Is rigorous redatuming feasible?

W.A. Mulder, Shell International Exploration and Production B.V., The Netherlands

Summary

No, not in a strictly formal sense, but surprisingly good results can be obtained with a suitable regularization. Here, redatuming is formulated as an inverse problem for the full wave equation, including multiples, without making any assumption such as down-going waves or primaries only. For simplicity, only the constant-density acoustic case is considered. The inverse problem is ill posed but can be solved with the proposed regularization. Some synthetic-data examples are included that show that the effect of a complex overburden can completely be removed.

Introduction

Redatuming [1,2,3,6] is an operation on seismic data that accounts for translations of the positions of sources or receivers, or both. It can be used, for instance, to preprocess seismic data for imaging algorithms that require a regular acquisition geometry or to remove the effects of an irregular topography. Another application is the simplification of the processing and interpretation of data recorded in areas with a complex near-surface geology. An example is sub-basalt imaging [4]. Here we will attempt to move sources and receivers to a depth below a complex overburden in such a way that the effect of the overburden is completely removed. To that end, redatuming is formulated for the "two-way" wave equation. This means that events such as multiples and refraction events are included. To keep the exposition simple, the method is described for constant-density acoustics only. In the following section, the basic equations that describe rigorous redatuming are presented. They lead to an ill-posed inverse problem. Its numerical solution is outlined next and requires regularization. The method was applied to a one-dimensional and a two-dimensional problem based on synthetic data.

Basic equations

Consider the wave equation for constant-density acoustics

$$Lp(t,\mathbf{x}) = f(t,\mathbf{x}), \quad L = c^{-2}(\mathbf{x})\partial_{tt} - \Delta,$$

on a domain C. Here $p(t,\mathbf{x})$ is the pressure, $f(t,\mathbf{x})$ is a source term, and L is the wave operator with velocity $c(\mathbf{x})$. The domain C is assumed to be surrounded by absorbing boundaries, except perhaps at $z = 0$ where we may impose the free-surface boundary condition $p = 0$. The domain can be split into two parts by cutting it along a given depth $z_0 > 0$, resulting in a shallow part A with $0 = z < z_0$ and a deeper part B with $z > z_0$, see Figure 1. We can define wave equations that have the same boundary conditions as the domain C but an absorbing condition at $z = z_0$. The wave equations and solutions inside these domains will be denoted by a superscript A, B, or C. After a Fourier transform in time, we consider the two problems

$$\hat{L}^C \hat{p}^C = \hat{w}\delta(\mathbf{x} - \mathbf{x}_C), \quad \hat{L}^C = -k_C^2 - \Delta,$$
$$\hat{L}^A \hat{p}^A = \hat{w}\delta(\mathbf{x} - \mathbf{x}_A), \quad \hat{L}^A = -k_A^2 - \Delta,$$

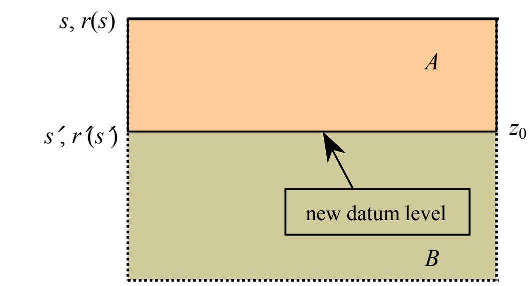

Figure 1: The domain C is the union of A and B.

where w is the wavelet, and $\mathbf{x}^A$ and $\mathbf{x}^C$ are the positions of sources. Now

$$\int_V d\mathbf{x}\left(\hat{p}^A \Delta \hat{p}^C - \hat{p}^C \Delta \hat{p}^A\right) = \hat{w}\left[\hat{p}^C(\mathbf{x}_A) - \hat{p}^A(\mathbf{x}_C)\right] + \int_V d\mathbf{x}\left(k_A^2 - k_C^2\right)\hat{p}^A \hat{p}^C .$$

If $V = A$, the integral on the right-hand side vanishes. If the pressure vanishes at the surface $z = 0$ and if the vertical boundaries are sufficiently far away so that we can neglect the pressure there, partial integration leads to

$$\hat{w}\left[\hat{p}^C(\mathbf{x}_A) - \hat{p}^A(\mathbf{x}_C)\right] = \int_{z=z_0} dx\,dy \left(\hat{p}^A \partial_z \hat{p}^C - \hat{p}^C \partial_z \hat{p}^A\right),$$

or, by denoting $\mathbf{x}_C$ with s and $\mathbf{x}_A$ with r and by using reciprocity,

$$\hat{w}\left[\hat{p}^C_{r(s)} - \hat{p}^A_{r(s)}\right] = \mathrm{B}(\hat{p}^A_{s'(s)}, \hat{p}^C_{s'(r)}) := \int_{z=z_0} dx'dy' \left(\hat{p}^A_{s'(s)}\partial_z \hat{p}^C_{s'(r)} - \hat{p}^C_{s'(r)}\partial_z \hat{p}^A_{s'(s)}\right) \qquad (1)$$

Here the subscript $r(s)$ indicates data recorded at a receiver r for a shot s. The points s' are located at (x', y', z_0'). This equation can be interpreted as redatuming. If data for the domain C have been recorded at $r(s)$ near the surface and the velocity is known inside domain A, we can compute $\hat{p}^A_{r(s)}$ with the same acquisition geometry as the measured $\hat{p}^C_{r(s)}$, and $\hat{p}^A_{s'(s)}$ for receivers at depth z_0 and the same shot positions. Reciprocity implies that we can use

$\hat{p}^A_{s(s')}$ for $\hat{p}^A_{s'(s)}$. Given these data, Eq. (1) defines an inverse problem for $\hat{p}^C_{s'(r)} = \hat{p}^C_{r(s')}$ and its z-derivative. We postpone the discussion on solvability until the next sections and only remark that the computation of the solution implies redatuming of the sources to depth z_0. The integral can be viewed as bilinear operator B(a,b) acting on two wave fields a and b, which is anti-symmetric: B(a,b) = –B(b,a) and B(a,a) = 0. As a consequence, we can replace Eq. (1) by

$$\hat{w} g_{r(s)} = \mathrm{B}\left(\hat{p}^A_{s'(s)}, g_{s'(r)}\right), \quad g = \hat{p}^C_{r(s)} - \hat{p}^A_{r(s)}. \qquad (2)$$

If we also want to redatum the receivers to the same depth, a far less straightforward derivation produces

$$\hat{w}\left[\hat{p}^C_{r(s')} - \hat{p}^A_{r(s')}\right] = \mathrm{B}(\hat{p}^C_{r'(r)}, \hat{p}^B_{r'(s')} - \hat{p}^A_{r'(s')}), \qquad (3)$$

where both s' and r' are positioned at the depth z_0. The contraction is over r'. The right-hand side can be written as

$$\mathrm{B}\left(\hat{p}^C_{r'(r)}, \left[\hat{p}^B_{r'(s')} - \hat{p}^{B,dir}_{r'(s')}\right] - \left[\hat{p}^A_{r'(s')} - \hat{p}^{A,dir}_{r'(s')}\right]\right),$$

where $\hat{p}^{B,dir}_{r'(s')} = \hat{p}^{A,dir}_{r'(s')}$ are the direct waves. With some effort, it can be shown that the term $\mathrm{B}\left(\hat{p}^C_{r'(r)}, \hat{p}^A_{r'(s')} - \hat{p}^{A,dir}_{r'(s')}\right)$ vanishes when Eqs. (2) and (3) are combined. We therefore have to invert Eq. (2) and

$$\hat{w} g_{r(s')} = \mathrm{B}\left(\hat{p}^C_{r'(r)}, \hat{p}^B_{r'(s')} - \hat{p}^{B,dir}_{r'(s')}\right). \qquad (4)$$

Using computed data for $\hat{p}^A$, Eq. (2) can be solved for $g_{r(s')}$. The solution of Eq. (4) requires $\hat{p}^C_{r(r')}$. There are a number of options. (**1**) Use the solution of the first equation to find $\hat{p}^C_{r'(r)} = \hat{p}^A_{r'(r)} + g_{r'(r)}$. Unfortunately, this is wrong as the contribution from $\mathrm{B}\left(\hat{p}^C_{r'(r)}, \hat{p}^A_{r'(s')} - \hat{p}^{A,dir}_{r'(s')}\right)$ is not included because it cannot be found by the inversion. (**2**) Solve Eq. (1) with $\hat{p}^A$ computed by using a dirichlet (zero) boundary condition at z_0, so that we get a less ill-posed problem in $\hat{p}^C_{r'(r)}$. Repeating the same with a neumann (zero z-derivative) boundary condition will provide $\partial_z \hat{p}^C_{r'(r)}$ after inversion. (**3**) Approximate $\hat{p}^C_{r'(r)}$ by $\hat{p}^A_{r'(r)}$. If z_0 is sufficiently deep, this approximation is exact for times shorter than the travel time from z_0 to the surface. The last approach is used here.

Discretization

If the velocity is known exactly inside the domain A – the shallow part of domain C – we can compute the wave field $\hat{p}^A$ for any source-receiver pair inside A, for instance with a finite-difference scheme. Suppose the computational domain has a uniform equidistant grid with vertical spacing Δz. The redatuming level z_0 can be chosen half way between two grid points. With the discretization $a(x, y, z_0) = ½[a(x, y, z_0–½\Delta z)+a(x, y, z_0+½\Delta z)]$, $\partial_z a(x, y, z_0) = [a(x, y, z_0+½\Delta z) - a(x, y, z_0–½\Delta z)]/\Delta z$, the discrete form of Eq. (1) becomes

$$\hat{w}\left[\hat{p}^C_{r(s)} - \hat{p}^A_{r(s)}\right] \propto \sum_{x_{s'}, y_{s'}} \left(\hat{p}^A_{s'_-(s)} \hat{p}^C_{r(s'_+)} - \hat{p}^A_{s'_+(s)} \hat{p}^C_{r(s'_-)}\right), \qquad (5)$$

Here $s'_\pm$ denotes a source at position ($x_{s'}$, $y_{s'}$, $z_0 \pm ½\Delta z$) and the constant is determined by the vertical grid spacing and the horizontal spacing of sources. For each frequency, we have to solve the singular linear system

$$\hat{w} g_{r(s)} = \sum_{s'_\pm} M_{s,s'_\pm} g_{r(s'_\pm)}, \text{with } M_{s,s'_\pm} = \pm Const. \hat{p}^A_{s'_\mp(s)}.$$

Numerical solution

First consider the one-dimensional case. Then Eq. (5) is clearly singular because for each frequency, there is one equation in two unknowns. If the equation is solved by an iterative method, for instance by formulating the associated least-squares problem and using the conjugate gradient method, or by a direct method such as a truncated singular value decomposition (TSVD), a minimum-norm solution is obtained. Unfortunately, this minimizes the size of $g_{r(s'_\pm)}$, leading to large errors in $g_{r(s')} = \frac{1}{2}\left[g_{r(s'_-)} + g_{r(s'_+)}\right]$. Better results were obtained by using another norm for the minimum-norm solution, namely $|g(z_0)|^2 + \beta^2|\partial_z g(z_0)|^2$. When using the TSVD, this can be accomplished by formulating the inverse problem with g and $\gamma^{-1}\partial_z g$ as unknowns. The linear system then has the form

$$\begin{pmatrix} m_1 & \gamma m_2 \end{pmatrix} \begin{pmatrix} g \\ \gamma^{-1}\partial_z g \end{pmatrix} = r,$$

and the use of its pseudo-inverse results in the minimum-norm solution

$$g = m_1^* r \Big/ \left(|m_1|^2 + \gamma^2 |m_2|^2\right), \partial_z g = \gamma^2\, m_2^* r \Big/ \left(|m_1|^2 + \gamma^2 |m_2|^2\right).$$

The asterisk denotes the complex conjugate. We can obtain $|g| / |\partial_z g| = |m_2| / |m_1| = \left|\hat{p}^A\right| \Big/ \left|\partial_z \hat{p}^A\right|$ by choosing $\beta = \gamma^{-1} = \left|\hat{p}^A\right| \Big/ \left|\partial_z \hat{p}^A\right|$.

In more than one space dimension, the linear system for shot redatuming involves a set of shot and receivers:

$$\hat{w} g_{r(s)} \propto \sum_{s'} m_{1,s'(s)} g_{r(s')} + m_{2,s'(s)} \partial_z g_{r(s')},$$

where $m_{1,s'(s)} = -\partial_z \hat{p}^A_{s'(s)}$ and $m_{2,s'(s)} = \hat{p}^A_{s'(s)}$. The singular behavior of this problem is less clear. Finite aperture

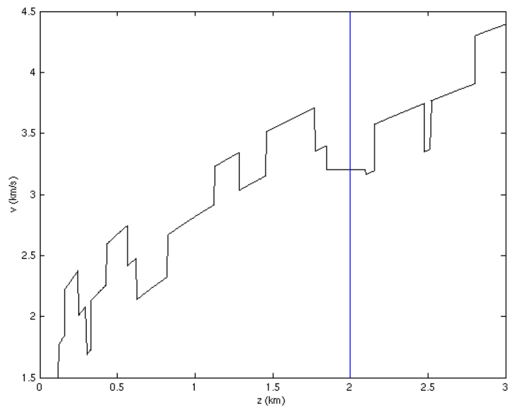

Figure 2: Velocity model for 1D case. The redatuming level is at 2 km depth.

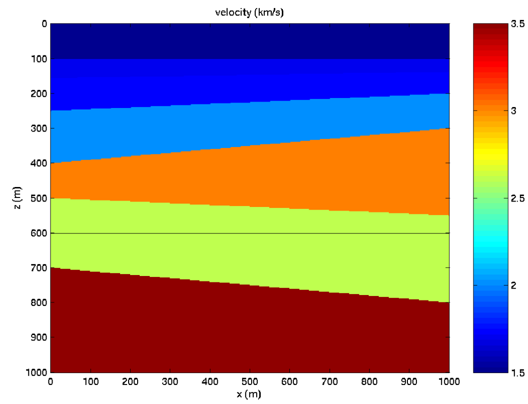

Figure 4: Velocity model for the 2D example.

effects will play a role in two ways. First, the assumption that the pressure vanishes at the sides of the domain will be violated. Secondly, illuminations effects will cause the solution for some receivers to be better defined that for others. This implies again that, strictly speaking, rigorous redatuming is not feasible, although useful results may still be obtained.

1D example

A one-dimensional example is based on the marine velocity model displayed in Figure 2. Data were computed with an eight-order time-domain finite-difference code [5] on a grid with 2.5 m spacing. The source and receiver were placed at 10 m depth below the free surface and 5 seconds of data were recorded.

For the redatuming, a depth z_0 = 1998.75 m was chosen and $\hat{p}^{A}_{r(s'_{\pm})}$ was computed at depths of 1997.5 and 2000 m, respectively, using the original model with the free-surface boundary condition down to 2 km and continuing with a constant velocity model at larger depths. First the sources were redatumed and then the receivers. For the TSVD, values smaller than 10^{-3} times the maximum singular value over all frequencies were set to zero. This mainly suppresses the effect of the higher frequencies that have small amplitude and contain more numerical noise than the lower frequencies.

The resulting redatumed trace is shown in Figure 3. For comparison, the "exact" p^B is shown as well. The latter has been computed by the finite-difference method in a model that is constant for depths less than 2 km and identical to the original model as larger depths. Also, the free-surface condition was replaced by an absorbing boundary condition. Apart from the direct wave, which was so strong that it has been clipped in the plot, the agreement between the redatumed result (black) and the reference solution (blue) is remarkably good. Also, the "exact" p^C–p^A is shown in green. That trace displays reflections and multiples due to waves that first traveled into part B of the domain, and then were reflected back and forth between the domains B and A. Note that these events have been removed by the redatuming.

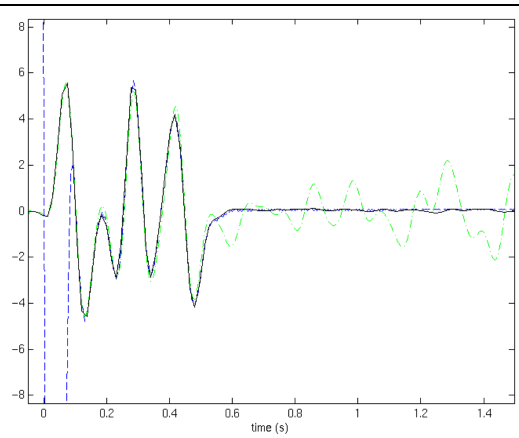

Figure 3: Result after redatuming both the source and receiver to 2 km depth. The blue dashed line is the exact result p^B, including the direct wave. The green dash-dotted line represents the exact p^C–p^A.

2D example

The method was further tested on a two-dimensional synthetic example. The velocity model is shown in Figure 4. A free-surface boundary condition was used at the top. We choose z_0 = 598.75 m, which is marked by the horizontal line in the figure. At the top, receivers were

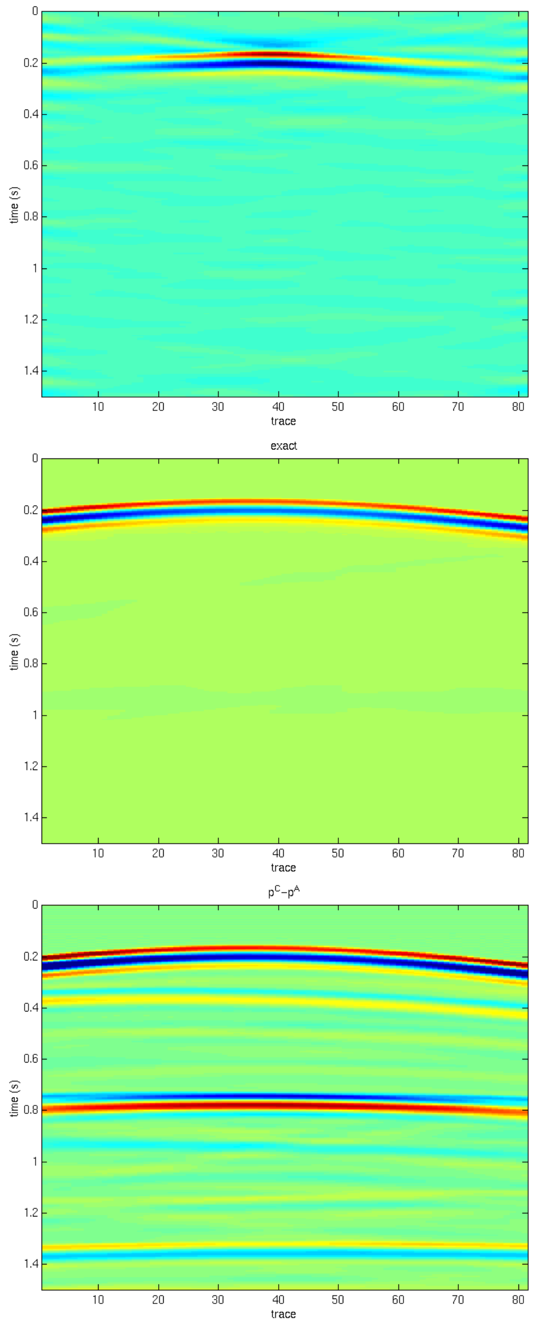

Figure 5: Redatumed shot panel (top), exact solution (middle), and exact $p^C - p^A$ (bottom).

placed at a depth of 5 m between x = 100 and 900 m with 10 m spacing. The sources were placed between 105 and 895 m with the same spacing and depth. Near z_0, we put receivers (or sources) between x = 100 and 900 m with 10 m spacing and at 597.5 and 600.0 m depth. A 2.5 m grid spacing was used and 1.5 s of data were recorded.

The result after redatuming is displayed in at the top of Figure 5, for a shot at x = 490 and z = 598.75. The picture in the middle displays the exact p^B, after subtraction of the direct wave which was computed in a constant-velocity model. The bottom panel shows the exact $p^C - p^A$ and still includes waves that have traveled downward from the redatuming level and were subsequently reflected back and forth between domains B and A. The redatuming operation has removed these waves.

The result is less accurate than in the one-dimensional case. The threshold value in the TSVD had to be chosen fairly large, at 0.1 times the largest singular value over all frequencies, to obtain acceptable results. This suggests that the two-dimensional problem is quite ill-posed.

Conclusions

The answer to the question posed in the title is *no*, in a strictly formal sense, but surprisingly good results can still be obtained with the proposed regularization.

The extension of the redatuming equations to variable-density acoustics and elastics is straightforward. Solving the inverse problem for the former is similar to the presented case. For the elastic case, the solution of the inverse problem is expected to be more difficult.

It remains to be seen if the current method can be used on real data. This requires a very accurate model between the surface and the redatuming level, which may be impossible to determine in practice. Also, its potential use in multiple removal and survey sinking remains to be investigated.

References

[1] Berryhill, J. R., 1979. Wave-equation datuming, *Geophysics* **44**, 1329 – 1344.

[2] Berryhill, J. R., 1984. Wave equation datuming before stack, *Geophysics* **49**, 2064 – 2067.

[3] Bevc, D., 1997. Imaging complex structures with semirecursive Kirchhoff migration, *Geophysics* **62**, 577 – 588.

[4] Martini, F. and Bean, C.J., 2002. Interface scattering versus body scattering in subbasalt imaging and application of prestack wave equation datuming, *Geophysics* **67**, 1593 – 1601.

[5] Mulder, W.A. and Plessix, R.-E., 2002. Time- versus frequency-domain modelling of seismic wave propagation, Extended Abstract E015, 64th EAGE Conference & Exhibition, Florence, Italy.

[6] Shtivelman, V. and Canning, A., 1988. Datum correction by wave equation extrapolation, *Geophysics* **53**, 1311 – 1322.