Getting around the ambiguity in attenuation imaging

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SUMMARY

Migration of seismic data in a constant-density visco-acoustic model provides reflectors that consist of velocity and of attenuation perturbations relative to a given background model. Earlier work has shown that simultaneous imaging of these perturbations leads to an ambiguity: a velocity perturbation may produce nearly the same data as an attenuation perturbation. There are various approaches that can circumvent the ambiguity. An acquisition geometry with sources and receivers all around the scattering object removes the ambiguity, but is useless in geophysical exploration. Including causality in the scattering model when using the Born approximation will help if data with a sufficient broad frequency band are available. Here we consider the question if nonlinear inversion is able to get around the ambiguity. A numerical experiment with a visco-acoustic frequency-domain finite-difference code suggests a positive answer if causality is included, although the number of iterations required to reach an acceptable solution is very large. Without causality, nonlinear inversion appears to provide better results than linearized inversion with the Born approximation, but it seems that the ambiguity still hampers convergence to the true solution.

INTRODUCTION

Migration of seismic data produces an image of the subsurface if a sufficiently accurate background velocity model is available. With careful weighting, it also provides quantitative information about the reflectors. Iterative migration can improve the accuracy of the result even if the weighting is fairly crude. The algorithm tries to minimize the error between the modeled and observed data. More precise migration weights will speed up the convergence. Iterative migration employs the Born approximation of the wave equation for forward modelling. As a result, the inverse problem is linear in the reflectivity and will be easier to handle than full waveform inversion, which is highly nonlinear and is plagued by local minima. The reconstructed reflectivity represents the difference between the reconstructed model parameters and the given background model.

Earlier (Hak and Mulder, 2008a,b), we applied a frequency-domain finite-difference code for 2D visco-acoustic modelling to the iterative migration problem and encountered unexpected results. In the constant-density case, the reflectivity consists of impedance perturbations, which become velocity perturbations as the density is assumed constant, and of perturbations in the parameter that describes attenuation, for instance, the quality factor (Ribodetti and Virieux, 1998). We found that migration not only reconstructs velocity perturbations but also leads to unphysically large attenuation variations. Further analysis revealed that we could actually construct two different scattering models that provided nearly identical data (Mulder and Hak, 2009). One model contained only velocity perturbations, the other involved attenuation perturbations. Because the problem is linear in its model parameters under the Born approximation, any linear combination of the two extreme cases will also produce nearly the same data, if the sum of the relative weights equals one. This means that the inverse problem does not have a unique solution and that it is impossible to simultaneously recover velocity and attenuation perturbations from seismic data, at least under the assumptions we made. These were: (1) we considered a typical surface acquisition geometry for measuring the seismic data; (2) the background model was visco-acoustic and contained a frequency-dependent correction for causality, but this correction was dropped in the perturbations; (3) the Born approximation, which makes the inverse problem linear. In a subsequent paper (Hak and Mulder, 2010a), we addressed the first assumption and considered different geometries. Sources and receivers on a circle around a point scatterer, not necessarily at the centre of the circle, removed the ambiguity in the 2D case. This confirms the result of Ribodetti et al. (2000). Unfortunately, it reappeared for other, more realistic geometries. Including the correction for causality in the perturbation proved more fruitful (Hak and Mulder, 2010b). In a numerical experiment, we found that the frequency-dependent correction term can apparently circumvent the ambiguity if the data have sufficient bandwidth. Still, the gradient of the least-squares cost function for the first update differed considerably from the desired result. Therefore, a large number of conjugate-gradient iterations, of the order of 10^5, was required to change the first migration image into the correct one.

This leaves the question if dropping the Born approximation will also remove the ambiguity. We may expect that visco-acoustic full waveform inversion of the data in a correct background model will do the job if causality is imposed. It is not clear if nonlinear inversion by itself can accomplish the same without the frequency-dependent correction for causality. We will address this question by a numerical experiment for the two-dimensional constant-density visco-acoustic wave equation in the frequency domain.

METHOD

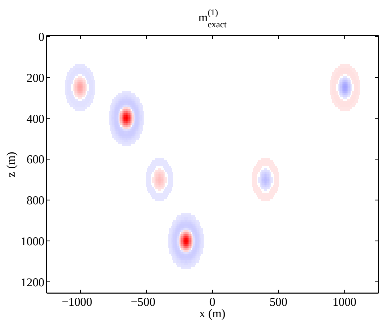
The constant-density visco-acoustic wave equation in the frequency domain reads

$$\omega^2 \tilde{m} p + \Delta p = s.$$

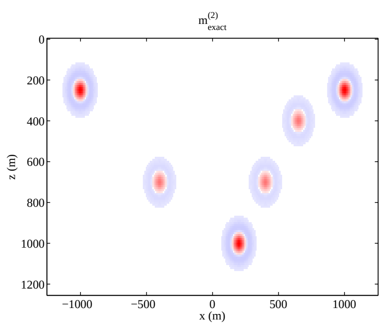
The pressure $p(\omega,\mathbf{x})$ depends on the angular frequency $\omega = 2\pi f$ and position $\mathbf{x}$, as does the source term $s(\omega,\mathbf{x})$. The model

$$\tilde{m}(\omega,\mathbf{x}) = \sigma^2(\mathbf{x})\left[1 + \frac{\mathrm{i} - a(\omega)}{Q(\mathbf{x})}\right], \quad a(\omega) = \frac{2}{\pi}\log\left(\frac{\omega}{\omega_r}\right),$$

contains the slowness $\sigma(\mathbf{x})$ and the quality factor $Q(\mathbf{x})$ that parametrizes the attenuation. The causality requirement leads to logarithmic term $a(\omega)$ and is defined relative to $\omega_r = 2\pi f_r$ (Aki and Richards, 1980). A common choice for the reference frequency f_r is 1 Hz. We will use $M^{(1)}(\mathbf{x}) = \sigma^2$ and $M^{(2)}(\mathbf{x}) = \sigma^2/Q$ as model parameters, implying that $\tilde{m}(\omega,\mathbf{x}) = M^{(1)} + [\mathrm{i} - a(\omega)]M^{(2)}$.



(a)



(b)

Figure 1: (a) First and (b) second component of the given model after subtraction of the background model. Positive values are red, negative blue.

The least-squares cost functional, which measures the difference between the modeled data p and observed data p^{obs}, is

$$J = \frac{1}{2}\sum_{\mathrm{s},\mathrm{r(s)},\omega}\left|S_{\mathrm{r(s)}}\,p(\omega,\mathbf{x}) - p^{\mathrm{obs}}_{\mathrm{r(s)}}(\omega)\right|^2.$$

The summation involves all shots, indexed by s, receivers for a given shot, indexed by r(s), and the selected frequencies. The operator $S_{\mathrm{r(s)}}$ samples the modeled wavefield $p(\omega,\mathbf{x})$ at the receiver position.

A 2D frequency-domain visco-acoustic finite-difference code (Mulder and Plessix, 2004) provided the synthetic data. We used the L-BFGS method for the minimization of the cost functional (Byrd et al., 1995). This method requires the gradient of the cost-functional, which is easy to compute in the frequency domain.

RESULTS

The 2D test problem has a homogeneous background with a velocity $c_b = 1500$ m/s and quality factor $Q_b = 100$. We denote the background model by $M_b^{(k)}$, $k = 1,2$, with $M_b^{(1)} = c_b^{-2}$ and $M_b^{(2)} = c_b^{-2}Q_b^{-1}$. To this model, we added eight perturbations in the form of sombrero functions, obtained by applying the Laplace operator to a gaussian. Its standard deviation was 40 m. The full model is denoted by $M^{(k)}$ and the difference with respect to the background model by $m^{(k)} = M^{(k)} - M_b^{(k)}$. The largest peak value of $m^{(1)}$ was 5×10^{-8} and 2×10^{-8} for $m^{(2)}$, both in (s/m)^2. The actual amplitudes of each the sombrero functions can be read off from the plots of vertical cross sections through their centers, to be presented later on. Figure 1 shows $m^{(1)}$ and $m^{(2)}$. We generated synthetic data with the frequency-domain code, using absorbing boundary conditions on all sides. The receiver positions ranged from -1900 to 1900 m in x at a 25-m spacing. The sources were placed halfway in between, between -1887.5 and 1887.5 m with the same spacing. All source and receivers had zero depth. The Ricker wavelet had a peak frequency of 15 Hz. We only considered frequencies between 4 and 20 Hz at an interval of 0.5 Hz.

Figure 2 displays the reconstructed model after 18690 iterations, when the least squares cost function had dropped by a factor 10^{-7} from its initial value obtained for the background model. We included the correction for causality. The reconstruction of both components of the model is quite satisfactory for shallower part of the model, but less so for the deeper part. Nevertheless, the numerical experiment shows that the correction for causality and the nonlinear inversion can circumvent the ambiguity in the simultaneous reconstruction of velocity and attenuation perturbations.

This still leaves the question if nonlinear inversion by itself, without the correction for causality, is able to get around the attenuation imaging ambiguity. For the next experiment, we did not include the correction for causality in generating the synthetic data nor in the inversion. It now took 28782 iterations to reduce the least-squares cost function by a factor of 10^{-7} from its initial value in the constant starting model. Figure 3 shows both components of the reconstructed model. Despite the substantial reduction of the cost function, a significant difference between the reconstructed and true model remains. This suggests that the nonlinear inversion by itself cannot fully remove the ambiguity in the simultaneous reconstruction of velocity and attenuation perturbations.

Even with causality included in the scatterer – the difference between the full model and the background model – convergence is slow. We observed the same for linearized inversion using the Born approximation (Hak and Mulder, 2010b). A large number of iterations was required before the reconstructed solution starts to approximate the true one. One reason for the slow convergence is the fact that the gradient of the least-squares cost function is still affected by the attenuation scattering ambiguity. The gradient controls the update of the current estimate of the model. Figure 4 shows the gradient for the first iteration, starting from the background model, for a problem with just a single, scatterer with zero second component at 400 m depth, again using a sombrero function. For this first iteration, the first component of the gradient resembles the desired answer. The second component should be very small, but is completely off.

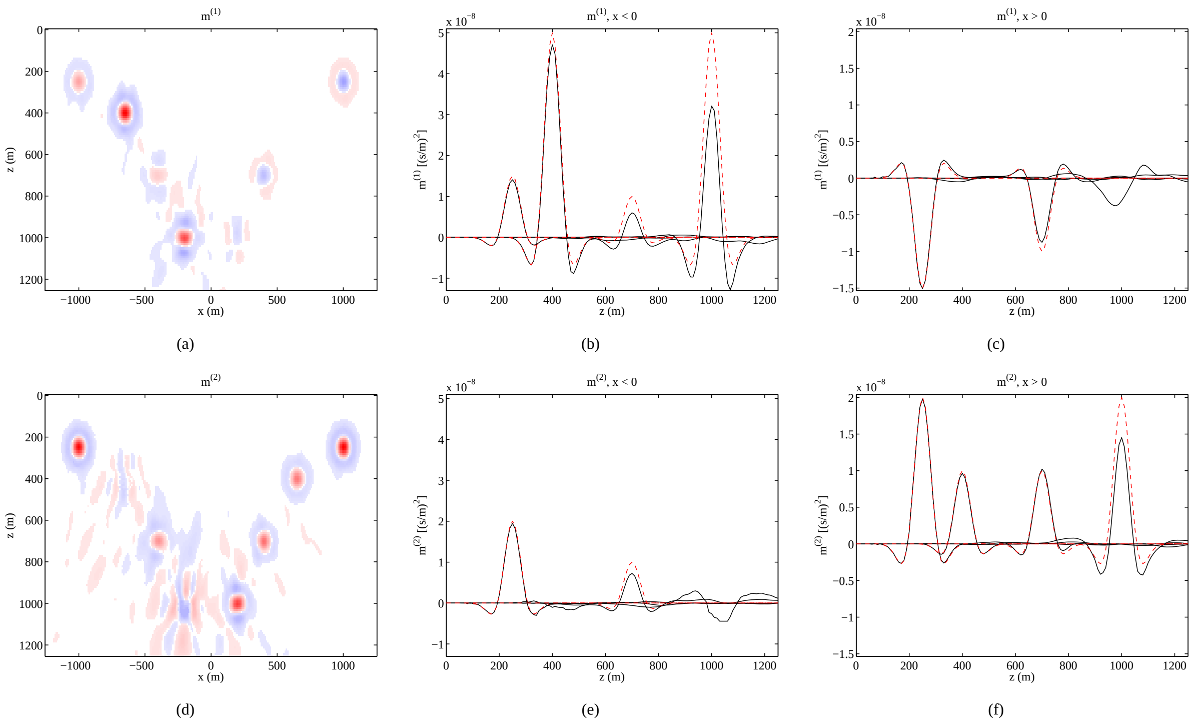


Figure 2: (a) First component of the reconstructed model, the square of the slowness, after subtraction of the background model. The correction for causality was applied. (b) Vertical cross-sections of the first component at $x = -1000$, -650, -400, and -200 m, and (c) at $x = 200$, 400, 650, and 1000 m. The original perturbations are drawn as red lines. (d), (e), and (f) display the same for the second component of the model, the squared slowness divided by the quality factor.

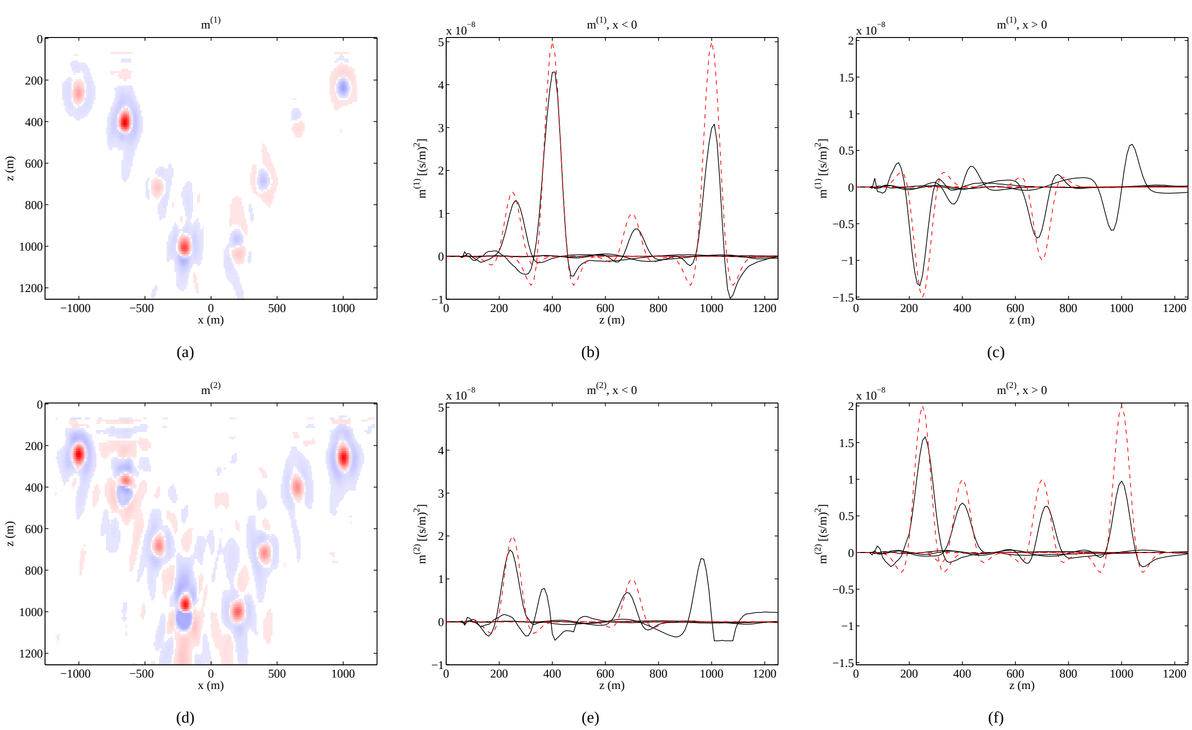


Figure 3: (a) First component of the reconstructed model, the square of the slowness, after subtraction of the background model. In this case, the correction for causality was not applied. (b) and (c) show vertical cross-sections at the same positions as in the previous figures. (d), (e), and (f) display the second component of the model.

Figure 5 shows a similar behavior, but now for a scatterer that has both components nonzero. The attenuation scattering ambiguity clearly has a detrimental effect on the shape of the gradient. To fix this, many iterations are required.

Convergence may be faster for a different model parametrization. If, for instance, we selected σ and $1/Q$ as the unknowns instead of σ^2 and σ^2/Q, we found a slightly better convergence rate in the nonlinear case.

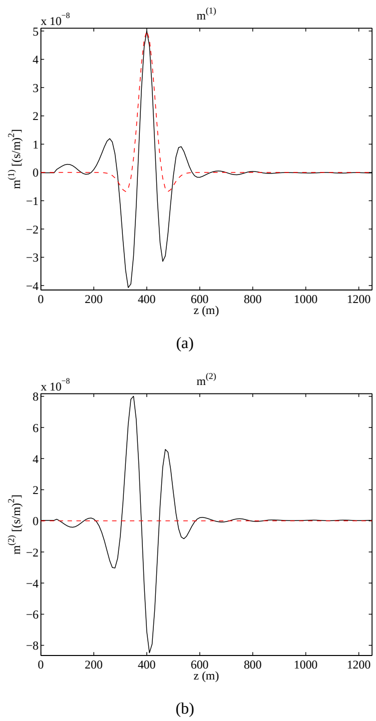


Figure 4: Vertical cross-sections of the gradient for the first update and the true model. The first (a) and second component (b) of the gradient have been scaled to the same magnitude as the difference between the true model and the background. In this case, the true model had a zero second component.

CONCLUSIONS

The attenuation scattering imaging ambiguity occurs in migration of seismic data when imaging simultaneously for velocity and attenuation perturbations in a given background model. Different perturbations or scattering models can produce nearly identical data. Including the correction for causality as part of the scattering model will remove the ambiguity if data with sufficient bandwidth are available, as demonstrated earlier for linearized inversion with the Born approximation (Hak and Mulder, 2010b). Here, we investigated if nonlinear inversion can accomplish the same. A numerical experiment of nonlinear inversion for a simple model reproduced the original model, at least in the shallow parts, when a correction for causality was included. Without causality, the reconstruction of the true model did not fully succeed, although the results are better than would be expected with linearized inversion. This

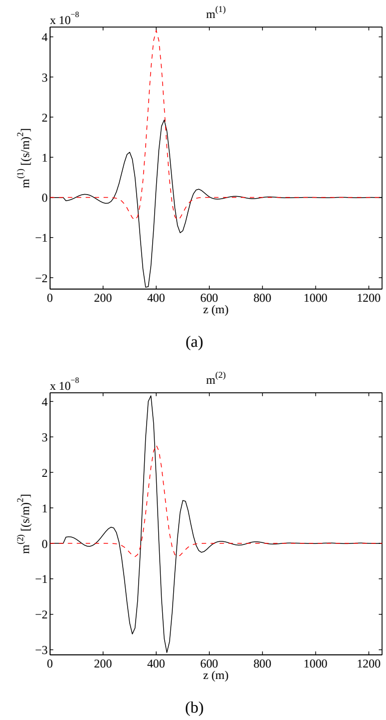


Figure 5: Vertical cross-sections of the gradient for the first update and the true model. The first (a) and second component (b) of the gradient have been scaled to the same magnitude as the difference between the true model and the background.

suggests that causality as part of the scattering model is the key factor in circumventing the ambiguity, consistent with the work of (Innanen and Weglein, 2007). In spite of that, the ambiguity still appears to have a detrimental effect on the convergence rate of the nonlinear inversion. The number of iterations is substantial and may be too large for practical applications.

Finally, we should emphasize that these results do not pertain to the problem of estimating the attenuation of the background model.

ACKNOWLEDGMENTS

This work is part of the research programme of the ‘Stichting voor Fundamenteel Onderzoek der Materie (FOM)’, which is financially supported by the ‘Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)’.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2010 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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