Velocity analysis with multiples – NMO modeling for layered velocity structures

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SUMMARY

Several method exist to invert for a velocity model from seismic data. Most commonly used methods are defined in the image domain. However, some methods that are defined in the data domain also exist. In this paper we review and compare a few methods on synthetic data, using the convolutional model and NMO traveltimes to model the data. Special attention will be given to the behavior of the different methods in the presence of multiples. In traditional MVA methods these usually pose a problem since multiples are not flattened or focused for the correct velocity model. In the data domain multiples do focus for the correct velocity model if they are correctly modeled. This is illustrated with a simple example. Another simple example illustrates how ideas from waveform inversion and data-domain velocity analysis can be combined to obtain the correct velocity model and reflectivity from synthetic data with multiples.

INTRODUCTION

In waveform inversion one tries to infer a set of model parameters, in particular velocity, from seismic data by fitting the data in a least-squares sense (Tarantola, 1984). While intuitively pleasing, this approach has several drawbacks. First, in order to get reliable results, the modeling used has to be accurate. Second, because of the absence of low frequencies in the data, it is difficult to obtain information about the slowly-varying components of the velocity model. The latter problem is addressed by Migration Velocity Analysis (MVA). However, this approach is usually based on high-frequency and/or one-way approximations of the wave equation. This prevents MVA to yield good results in the presence of strong multiples, which are not accounted for by these approximations. However, there are that try to use the multiples by removing them or transforming them into primaries (Verschuur and Berkhout, 2007).

Velocity analysis can also be performed in the data domain by generating data for the estimated reflectivity and comparing them to the observed data (Chavent et al., 1994; Plessix et al., 1999). Recently a method has been proposed that uses the correlation of observed and predicted data (van Leeuwen and Mulder, 2007a). This correlation will 'focus' for the correct velocity model. By automatically updating the velocity model to optimize the amount of focusing it is indeed possible to obtain a good NMO velocity model (van Leeuwen and Mulder, 2007b). We will refer to this velocity analysis method as DVA in the rest of the paper.

In principle, it should be possible to obtain a good velocity model with DVA in the presence of multiples, if the multiples are modeled correctly. But to model the multiples correctly, the migrated image, from which the synthetic data are modeled, has to be correct. In general the migrated image will contain spurious events due to multiples in the data. However, by iteratively updating the migrated image, it should be possible to obtain the correct reflectivity if the velocity model is correct. This leads to two extremes: (1) given the correct velocity model, we can obtain the correct reflectivity with least-squares inversion; (2) given the correct reflectivity, we can obtain the correct velocity model with the data-correlation method. In practice, neither of these approaches is useful since both the reflectivity and the velocity model are unknown.

In the current paper we test a method that combines ideas from waveform inversion and DVA to alternately update the reflectivity and the velocity model. The convolutional model and NMO traveltimes are used to generate the data. First-order surface multiples are included by convolving the data with itself.

CONVOLUTIONAL MODEL

Given a reflectivity $r(t_0)$ and NMO velocity $v(t_0)$, both as function of two-way traveltime t_0, and a wavelet $w(t)$, the data $p(t,h)$ as a function of time and offset are modeled as

$$p[r,v](t,h) = \int \mathrm{d}t'\, r\big(\tau_0[v](t',h)\big)\, w(t-t'), \tag{1}$$

where $\tau_0[v](t',h)$ is the inverse of the traveltime

$$\tau[v](t_0,h) = \sqrt{t_0^2 + \big(h/v(t_0)\big)^2}. \tag{2}$$

The true reflectivity and velocity model will be denoted with a hat, likewise, the measured data are denoted as $\hat{p}(t,h) = p[\hat{r},\hat{v}](t,h)$. This describes the kinematics of acoustic data accurately for layered media and offsets $h \lesssim 10^3 t$ (Symes, 1999). Amplitude effects, such as geometrical spreading and attenuation are neglected. Multiples can be modeled as well. Data with first-order surface multiples are given by

$$p[r,v](t,h) = \int \mathrm{d}t' \int \mathrm{d}t''\, r\big(\tau_0[v](t',h)\big) \Big(1 - \alpha r\big(\tau_0[v](t'-t'',h)\big)\Big) w(t-t'), \tag{3}$$

where α determines the relative amplitude of the multiples. Note that we are in fact auto-convolving the data. Figure 1 depicts an example of such data with and without multiples.

ITERATIVE MIGRATION

In linearized waveform inversion, the reflectivity is obtained by minimizing the least-squares error (Plessix and Mulder, 2004). This concept can be carried over to the convolutional model.

$$r^*[v](t_0) = \underset{r}{\operatorname{argmin}} \int \mathrm{d}t \int \mathrm{d}h \left(p[r,v](t,h) - p^{\mathrm{obs}}(t,h) \right)^2. \tag{4}$$

Usually, the reflectivity is iteratively updated with a gradient-based optimization method (Pratt and Hicks, 1998). Starting from a zero initial guess of the reflectivity (i.e., $r^{(0)} = 0$), the first update is equivalent to a traditional stack (Lailly, 1983)

$$r^{(1)}(t_0) \propto \int \mathrm{d}h\, p^{\mathrm{obs}}\big(\tau[v](t_0,h),h\big) \tag{5}$$

Of course, any multiples present in the data will give rise to spurious reflectors in the stack, even if the true velocity model is used. Figure 2 (a) shows an example. The migrated image can be greatly improved by iterating. Given the true velocity model the true reflectivity can be obtained in this way, as is shown in figure 2. All spurious reflectors, caused by multiples in the data have disappeared.

MVA

To update the velocity, two generalizations of the migrated image are frequently deployed. The first uses offset as redundant variable. In the current context this image is given by

$$I[v](t_0,h) = \hat{p}\big(\tau[v](t_0,h),h\big). \tag{6}$$

For the correct velocity model, the primaries in the image line up such that $I[\hat{v}](t_0,h)$ is independent of h. The Differential Semblance Optimization (DSO) method exploits this to update the velocity model automatically (Symes, 1999; Mulder and ten Kroode, 2002). If there are multiples present in the data, these will not be flattened for the correct velocity model. Hence, multiples have to be removed before using

this method. If the velocity increases with depth, multiples will have a lower apparent velocity and they can be filtered out using a dip filter on $I[v](t_0,h)$ (Mulder and ten Kroode, 2002; Li and Symes, 2007). Figure 3 shows an example of this image for the correct velocity models with and without multiples.
Another generalization of the migrated image introduces an offset at depth (MacKay and Abma, 1992). In the current context this image is given by

$$I[v](t_0,\Delta h) = \int \mathrm{d}t \int \mathrm{d}h\, \hat{p}\big(t+\tau[v](t_0,h+\Delta h),h\big)\, w\big(t\big) \tag{7}$$

For the correct velocity model, primaries will focus at zero shift in this image (de Hoop and Stolk, 2006). This is illustrated in figure 4 (a). Again, multiples will not focus for the correct velocity model and will have to be removed before applying this method. As can be seen in figure 4 (b), the multiples are present at negative shifts. This is due to the fact that the NMO velocity increases with depth and that the data contain only positive offsets. Like the dip filtering in the previous case, this focusing behavior gives us the opportunity to discern multiples from primaries in the image.

DVA

As demonstrated by van Leeuwen and Mulder (2007b) it is possible to obtain a reasonable NMO velocity model by maximizing the amount of focusing of the data correlation for data with primary reflections. Here, we test the concept on synthetic data with first-order multiples. The data-correlation is given by

$$C[p,\hat{p}](t,\Delta h) = \int \mathrm{d}h\, p[r,v](t,h)\hat{p}(t,h+\Delta h). \tag{8}$$

To get an idea of how the data correlation is expected to behave in the presence of multiples, we auto-correlate data containing only primaries and data containing multiples. Figure 5 (a,b) illustrates that both will focus for the correct velocity model. It is of importance here that the multiples are modeled correctly. To illustrate this, we show the correlation of data with multiples with primary data in figure 5 (c). The energy is not properly focused at zero shift as it was in the previous two cases.
To obtain a velocity the focusing is measured with a weighted norm, and the velocity model that maximizes this is sought.

$$v^*[r](t_0) = \underset{v}{\mathrm{argmax}} \int \mathrm{d}t \int \mathrm{d}\Delta h\, \exp\left[-\gamma \Delta h^2\right] \big(C[p[r,v],\hat{p}]\big)^2. \tag{9}$$

The velocity model is represented on a spline grid and a gradient-based optimization method is used to obtain the velocity model. Figure 2 (b) shows that, given the true reflectivity, the true velocity model can be recovered in this way.

JOINT INVERSION

We have seen that, given the true velocity model, the true reflectivity can be obtained by minimizing the LS-error and that, given the true reflectivity, the true velocity model can be obtained DVA. Here, we combine both ideas. The reflectivity and velocity model are updated alternately to obtain the true reflectivity and velocity model. It proved beneficial to use the LS error as weight for the DVA updating. This ensured that the velocity was not updated at places where the data was already explained. Below is an algorithmic description of the above outlined approach. The LS misfit function J_{LS} is as defined in equation 4. The LS weight w is applied to the observed data. This has the same effect as weighting the data correlation (equation 8) since it is linear in the observed data. The DVA misfit function is as defined in equation 9. For the optimization we use a BFGS algorithm (Vogel, 2002). The NMO velocity is represented with cubic splines and the gradient is calculated with the adjoint state method (Plessix, 2006). We stop updating when the data misfit becomes smaller than some threshold ε.

Algorithm 1 Algorithmic description of the joint inversion scheme used.

while $||w^{(k-1)}|| < \varepsilon$
  $r^{(k)} = \underset{r}{\mathrm{argmin}}\ J_{\mathrm{LS}}[r, v^{(k)}, \hat{p}]$;
  $\delta p^{(k)} = \hat{p} - p^{(k)}$;
  $w^{(k)} = \int \mathrm{d}h\, \delta p^{(k)}\big(\tau(t_0,h),h\big)$;
  $v^{(k+1)} = \underset{v}{\mathrm{argmax}}\ J_{\mathrm{DVA}}[r^{(k)}, v, w^{(k)} \cdot \hat{p}]$;
  $k{+}{+}$
end

The results are depicted in figures 6 – 9. Figure 6 shows the initial velocity model and the corresponding reflectivity, obtained from a LS inversion. This reflectivity was then used to update the velocity with the data correlation method. The resulting velocity model can be seen in figure 7, where also the corresponding reflectivity is depicted. Figure 8 and 9 show the subsequent iterations. It shows that the velocity model and reflectivity gradually improve, moving from the shallow to the deeper part.

DISCUSSION

When applying conventional velocity analysis methods, multiples have to be removed from the data. This is because multiples do not exhibit the same focusing behavior as primaries. Hence, their focusing cannot be used to select a suitable velocity model. In some cases the multiples can be taken care of during the imaging step by dip filtering or muting part of the image. The example suggests that in the data correlation, multiples do focus *if they are correctly modeled*. With correct modeling here we mean that the reflectivity contains no spurious reflectors. A test of the data correlation method using the true reflectivity seems to confirm this.
Further tests on synthetic data suggest that it is possible to combine ideas from waveform inversion and DVA to alternately update the reflectivity and velocity model to obtain the correct velocity model.
In some cases conventional MVA combined with filtering or muting will work fine and there is no need for more sophisticated methods. These firsts tests with DVA in the presence of multiples encourages us to develop the method further and test it in more complex settings where conventional MVA techniques are bound to fail.

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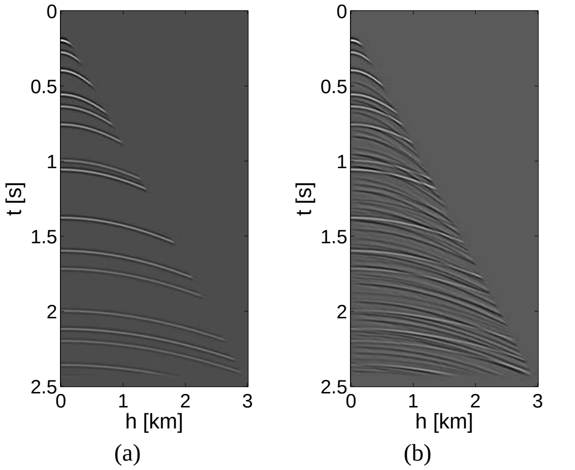

Figure 1: *Synthetic data for the true velocity model and reflectivity. (a) depicts the primaries-only data and (b) the data with first order surface multiples.*

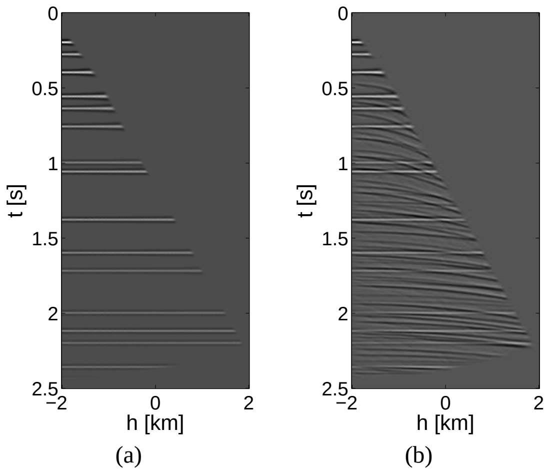

Figure 3: *Example of generalized images as a function of surface offset for the true velocity model. (a) depicts the image for primary data. (b) depicts the image for data containing also first order surface multiples. For the correct velocity model not all the events are flattened.*

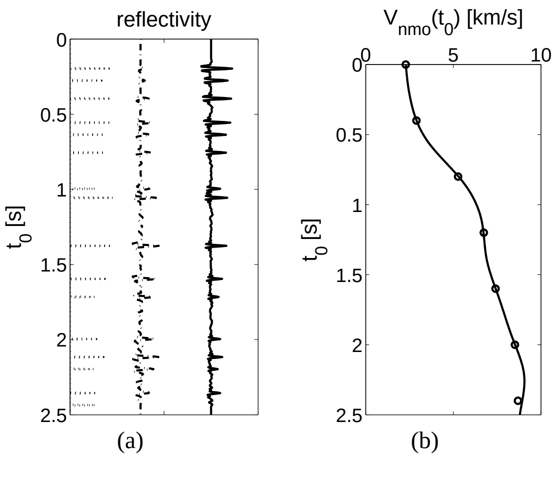

Figure 2: *Results of separate inversion. (a) True reflectivity (dots), traditional stack (dashed) and reflectivity obtained with LS inversion for the true velocity model (solid). (b) True velocity model (circles) and final velocity obtained with the data-correlation method for the true reflectivity.*

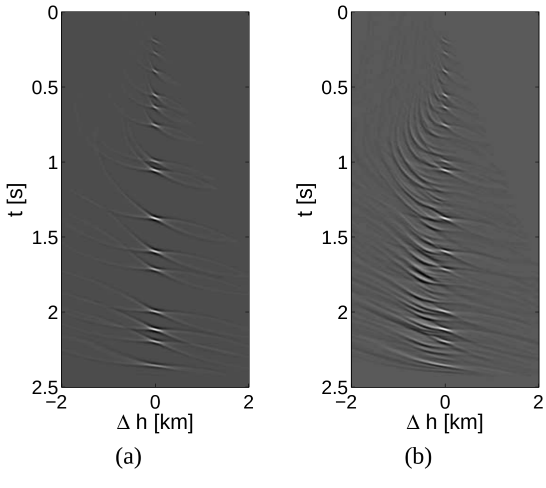

Figure 4: *Example of generalized images as a function of depth offset for the true velocity model. (a) depicts the image for primary data. (b) depicts the image for data containing also first order surface multiples. For the correct velocity model not all the events focused.*

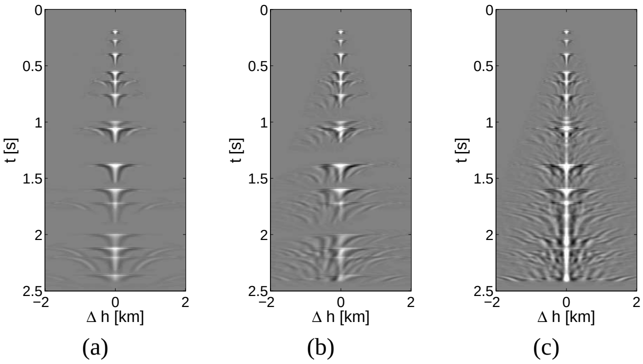

Figure 5: *Example of data correlation. (a) Shows the correlation of data with only primaries. It shows that the energy is focused for the correct velocity model. (c) Depicts the correlation of data with multiples. Here, like in (a) the energy does focus. Finally, (b) Shows the correlation of data with primaries and data with multiples. This indicates that if the multiples are not properly modeled, the energy will not focus for the correct velocity model.*

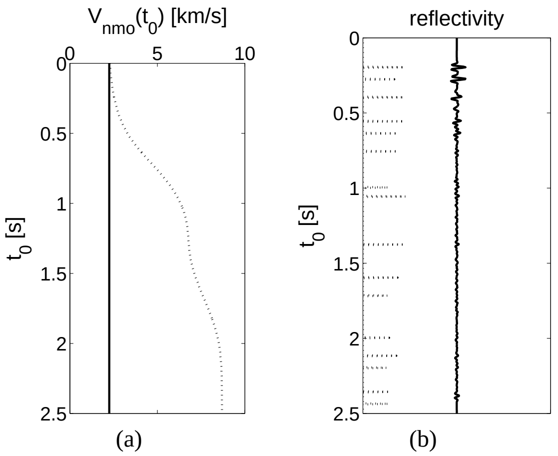

Figure 6: *(a) initial velocity model, (b) reflectivity obtained for the initial velocity model.*

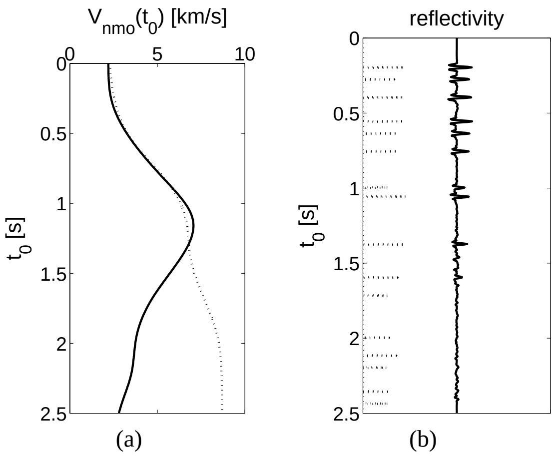

Figure 7: *(a) velocity model obtained for reflectivity from figure 6, (b) reflectivity obtained for this velocity model.*

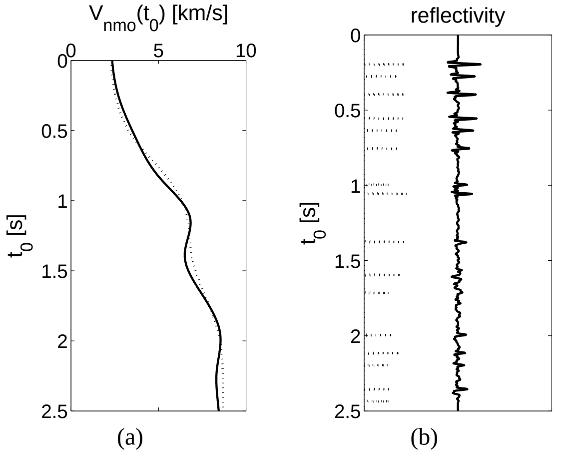

Figure 8: *(a) velocity model obtained for reflectivity from figure 7, (b) reflectivity obtained for this velocity model.*

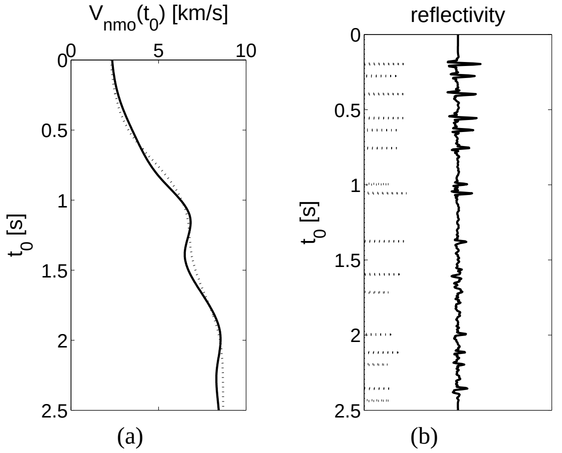

Figure 9: *(a) velocity model obtained for reflectivity from figure 8, (b) reflectivity obtained for this velocity model.*

EDITED REFERENCES

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