Coupling of Elastic Isotropic Medium Parameters in Iterative Linearized Inversion

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An elastic isotropic medium is described with three parameters. In seismic migration the perturbation of one elastic parameter affects the images of all the three, which means that these parameters are coupled. For an effective quantitative reconstruction of the true elastic medium reflectivity one can apply an iterative linearized migration/inversion, where minimization of the misfit functional is done by the conjugate gradient method. The final result of the iterative approach can be obtained directly by Newton's method, using the pseudo-inverse Hessian matrix.

Calculation of this matrix for a realistic model is an extremely resource-intensive problem, but for a model of a scatterer in a homogeneous elastic medium it is quite feasible. This paper presents the numerical results of elastic linearized inversion for this simple model, calculated both with iterative approach and Newton's method. Experiments show that in the both cases the elastic parameters have coupled weaker than in the case of migration. The iterative approach allows achieving acceptable quality of the inversion, but requires a large number of iterations. For faster convergence it is necessary to use the preconditioned conjugate gradient method. The optimal preconditioning will improve the convergence of the method as well as the quality of inversion.

Key words: *Linearized inversion, seismic migration, elastic isotropic medium, coupling of elastic parameters.*

Introduction

A classic seismic migration (Claerbout 1971) allows for qualitative estimation of reflection capability in an elastic medium, but the true values of the medium in this case remain unknown (Zhu *et al*. 2009). Moreover, the migration result gives information about heterogeneities of each of the three parameters used to describe an elastic isotropic medium, so the true perturbation of one of them in a migration image can look like as perturbation in each of the three parameters. There have been a number of optimization methods to estimate the elastic parameters both qualitatively and quantitatively (Ампилов *et al*. 2009; Virieux and Operto 2009). Their application gives more accurate estimations of true characteristics of the medium and reduces the coupling effect of the parameters. Uncoupling the elastic parameters mathematically is an extremely resource-intensive problem due to the optimized character of the solution (Gelis *et al*. 2007; Virieux and Operto 2009). That is the reason inversion methods are applied first-hand, so the elastic characteristics could be properly accessed with account for the required computation time and operation memory. For instance, one can apply an iterative linearized migration/inversion (Beydoun and Mendes 1989; Jin *et al*. 1992; Tura *et al*. 1993), which advantages are described in Ostmo *et al.* (2002), where the method was applied to an acoustic wave equation with constant density in the frequency domain. If computations are considered, this approach has proved more effective than a full waveform inversion (Tarantola 1986; Mora 1987; Fichtner 2010).

The objective of this research has been studying the mutual effect of elastic parameters when the iterative linearized inversion is applied. This paper presents the method's theoretical basis and a number of numerical experiments, using a point scatterer in a homogeneous isotropic elastic medium as the model. For such a simple case the solution can be obtained by Newton's method (Fichtner 2010), which allows for direct estimation of the elastic parameters' maximum inversion quality, available for the iterative inversion, so one can make a conclusion about applicability of this method for a particular case.

Effectiveness of the iterative approach mainly depends on initial problem's conditionality. Its optimal preconditioning has significantly improved the iterative approach's convergence [Axelsson, 1996], so one of the objectives of this study has also been finding an efficient preconditioning method for the iterative linearized inversion.

Method

A system of motion equations for an elastic isotropic medium in the frequency domain can be expressed as $L\mathrm{u} = \mathrm{f}$, where the wave operator L affects the displacement vector u in the following way:

$$Lu = -\omega^2 \rho \mathbf{u} - \nabla \cdot \left[\lambda (\nabla \mathbf{u}) \mathbf{I} + \mu \left(\nabla \mathbf{u} + [\nabla \mathbf{u}]^T\right)\right] = \mathbf{f}. \quad (1)$$

In the equation (1) the right part f corresponds to the source; ρ is the density; λ and μ are the Lame parameters; ω is the angular frequency; I is the unit tensor. The medium parameters are expressed as a sum of the reference value and the perturbation: $\rho = \rho_0 + \rho_s$, $\lambda = \lambda_0 + \lambda_s$ and $\mu = \mu_0 + \mu s$. Linearization of the problem, followed by the Born approximation results in two equations: $L_0\mathbf{u}_0 = \mathbf{f}$ and $L_0\mathbf{u}s = -Ls\mathbf{u}_0$, where $\mathbf{u}_0$ is the falling field from the source in the reference model and $\mathbf{u}s = \mathbf{u} - \mathbf{u}_0$ is the field, scattered by perturbated parameters. The symbol $L0$ marks the elastic wave operator with the reference values ρ_0, λ_0, μ_0, while Ls is the scattering wave operator with the characteristics ρs, λs and μs.

Instead of the absolute value of some elastic parameter *ms,* it is convenient to apply its relative perturbance, written as:

$$\tilde{m} = \frac{m}{m_0} - 1 = \frac{m_s}{m_0}. \quad (2)$$

The value $\tilde{m}$ in the expression (2) is the reflection capacity relative to the parameter *ms*. in this case,the inversion problem is solve in order to find the optimal model $\tilde{\mathbf{m}}_{opt}$ among the other possible models *m* for certain reference values of the medium m_0, as well as for known wave field $\mathbf{u}_{obs}$. It has been done by minimizing the residual quadratic functional:

$$\mathbf{J}(\tilde{\mathbf{m}}) = \frac{1}{2} \sum_{\omega} \sum_{x^s, x^r} \|\mathbf{u} - \mathbf{u}_{obs}\|^2. \quad (3)$$

The optimal model $\tilde{\mathbf{m}}_{opt}$ should correspond to the functional minimum **J** and, as the result, the zero gradient for the parameters . A linear approximation of such a gradient in the vicinity of the initial model results in Newton's method (Fichtner 2010):

$$\mathbf{H}\tilde{\mathbf{m}}_{opt} = -\nabla \mathbf{J}, \quad (4)$$

where $\nabla \mathbf{J}$ marks the gradient (first derivative vector) and **H** is the Gesse matrix (second derivative matrix) of the residual functional **J**. Hence, minimization of **J**, determined by the expression (3) requires the full Gesse matrix to be calculated, and since it is very close to a degenerate one, we have applied the pseudoinversion (Golub amd Kahan 1965). In case of a real model such a matrix will have a very big size even for modern computational systems both in the terms of computational time and data amount (Gelis *et al*. 2007). On the other hand, to solve the equation (4) one can apply the conjugate gradient method (Axelsson 1996), which does not require explicit calculation of the matrix H, but for each of the iterations it multiplies the matrix H and a certain vector. Hence, using a finite number of the iterations, theoretically, one is able to obtain a result that is close to one obtained by Newton's method.

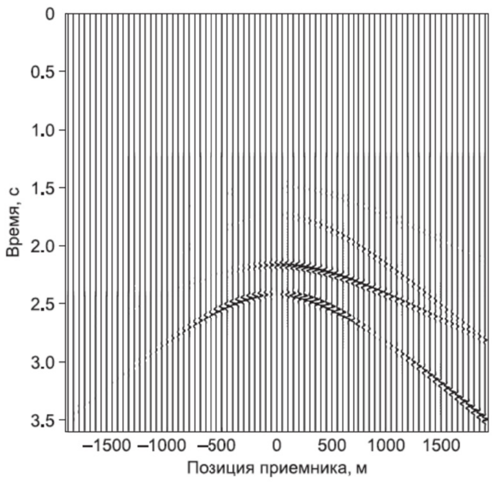
But when the amount of iterations is too big, the preconditioning has to used, which speeds up the method's convergence (Axelsson 1996). The preconditioning can be interpreted as a problem equivalent to the problem (4) by, for instance, multiplying of the initial system (left) by the symmetric positively- defined preconditioner-matrix P (Капорин 2011):

$$\mathbf{PH}\tilde{\mathbf{m}}_{\mathrm{opt}} = -\mathbf{P}\nabla\mathbf{J}. \qquad (5)$$

But the convergence rate has not been the only preconditioning aspect. The matrix **P** should be close to the inversed matrix **H**, but it obvious, if the matrix **H**-1 is used as the preconditioner, the CG method will converge after one iteration, so the iteration method becomes unnecessary. Thus, it is important one provides the best joint quality of preconditioning, which is a combination of such factors as: costs of calculating the preconditioner, its application for each of the iterations, reduction of the iterations' number, required to obtain a solution of appropriate quality (Капорин 2011).

One of the most commonly applied and easiest methods have been the Jacoby iteration technique, which preconditioner is an inverted diagonal part of the Gesse matrix (Kelley 1995; Axelsson 1996). Another obvious choice has been a block-diagonal approximation of the Gesse matrix (Beylkin and Burridge 1990). Each block of its preconditioner is an inverted 3 × 3 matrix, composed of the residual-functional second derivatives, calculated for one spatial point. For such approximation one has to account for the interconnections of the tree parameters at this point, while all the other spatial nodes can be neglected (Beydoun and Mendes 1989). In case, all the simple preconditioning methods listed are not effective, one can opt for more complex approaches such as incomplete LU-factorization, incomplete Cholesky factorization and approximate inverted triangular decomposition.

The full wave inversion uses a nonlinear method of conjugate gradients or the BFGS (Broyden–Fletcher–Goldfarb–Shanno, (Mulder and Plessix 2004; Metivier *et al*. 2012), a quasi-Newton method to minimize the residual functional. The last together with its modification (L-BFGS) can be applied to find solutions for linear algebraic systems as the equation (4). Moreover in (Nazareth 1979) it has been shown that a preconditioned method of conjugate gradients is a special case of BFGS. The difference has been that BFGS renovates inverted Gesse matrix approximation at each of the iterations, while in the CG method, as a rule, uses a constant approximated matrix. On the other hand BFGS is very sensitive to initial approximation of the inverted Gesse matrix that, in this case, is interpreted as a preconditioner matrix. Hence, any conclusion about optimal CG- method preconditioning can easily generalized for BFGS. That’s why such research is important not only for the linearized approach, but also for the nonlinear inversion.



Time, s Source position, m

Figure 1 *Example of a wave field scattered on density heterogeneity (vertical component of displacement vector).*

Numerical results

In the experiments one used a model of point scatterer in a reference homogeneous isotropic medium with receivers on the half-space surface with the medium density ρ = 2 g/cm^3, the P-wave velocity α = 2 km/s and the S- wave velocity β = 1.2 km/s. The scatterer presented a point perturbation of one of the elastic parameters with singular reflectivity, i.e. the parameter’s value at the perturbation point exceeded 2 times the reference one. One considered three types of the point scatterer with coordinates xp = 0, yp = 0 and zp = 0.75 km: the first one was the density perturbation; second- the impedance perturbance of P-waves $Z\alpha$

= ρα and third- the impedance perturbance of S-waves Zβ = *ρβ*. In total, 152 sources were modeled on the surface, all of them located in the same plane *y* = 0 km, as the scatterer. That's why in the experiment only *P*- and *SV*-waves were utilized. The sources were located along x-axis between 1.8875 and 1.8875 km with 0.025- km lag. The 153- receiver profile was located on the surface as well, along x-axis between - 1.9 and 1.9 km with the same lag. The experiment was limited to a vertical-force source, so one used only the vertical component of the seismic data modeled. The scattered wave field was built in the frequency domain using the Grin 3D function for homogeneous medium (Wu and Aki 1985). As the source function one applied the Ricker signal with the central frequency of 15 Hz. To model the wave field 166 discrete frequencies were used from 0 to 42 Hz.

In Fig. 1 you can see an example of the wave field scattered on density heterogeneity. The figure shows its vertical component of the displacement vector. To estimate the medium model in the vicinity of the scatterer, in the work plane *y* = 0 km one selected a 0.4 by 0.4 – km zone with the discretization of 0.01 km and corresponding grid size of 41 × 41. Computation of the gradients and Gesse matrix for this grid was performed using 32 data flows of the SMP cluster of Saint-Petersburg State University. The computation time was around 14 hours. The pseudoinversion was performed using singular decomposition (Golub and Kahan 1965) with standard cutoff threshold.

In Fig. 2 one can see a table of nine components of the residual functional gradient. Each component is a derivative for one of the elastic parameters, presented for each point of the spatial domain. The columns correspond to the three cases of the initial model, i. e. when the values $\tilde{\rho}$, $\tilde{Z}_\alpha$ и $\tilde{Z}_\beta$ at the scatterer point (*xp*, *yp*, *zp*) are equal to one alternately. The rows show the corresponding gradient components – derivatives for $\tilde{\rho}$, $\tilde{Z}_\alpha$ and $\tilde{Z}_\beta$. The value over each image is the derivative maximum amplitude in absolute value. The colored scale makes it easy to distinguish the positive (red color) and negative (blue color) gradient values.

In Fig. 2 you can see that the non-zero values are concentrated mainly in the central area in the vicinity of the scatterer. At the same time, all the three components of the gradient are simultaneously and equally sensitive to perturbation of any of the elastic characteristics. It means the three parameters are interrelated. The reason for it is that correlated wave fields are similar in their structure, so perturbation of one of the parameters is reflected in the image of each of them. Moreover, P-and S-waves, produced by the source and scatterer are mutually correlated when calculating the gradient, giving birth to fault images (Hak and Mulder 2007).

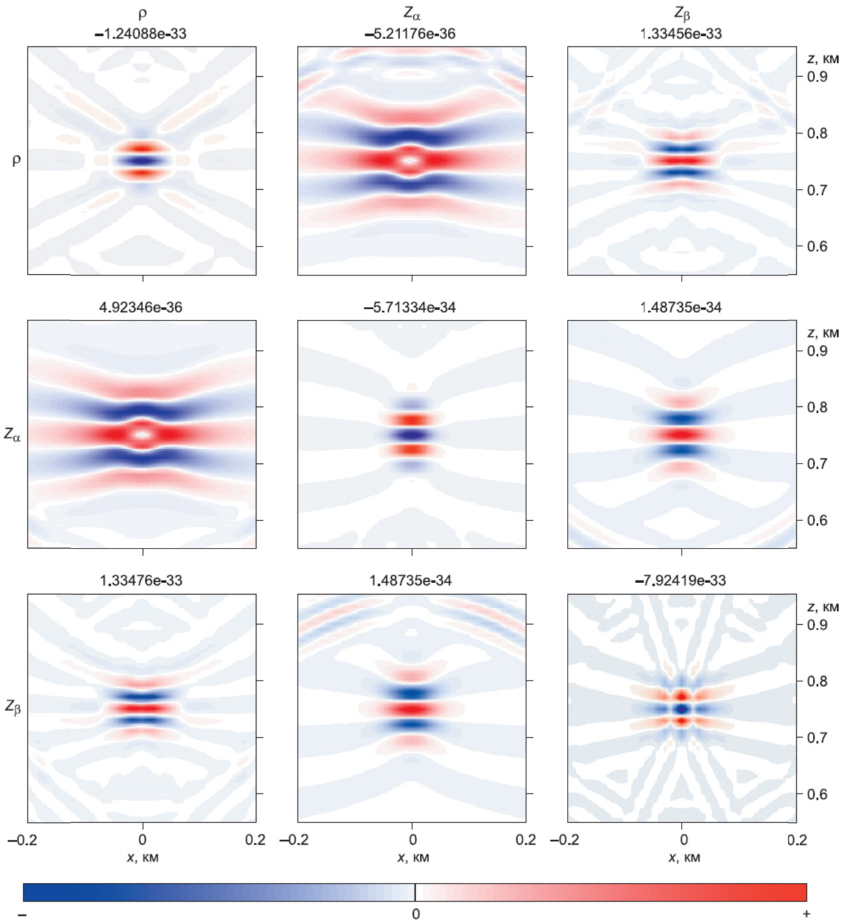


Figure 2 *Residual functional gradient for the three cases of initial model. The column captures show, which of the elastic parameters demonstrates the true perturbation. The row captures show gradient components, i. e. The parameters used for differentiation.*

In case of the linearized inversion, when the equation (4) is solved using the pseudoinverted Gesse matrix, the interrelation of the parameters is less expressed. It can be seen in Fig.3, showcasing restoration of the model parameters by Newton's method. The columns here correspond to the initial models as well. The rows demonstrate the restored reflection capacities $\tilde{\rho}$, $\tilde{Z}_\alpha$ и $\tilde{Z}_\beta$. The color diagram is similar to one in Fig. 2. The value over each image is maximum reflection capacity in absolute value. If there was no mutual effect, the diagonal amplitudes would be equal to one and non-diagonal ones – to zero.

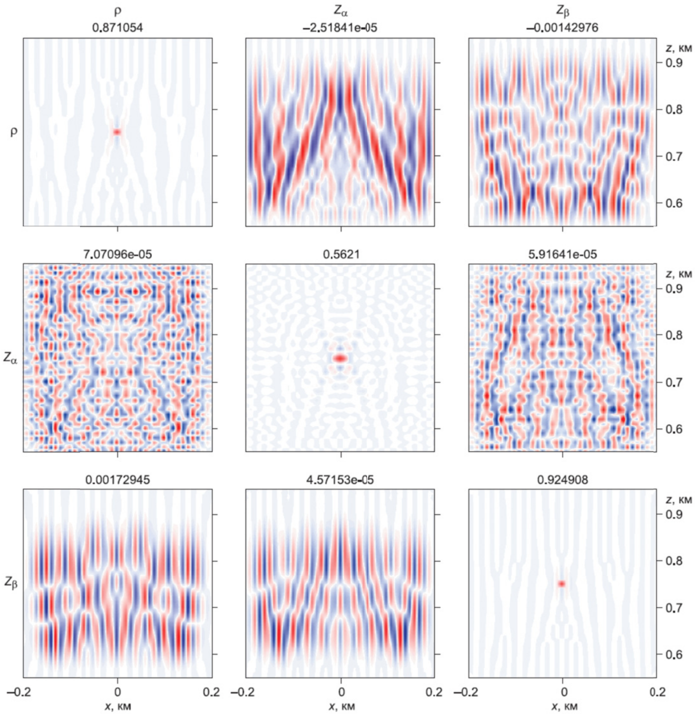
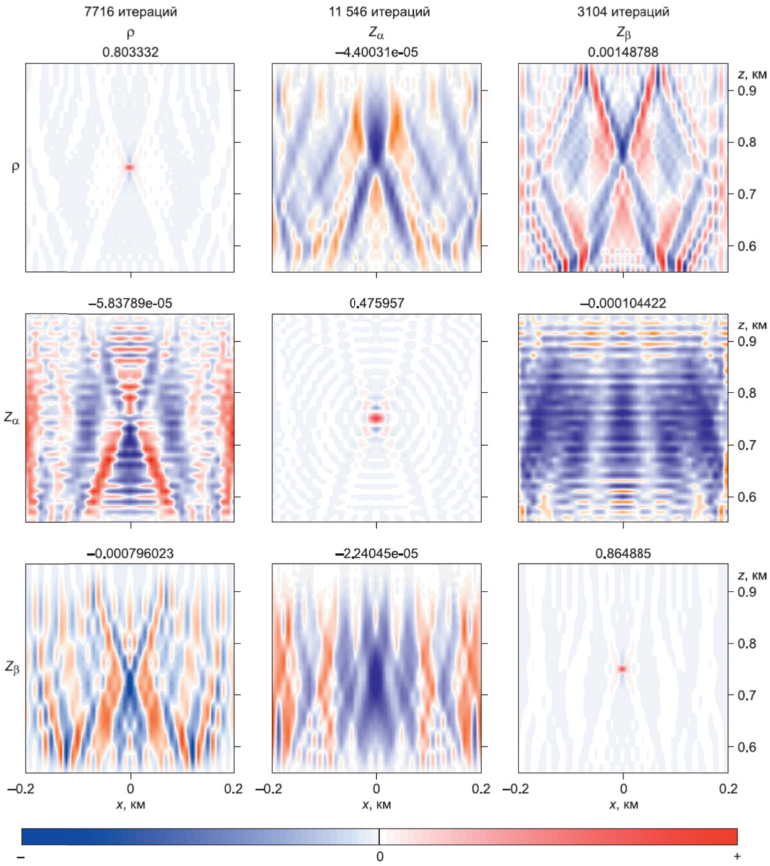


Figure 3 *Inversion results by Newton's method and the pseudoinverted Gesse matrix. The column captures show, which of the elastic parameters has the true perturbation. The rows correspond to the reconstructed parameters.*

In order to estimate interconnection of the elastic parameters the following qualitative characteristic has been introduced:

$$Q_a = \frac{|\tilde{M}_a| - \max\left(|\tilde{M}_b|, |\tilde{M}_c|\right)}{|\tilde{M}_a|} \cdot 100\ \%,$$

where $\tilde{M}_a$ is the maximum reflection capacity of a with true perturbation; $\tilde{M}_b$ и $\tilde{M}_c$ are the maximum reflection capacities of the two nonperturbed parameters b and c. Fig. 3 demonstrates that the parameters $\tilde{\rho}$ and $\tilde{Z}_\beta$ are interrelated, while $\tilde{Z}_\alpha$ does not effect that much on $\tilde{\rho}$ and $\tilde{Z}_\beta$. The qualitative characteristic, determined by the equation (6) gives: $Q_\rho \approx 99.8$, $QZ_\alpha \approx 100$ и $QZ_\beta \approx 99.8$ %. To obtain appropriate accuracy by the CG method one requires bigger amount of iterations.



Итераций –iterations Км-km

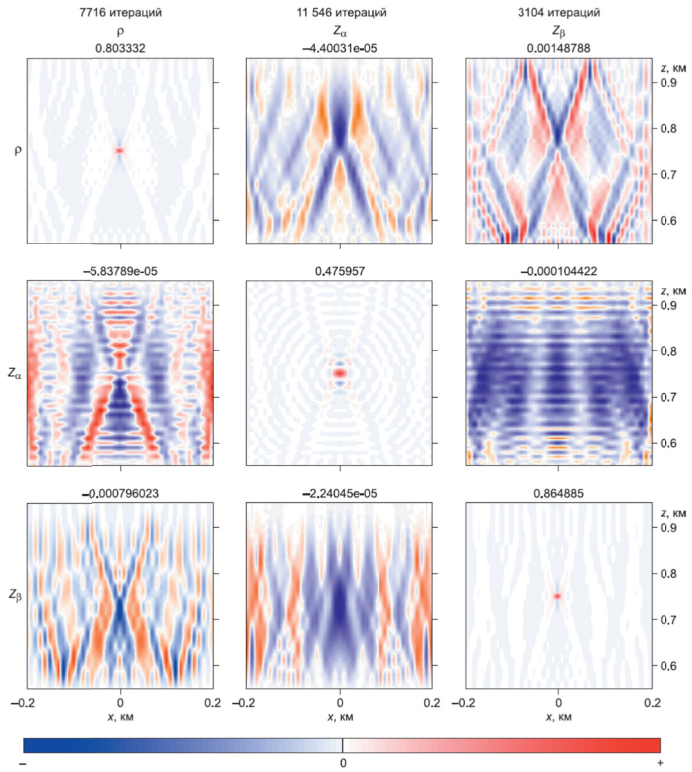
Figure 4 *Inversion results by the CG method. The column captures show, which of the elastic parameters demonstrates the true perturbance. The rows correspond to the reconstructed parameters.*

The approximate upper boundary for the number of iterations necessary can be determined with (Капорин 2011, 2012):

$$n \le \log_2 K + \log_2 \varepsilon^{-1}, \qquad (7)$$

where ε sets required reduction of Euclidean residual norm (Капорин 2012), while K is the so-called K- conditioning number of the matrix H, which has a number of advantages compared with the standard spectral conditioning number (Капорин 2011; Axelsson 1996). In the case considered an estimation gives no more than 60 000 iterations for ε = 10–6.

The inversion results by the CG method can be seen in Fig. 4. The value over each column indicates the required number of iterations for each corresponding model: 7716 iterations for $\tilde{\rho}$, 11 546 iterations for $\tilde{Z}_\alpha$ and 3104 iterations for $\tilde{Z}_\beta$, which has been a good match to the estimation given. In terms of quality, the solution obtained is comparable to one, obtained by Newton's method: $Q_\rho \approx 99.9$, $QZ_\alpha \approx 100$ и $QZ_\beta \approx 99.8$ %.



iterations

Figure 4 *CG-inversion results. The column captures show which of the elastic parameters demonstrates the true perturbance. The rows correspond to the reconstructed parameters.*

Such a big number of iterations have been due to the poor conditioning of the initial problem. But its practical application is possible only when one has to perform a few, but not thousands of iterations. That is why it is very important to estimate the quality of the solution obtained at initial iterations. For that purpose one has applied a criterion, connected with the qualitative estimation, determined by the equation (6): the solution has been at adequate quality level, if no iteration is needed at $Q \geq 95$ %. In Fig. 5 you can see the inversion results after application of the criterion. Hence, to obtain the adequate quality the CG-solution has required 39 iterations in the case of true perturbation $\tilde{\rho}$, 5 iterations- in the case of $\tilde{Z}_\alpha$ and 14 – in the case of $\tilde{Z}_\beta$. Based on this result we may conclude that the CG method can give appropriate result for a limited number of iterations. But even 39 iterations is too complex problem in case of realistic model, so there should be an effective way to increase the convergence rate. It can be done with preconditioning. In Fig. 6 one you can see CG-inversion results, preconditioned by the inverted diagonal part of the Gesse matrix. To limit the number of iterations the same quality criterion has been applied. The preconditioning has resulted in the number of iterations reduced to eight in the first case and to one – in the second, while the true perturbation $\tilde{Z}_\beta$ required 56 iterations. Thus, depending on problem conditions, the Jacobi preconditioning can both speed up the convergence and slow it down.

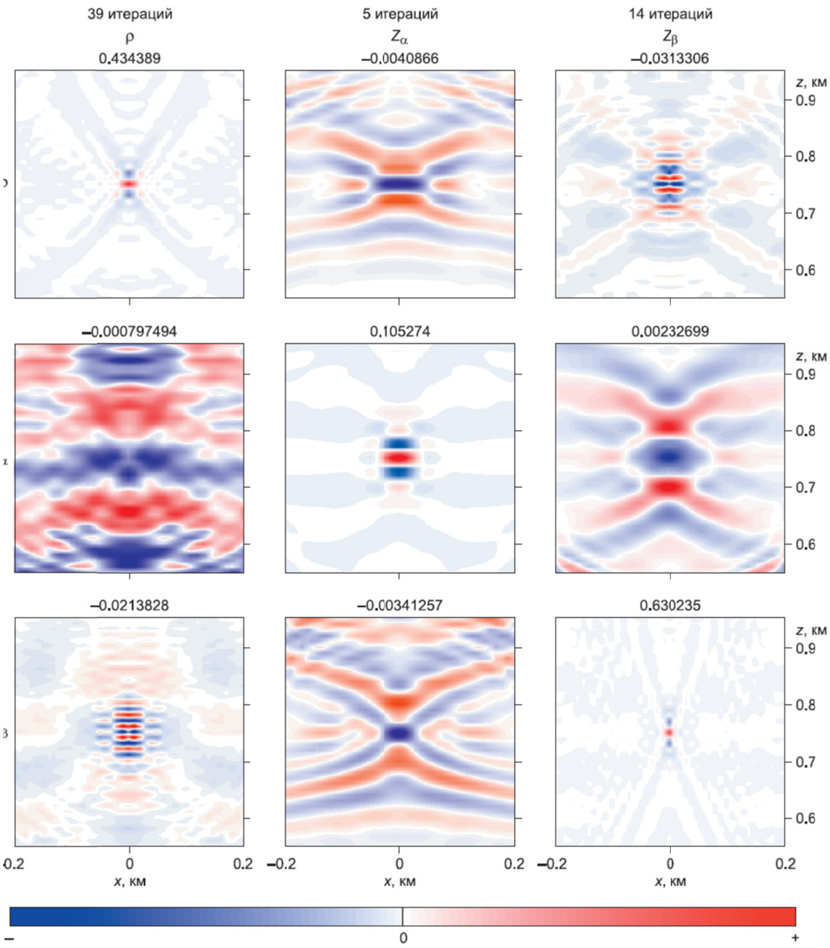


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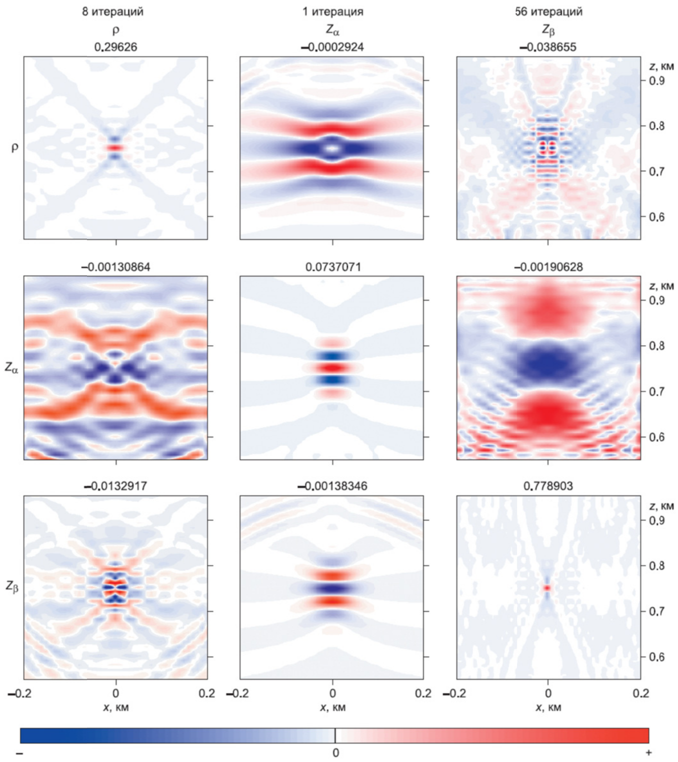
Figure 5 *Inversion results after a few initial CG iterations. For column and row captures see Fig. 4.*



8 iterations

1 iteration

56 iterations

Figure 6 *Inversion results by CG method preconditioned by inverted diagonal approximation of the Gesse matrix. For column and row captures see Fig. 4.*

Analogous situation takes place when one deals with CG preconditioning of inverted block-diagonal approximation of the Gesse matrix. Figure 7 shows the results of such inversion. The number of iterations has reduced to six in the first case and to one –in the second. On the other hand a significant number of iterations have been required in the case of true S-wave impedance perturbance due to the desired solution quality $Q \geq 95$ % has not been obtained (for the third column in Fig. 7 the solution quality has only reached 88%), so the iterations continued until the common convergence criterion was reached. This process was accompanied by explicit mutual effect of the $\tilde{Z}_{\alpha}$ and $\tilde{Z}_{\beta}$. parameters. So a conclusion can be made that the both preconditioners that are based on diagonal and block-diagonal approximation of the Gesse matrix have not been optimal.

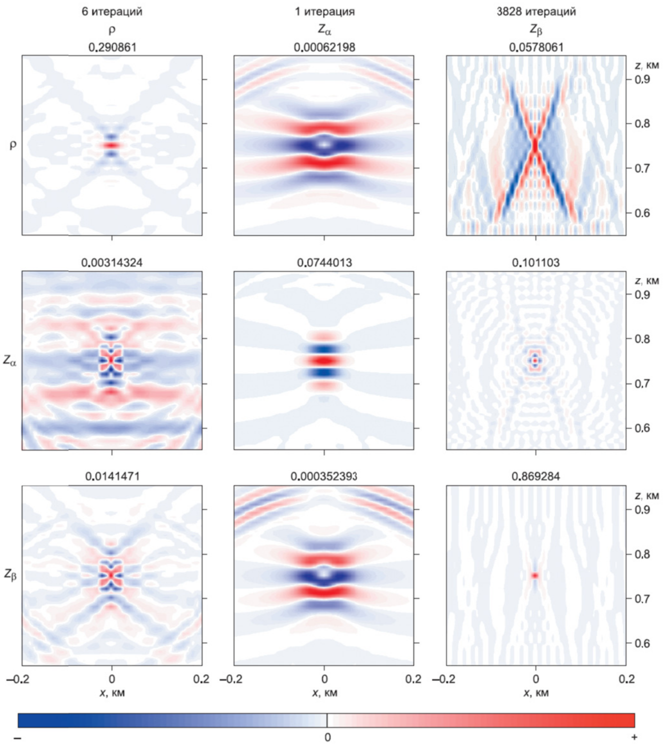


***Figure** 7 Results of CG inversion, preconditioned by inverted block-diagonal approximation of the Gesse matrix. For column and row captures see Fig. 4.*

Conclusions

In this paper a numerical study of interrelation of the three parameters of elastic isotropic medium while migration and inversion has been presented. The iterative linear inversion has proved to be qualitatively different from the classic migration, since the first allows one to estimate not only the spatial location of reflection borders, but also medium's physical parameters. The numerical results of elastic parameters' restoration have been presented by the example of a point scatterer in a homogenous isotropic medium with vertical-force sources and a vertical component of displacement vector as the data for an inverse problem.

Newton's inversion with calculation of the full Gesse matrix followed by its pseudoinversion has allowed one to estimate the maximum possible quality of the final result and showed the three - parameter system have interrelations that are insignificant. So, applying the iteration approach one can obtain an appropriate result of inversion. But for the solution to be of appropriate quality a significant amount of iterations of conjugate elements is required. To improve the convergence rate one has to use an appropriate preconditioning. In the paper it has been shown that the obvious solutions including the inverted diagonal and block-diagonal parts of the Gesse matrix are far from being optimal, since they are strongly dependant on the problem conditions. So, finding optimal preconditioning has been an important scientific problem that determined effectiveness of both linearized and nonlinear inversion. The further study should be aimed at finding an optimal CG preconditioner in the context of a linear iterative inversion, in such a way it could be applied as an effective and accurate tool for estimation of the parameters of elastic isotropic medium.

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