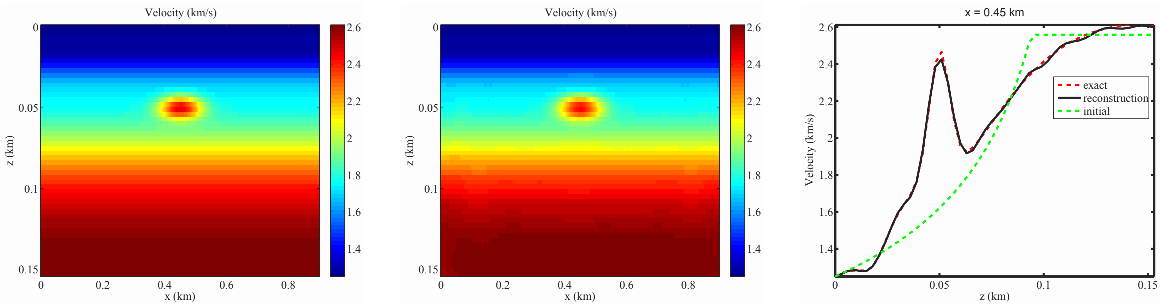
Conclusions

We have investigated the use of a constant squared-slowness gradient model obtained by dispersion curve analysis as a starting model for FWI. The main advantage of using the squared-slowness gradient approximation is the availablity of an analytical solution, which allows us to easily obtain the dispersion equation and invert the picked dispersion curves for the model parameters. We examined several syn-



(a) (b) (c)

Figure 5 *(a) Initial model obtained by dispersion curve inversion; (b) reconstructed model after 51 iterations; (c) vertical cross section at* $x = 450\,m$ *of the true (dashed red), initial (dashed green) and reconstructed (solid black) model.*



(a) (b) (c)

Figure 6 *Reconstruction of the inclusion: (a) true model; (b) after 180 iterations; (c) vertical cross section at* $x = 450\,m$ *of the true (dashed red), initial (dashed green) and reconstructed (solid black) model.*

thetic examples for which the smooth initial model estimated from the dispersion curves allowed for a reasonable FWI reconstruction of both a high-velocity inclusion as well the background velocity model.

Acknowledgements

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