Finite-difference iterative migration by linearized waveform inversion in the frequency domain

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Summary

We present an iterative migration technique for mapping seismic data to reflector amplitudes, obtained by formulating migration as an optimization problem. The method can be based on any kind of seismic modeling and provides true-amplitude images in a natural way. In this paper, we use a finite-difference solution of the linearized constant-density wave equation in the frequency domain. Because the constant-density acoustic equation cannot handle impedance contrasts, we use its linearized form, which is equivalent to the Born approximation. Examples are given for synthetic and real data and show that the iterative technique improves the amplitudes and resolution of reflectors.

Introduction

In the 1980s, it was recognized that migration could be formulated as an inverse problem (Lailly, 1983; Tarantola, 1984). If the difference between observed and modeled data is measured by a least-squares error functional, the gradient of the functional with respect to the model parameters resembles an un-weighted migration image. A gradient-type optimization algorithm can be used to find the minimum of the error, which ideally should converge to a true-amplitude migration image. In the context of optimization, true-amplitude migration weighting can be interpreted as an approximation to the diagonal of the inverse of the Hessian of the error functional. A suitable approximation will accelerate the convergence of the iterative method.

Iterative migration can be based on any kind of seismic modeling. Ray-based approaches have been presented by Lambaré *et al.* (1992), Nemeth *et al.* (1996), and Duquet *et al.* (2000) and show that iterative migration produces better-focused images. At the same time, the method can handle illumination problems and incomplete data sets.

Nowadays, there is a growing interest in wave-equation migration because this overcomes some of the difficulties of the ray-based migration, for instance in the presence of hard layers and salt bodies. Wave-equation migration algorithms are usually based on the imaging principle, among them reversed-time migration (Baysal *et al.*, 1983). On present day computer hardware, it is feasible to apply iterative migration to wave-equation migration to obtain better reflector amplitudes, certainly for two-dimensional problems. Here, we present results for iterative migration based on the finite-difference solution of the linearized wave equation in the frequency domain. Note that we use the full wave equation and not a one-way approximation. There are several advantages to working in the frequency domain, especially in two space dimensions (Marfurt *et al.*, 1989; Pratt, 1999; Plessix *et al.*, 2001). For each frequency, the linear algebra can be performed once and be reused for all shots and also for the migration. A subset of the available frequencies can be used under certain conditions (Plessix *et al.*, 2001). Coarser grids can be used for the lower frequencies without noticeable loss of numerical accuracy. Whereas the computation of the gradient in the time-domain requires the correlation of forward and backward propagated wave fields over all times, this operation reduces to a multiplication of two wave fields in the frequency domain. All these factors lead to a cost reduction of a least one order of magnitude compared to time-domain finite-difference migration.

Here we model wave propagation with a frequency-domain finite-difference method for the constant-density acoustic wave equation. Because this equation cannot properly handle impedance contrasts, we have taken the linearized version of this equation. This is the equivalent of the Born approximation. The earth is now represented by two independent parameters: a smooth background velocity and a rough reflectivity. The velocity is assumed to be given and is kept fixed. The reflectivities are taken as unknowns in the iterative migration.

The linearized approach has some advantages over the full acoustic, or even elastic wave equation. First of all, the constant-density equation is less costly to solve. Secondly, we only have to solve for one set of parameters. Thirdly, the problem is linear in the reflectivities, making it easier to solve the optimization problem. Fourthly, the full nonlinear optimization problem has many local minima, making full waveform inversion a difficult problem.

Here, we first describe the least-squares finite-difference migration based on the linearized equation in the frequency domain. Then a very simple synthetic example for a horizontally layered earth is presented to demonstrate the capabilities of the method. Finally, results for a marine line are shown.

Method

We start with the constant-density acoustic wave equation $L\,u = s$ with $L = -\sigma^2\omega^2 - \Delta$. Here $\sigma(\mathbf{x})$ is the slowness, $\omega = 2\pi f$ the angular frequency, f the frequency, $s(\omega,\mathbf{x})$ a source term, and Δ the Laplace operator. The slowness can be decomposed into a smooth component σ_0 and a rough component $\sigma_0 r$, where $r = (\sigma/\sigma_0) - 1$ will be called the reflectivity. The Born approximation leads to two equations:

$$L_0 u_0 = s \quad \text{and} \quad L_0 u_1 = 2\sigma_0^2 r \omega^2 u_0\,,$$

with $L_0 = -\sigma_0^2\omega^2 - \Delta$. The pressure field $u = u_0 + u_1$ is the sum of the incident and the scattered field.
The synthetic data are

$$c(\mathbf{x}_s, \mathbf{x}_r) = R(\mathbf{x}_r)(u_0(\mathbf{x}_s) + u_1(\mathbf{x}_s)) ,$$

with R a projection operator onto the receiver position $\mathbf{x}_r$ and $\mathbf{x}_s$ the source position.
The optimization problem is: find the best reflectivity, r, for a given background slowness σ_0 and the observed data, d, by minimization of the least-squares error functional

$$J(r) = \frac{1}{2}\sum_{\omega}\sum_{\mathbf{x}_s,\mathbf{x}_r}\left\| c(\mathbf{x}_s, \mathbf{x}_r)(r) - d(\mathbf{x}_s, \mathbf{x}_r)\right\|^2 .$$

The minimization of J with respect to r is performed by a preconditioned conjugate gradient method. Preconditioning is necessary for fast convergence. Here we use an approximation to the diagonal of the inverse of the Hessian, corresponding to the inverse of the square of the sum over the shots of the square of the incident field. This approach differs from the classic migration weights based on the imaging principle, where the images are preconditioned by shot.

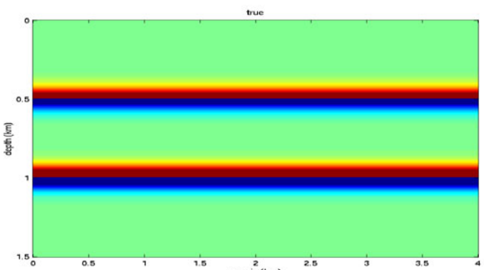

Figure 1: true reflectivity.

Synthetic example for a simple 1D model

As a first illustration of the method, we have chosen a simple 1D model with a velocity that increases linearly with depth from 2000 m/s at the surface to 3000 m/s at 1.5 km depth. There are two reflectors, shown in Figure 1. For the computations, we use 157 sources and 158 receivers at the surface equally distributed with 25 m spacing over the 4 km width of the model. We have committed the inverse crime of generating data with the same code as used for the inversion, computing 16 frequency panels between 15 and 30 Hz.
The results of the minimization after 1, 2, and 10 iterations are displayed in Figs 2–4. Vertical cross-sections through the central part are displayed in Fig. 5. The iterations clearly improve the relative amplitudes. Also, the resolution improves, that is, the image is better focused and the reflectors are sharper. Note that the biggest improvements occur during the first two iterations.

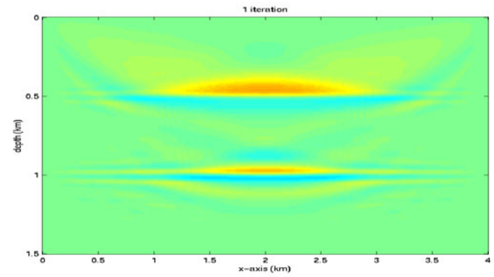

Figure 2: reflectivity after one iteration.

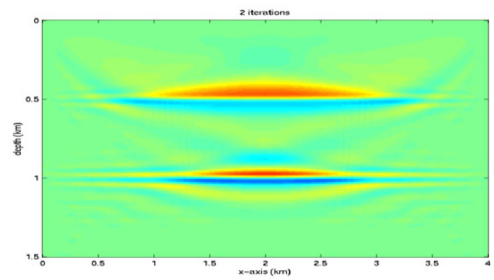

Figure 3: reflectivity after 2 iterations.

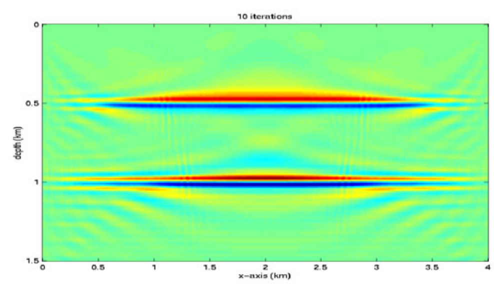

Figure 4: reflectivity after 10 iterations.

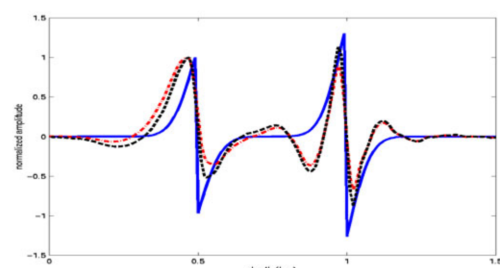

Figure 5: comparison between the true reflectivity (solid blue line), the result of the first iteration (dashed-dotted red line), and the result of the second iteration (dashed black).

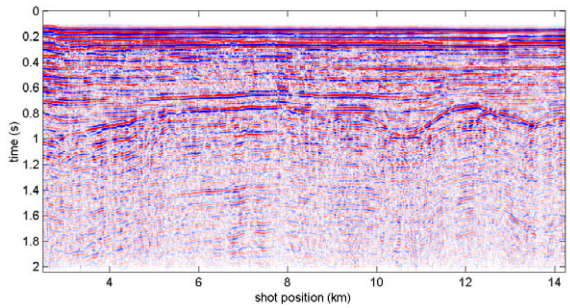

Figure 6: shortest-offset gather.

Real-data example

The second example is based on a marine line having 399 shots with 89 receivers for each shot. The distance between subsequent shots is 25 m, which is the same as the receiver spacing. The direct arrival, sea-bottom reverberations, and short-period surface multiples have been removed from the data. A shortest-offset gather is plotted in Fig. 7. Note the strong reflector at about 0.8 s, below which few events are visible and are mixed with long-period multiples.

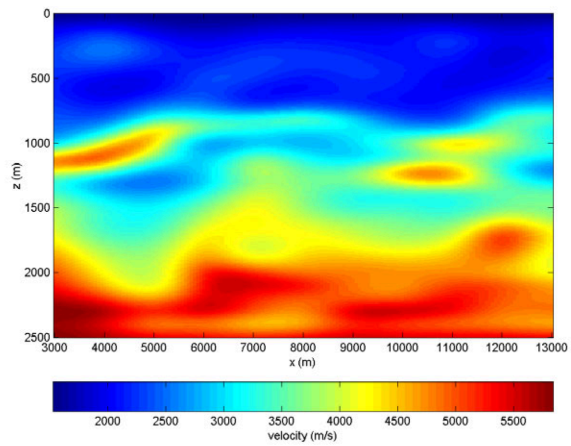

Figure 7: Velocity found by Differential Semblance Optimization.

Because the direct arrival has been removed from the data, it is excluded from the synthetics by simply dropping the incident field, u_0. A smooth velocity model was determined by Differential Semblance Optimization (Mulder and Ten Kroode, 2002) and is shown in Fig. 7. In order to reduce the noise level in the iterative results, a smoothing in the x-direction is applied. This allows us to obtain more continuous reflectors. The results after 1, 2, 5, and 10 iterations are plotted in Figure 8. At depths of less than 1 km, the iterations improve the resolution of the reflectors. In the reflectivity image at the fifth iteration, the lateral distribution of the amplitudes seems to be more realistic than after the first iteration.

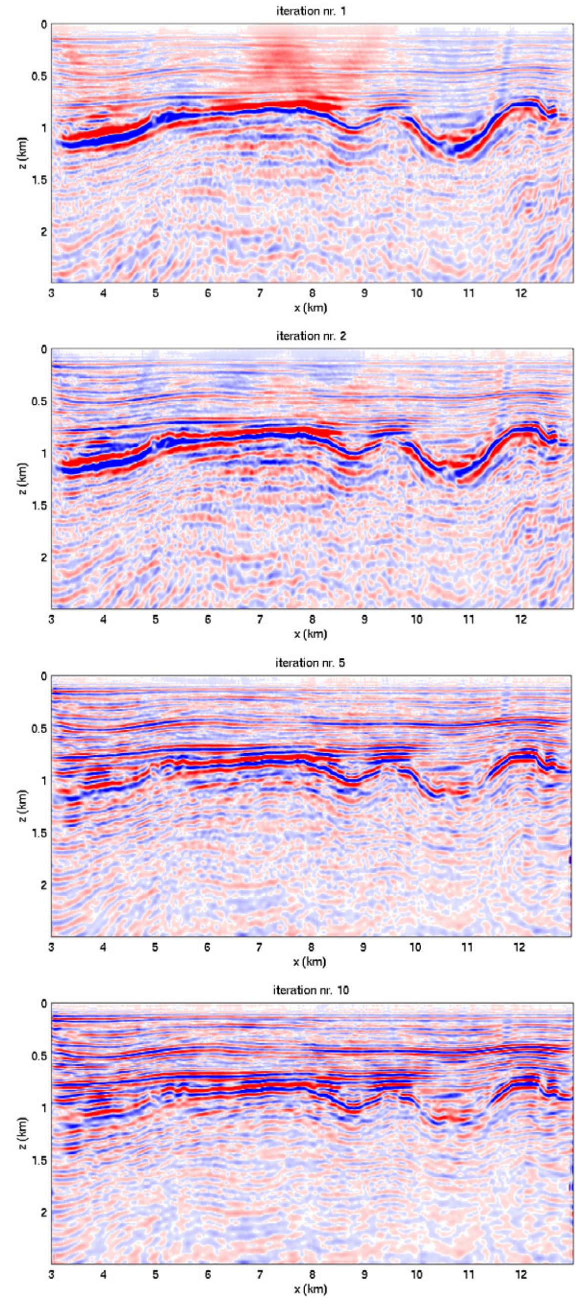

Figure 8: Reflectivity after 1, 2, 5, and 10 iterations, from top to bottom.

Beneath the hard layer, it is difficult to interpret the image even if some coherent events appear during the iterations. This may be due to the errors in the background velocity model that may not allow for constructive stacking, making it impossible for the iterative migration method to improve the image. The presence of multiples and a poor estimate of the source wavelet may also cause problems.

Discussion and conclusions

We have presented an iterative migration method based on frequency-domain finite-difference modeling. Because frequency-domain modeling is at least one order of magnitude faster than time-domain modeling, seismic lines can be process on a workstation in a few days. Iterative migration improves the focusing and resolution of migration images, a fact already noticed by several authors for ray-based migration.

It is difficult to obtain similar results with the classical migration approach, which can be considered as a single iteration preconditioned by suitable migration weights. These weights represent an approximation to the diagonal of the inverse of the Hessian. However, the use of band-limited data causes the Hessian to be non-diagonal. It may even not be diagonally dominant for certain cases. This means it is impossible to undo the effect of band-limited data by using a diagonal approximation.

Iterative migration provides a better reconstruction of the amplitudes in the numerical examples considered here. This can be important for cases in which it is difficult and costly to build appropriate migration weights, for instance in complex geometries.

The success of iterative migration depends on the quality of the background velocity model and the estimated source wavelet. Some of the difficulties encountered in the processing of the real-data example may be related to errors in the estimation of these parameters. An estimation of the source wavelet could be included in the minimization process. It does not make sense to include the estimation of the background velocity because the least-squares error functional has many local minima with respect to the background velocity. Therefore, a gradient-type optimization algorithm cannot find the absolute minimum, unless the starting model lies in the valley of the global minimum.

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