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Developing LRFD Design Specifications for Bridge Shallow Foundations

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ABSTRACT: An ongoing project, supported by the National Cooperative Highway Research Program, NCHRP Project 24-31 is aimed to develop LRFD procedures and to modify the current AASHTO design specifications for Ultimate Limit State (ULS) design of bridge shallow foundations. The current study utilizes a comprehensive database of 549 cases of shallow foundation load tests under various loading conditions (i.e. vertical-centric, vertical-eccentric, inclined-centric and inclined-eccentric). In this paper, the procedure to establish the LRFD design adopted in the research study is introduced. The design methods used for ULS design of bridge shallow foundations are presented and the uncertainty in the estimation of the ultimate bearing capacity has been expressed in terms of a bias, defined as measured over calculated capacities. The biases in the estimation of the ultimate bearing capacity have been appraised based on the database. Typical bridge foundation loadings and their uncertainties are defined and utilized along with the resistance uncertainties to establish resistance factors. The investigations lead to the conclusion that one single resistance factor for the bearing capacity is not sufficient, as different loading conditions result in different levels of uncertainties. Hence, different resistance factors have been established based on the First Order Second Moment (FOSM) method, and the Monte Carlo simulations (MCS), each for the vertical-centric, vertical-eccentric, inclined-centric and inclined-eccentric loading conditions. The recommended preliminary resistance factors thus obtained in the study are presented.

1 INTRODUCTION

An ongoing project, NCHRP Project 24-31: LRFD design specifications for shallow foundations, is aimed at developing LRFD procedures and modifying the current AASHTO design specifications for the Ultimate Limit State (ULS) design of bridge shallow foundations. It is supported by the National Cooperative Highway Research Program (NCHRP) under the Transportation Research Board (TRB) of the National Academy of Science (NAS). The AASHTO specifications are traditionally observed as a National Code of the US highway practice on all federally aided projects, hence, they influence the construction of highway bridge and other structure foundations across the USA.

The current AASHTO specifications as well as other existing codes based on Load and Resistance Factor Design (LRFD) principles were calibrated using a combination of reliability theory, fitting to ASD (allowable stress design) and engineering judgment. The main objectives of this project therefore are the compilation of a database of load tests on shallow foundations and the calibration of resistance factors based on the reliability analysis of the data to obtain more rational designs with consistent levels of reliability. The challenges for the requirement of the second objective include overcoming generic difficulties applying the LRFD methodology to geotechnical applications, i.e. the evaluation of uncertainty in the geotechnical model incorporating e.g. indirect variability (site or soil parameters interpretation), load dependency of the geotechnical resistance (especially in the case of shallow foundations, where a strict separation between load and resistance is not possible), judgment (e.g. a previous experience), and other similar factors.

2 EVALUATION OF BEARING CAPACITY UNCERTAINTY

2.1 *Database*

This research study utilizes a comprehensive database of load tests on shallow foundations, UML-GTR ShalFound07, for the evaluation of uncertainties in bearing capacity (BC) estimation. It contains 549 cases of load tests, mostly performed in Germany and the USA. It has been compiled from various publications noticeably using four major sources:

(a) ShalDB Ver5.1 (Briaud & Gibbens 1997), (b) Lutenegger & DeGroot (1995), (c) German test database in a set of volumes (e.g. Muhs & Weiss 1972) compiled by DEGEBO (Deutsche Forschungsgesellschaft für Bodenmechanik) and (d) tests carried out or compiled by the University of Duisburg Essen, Germany (some of which are presented in Perau 1995). The database summary is presented in Table 1.

Most cases relate to foundations subjected to vertical-centric loading in or on granular soils. Tests of foundation subjected to combined loadings (vertical-eccentric, inclined-centric and inclined-eccentric) were mainly small scale model tests performed in *controlled soil conditions* (in laboratories using soils of known particle size and controlled compaction).

Table 1. Summary of UML-GTR ShalFound07 database

Foundation type	Predominant soil type				Total
	Sand	Gravel	Mix	Others	
Plate load tests B ≤1m	346	46	2	72	466
Small footings 1 < B ≤ 3m	26	2	4	1	33
Large footings 3 < B ≤ 6m	30	--	1	--	31
Rafts & Mats B > 6m	13	--	5	1	19
Total	415	48	12	74	549

Notes:
"Mix": alternating layers of sand or gravel and clay or silt
"Others": either unknown soil types or other granular materials like loamy Scoria

2.2 *Failure load criterion and measured bearing capacity*

Vesić (1975) suggested the failure (ultimate) load to be the load which corresponds to the point where the slope of the load-settlement curve first reaches zero or a steady, minimum value. The interpreted ultimate loads for different load tests are shown in black dots in Figure 1 (Vesić 1963). In soils with higher relative densities, there is a higher possibility of failure in general shear mode and the failure load can be clearly identified, e.g. for test number 61. There are also cases when the identification of the "minimum value" becomes subjective, according to the soil relative densities D_r, e.g. for test number 64 in the figure. Hence, the interpretation of failure load of a shallow foundation from a load test is complex as the failure modes (general, local or punching) not only depend on the type of soil (categorized according to the relative density or the relative stiffness), but also on the footing embedment and loading type. Except for the case of general shear failure, in which the failure load is clearly defined by a peak in the load-settlement curve, judgment is often required to interpret a unique failure load. The examination of the load test results in the database reveal that the failures for which the load-settlement curves do not show a clear peak prevail, hence the interpretations of the failure loads for footings in most cases become difficult. In addition, many load tests in the database have been found to be unsuitable for use in the present study as they have not been carried out to failure. This is especially the case for larger sized foundations for which failure would be associated with very large loads and excessive displacements well beyond the service limits.

It was found from the comparison of the failure loads for 195 cases of database UML-GTR ShalFound07 that the Minimum Slope failure criterion provided the most consistent interpretation when compared to interpreted loads using two other methods: log-log load-settlement curve method (De Beer 1967) and two-slope criterion described in NAVFAC (1986). Hence the measured bearing capacity has been established using the Minimum Slope failure criterion.

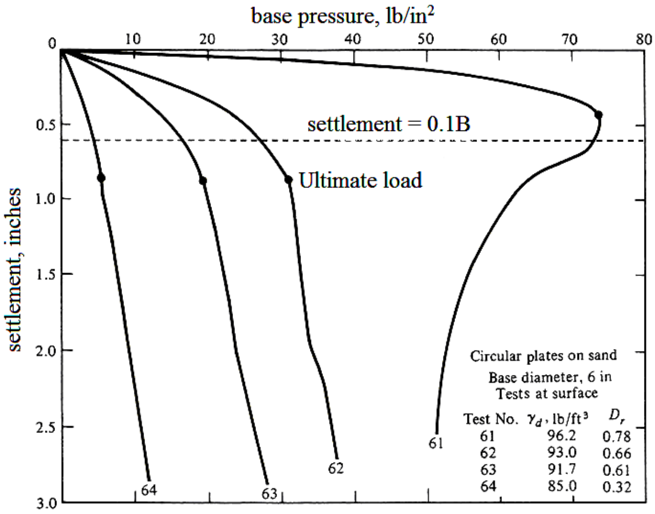

Figure 1. Failure criterion based on the minimum slope of the load-settlement curve (Vesić 1963; modified to show settlement of 0.1B); 1psi ≈ 6.9kPa, 1pcf = 0.157kN/m^3, 1in = 25.4mm

A total of 267 load test cases in/on granular soils could be used for the evaluation of uncertainties in the bearing capacity employing the Minimum Slope failure criterion. Out of the 267 cases, 172 foundations are under vertical-centric, 42 under vertical-eccentric, 39 under inclined-centric and 14 under inclined-eccentric loadings.

Fourteen out of the 172 footing cases under vertical-centric loadings are in *natural soil conditions* (for which SPT blow counts are available) from 8 sites. The average friction angles and unit weights are in the ranges 29.8° to 39° and 14.5kN/m^3 to 19.7kN/m^3, respectively. There are 158 cases in soils with *controlled soil conditions* in 7 sites. The average friction angles of these soils range from 34.6° to 46.0°, with most cases lying between 42° and 46°.

The average unit weights range from 10.2kN/m^3 to 18.4kN/m^3. The width of footings under vertical-centric loading ranges from 5cm to 3m, with nearly half of the footings of size 9cm × 9cm; 104 of the footings are square, 63 are rectangular and 5 are circular.

The load tests under all other loadings, i.e. vertical-eccentric, inclined-centric and inclined-eccentric, have been carried out in *controlled soil conditions*. The averages of the soil friction angles for vertical-eccentric, inclined-centric and inclined-eccentric load tests are 42.0°, 43.4° and 44.9°, respectively, while the averages of the soil unit weights are 16.8kN/m^3, 17.2kN/m^3 and 17.4kN/m^3, respectively. The footing widths range from 5cm to 1.0m for vertical-eccentric and inclined-centric load tests while the inclined-eccentric load tests are carried out on footings 9cm wide.

The loadings in the inclined-eccentric load tests have been applied in two ways: (a) in radial load path and (b) in step-like load path. In the radial load path, the load components are mutually increased such that both the load eccentricity and the load inclination angle are kept constant until failure. In the step-like load path, the vertical loading is kept constant while the horizontal loading is gradually increased till failure. Hence, for the step-like path loading, the failure load can be interpreted from the load-displacement curve for the horizontal direction only. In addition, for the case of inclined-eccentric loading, depending on the relative directions of load eccentricity and inclination as shown in Figure 2, the bearing capacity of the foundation may increase or decrease. The upper combination in Figure 2 is the case of positive or reversible moment, and the lower is the case of negative moment. The foundation BC is greater in the case of positive moment as compared to that with a negative moment for the same magnitude of load eccentricities and inclinations.

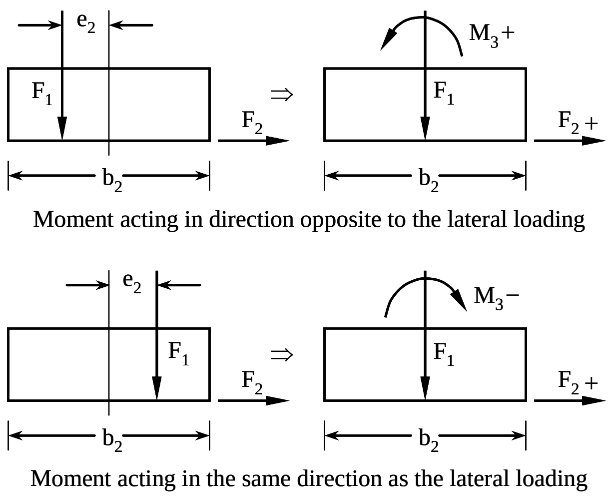

Figure 2. Positive moment (upper) and negative moment (lower) for inclined-eccentric loadings.

2.3 *Calculated bearing capacity*

The equation specified in AASHTO (2007) given in Equation 1, which is based on Vesić (1975), has been used to calculate the bearing capacity of a footing of length L and width B supported by a soil with cohesion c, average friction angle ϕ_f and average unit weight γ.

$$q_u = cN_{cm} + \gamma D_f N_{qm} C_{wq} + 0.5\gamma B N_{\gamma m} C_{w\gamma} \tag{1}$$

where q_u is the calculated bearing capacity, $N_{cm} = N_c s_c i_c$, $N_{qm} = N_q s_q d_q i_q$, $N_{\gamma m} = N_\gamma s_\gamma i_\gamma$ and C_{wq} and $C_{w\gamma}$ are the reduction coefficients for the presence of groundwater. For the depth of groundwater table from the ground surface $D_w = 0.0$, $C_{wq} = 0.5$ and $C_{wq} = 1.0$ when D_w = footing embedment depth (D_f) or below. $C_{w\gamma} = 0.5$ for $D_w \le D_f$, and 1.0 when D_w is greater than $1.5B + D_f$, with the values for intermediate groundwater location depths interpolated.

For granular soils, $c = 0$ and hence only the terms with N_{qm} and $N_{\gamma m}$ in Equation 1 come into the picture. The equations for the bearing capacity factors N_q and N_γ, based on proposals by Reissner (1924) and Vesić (1973, 1975) are respectively.

$$N_q = \exp\left(\pi \tan\phi_f\right)\cdot \tan^2\left(45 + \phi_f / 2\right) \tag{2}$$

$$N_\gamma = 2\left(N_q + 1\right)\cdot \tan\phi_f \tag{3}$$

The shape factors s_i used in the present calculation are those proposed by De Beer (1961, 1970) and Vesić (1973), which are also used in AASHTO (2007).

$$s_q = 1 + (B/L)\cdot \tan\phi_f \tag{4a}$$

$$s_\gamma = 1 - 0.4(B/L) \tag{4b}$$

For depth factors d_i, it is logical to use a consistent set of equations given by the same author, at present given by Vesić (1973, 1975). Hence, the proposal by Brinch Hansen (1970) and Vesić (1973) for depth factor d_q are used instead of the discrete values provided in AASHTO (2007).

$$\begin{aligned} d_q &= 1 + 2\tan\phi_f \left(1 - \sin\phi_f\right)^2 \cdot \left(D_f / B\right) && \text{for } D_f / B \le 1 \\ d_q &= 1 + 2\tan\phi_f \left(1 - \sin\phi_f\right)^2 \cdot \tan^{-1}\left(D_f / B\right) && \text{for } D_f / B > 1 \end{aligned} \tag{5}$$

For inclined loading cases, the following load inclination factors i_i proposed by Vesić (1975) have been used.

$$i_q = \left(1 - \frac{H}{\left(V + B\cdot L\cdot c\cdot \cot\phi_f\right)}\right)^n \tag{6a}$$

$$i_\gamma = \left(1 - \frac{H}{\left(V + B\cdot L\cdot c\cdot \cot\phi_f\right)}\right)^{n+1} \tag{6b}$$

where H and V are the horizontal and vertical components of the applied inclined load P, c is soil cohesion; and

$$n = \left[\frac{(2+L/B)}{(1+L/B)}\right]\cos^2\theta + \left[\frac{(2+B/L)}{(1+B/L)}\right]\sin^2\theta \quad (6c)$$

where θ is the projected direction of load in the plane of the footing, measured from the side of length L in degrees. In case of eccentric loading, the effective footing dimensions $L' = L - 2e_L$ and $B' = B - 2e_B$ are are to be used in Equations 1 through 6 instead of the full footing dimension $B \times L$.

For the bearing capacity calculations, the soil strength parameters are taken as the weighted averages of the strength parameters to a depth of twice the footing width, below the footing base. For footing cases with missing reported soil parameters, correlations of the parameters with SPT values have been used for estimation. A correlation given by Peck, Hansen and Thornburn as modified by Kulhawy & Mayne (1990) has been used to estimate the soil friction angles, while for the soil unit weight, a correlation proposed by Paikowsky et al. (2005) has been used as given by Equations 7 and 8, respectively.

$$\phi_f \approx 54 - 27.6034 \cdot \exp\left(-0.014 (N_1)_{60}\right) \quad (7)$$

$$\gamma = 0.138 (N_1)_{60} + 15.54 \;\; (\mathrm{kN/m^3}) \quad \text{for } \gamma \le 22.9 \mathrm{kN/m^3} \quad (8)$$

where the corrected values of N_{60} for overburden, $(N_1)_{60}$, have been obtained based on the proposal by Liao & Whitman (1986):

$$(N_1)_{60} = \sqrt{\frac{p_a}{\sigma'_v}} \cdot N_{60} \quad (9)$$

where p_a is the atmospheric pressure (≈100kPa or 1tsf) and σ'_v is the effective overburden pressure in the same unit as that of the atmospheric pressure.

2.4 *Summary of mean bias values*

The uncertainties in the aforementioned design method are expressed as a bias, defined as the ratio of the measured over the calculated bearing capacities. This lumped value includes all sources of uncertainties in the BC prediction such as the model uncertainties (e.g. BC factors, foundation scale effects etc), variation of soil properties and their interpretation, capacity interpretation etc. The biases in this present research have been studied according to the loading types, namely vertical-centric, vertical-eccentric, inclined-centric and inclined-eccentric loadings as well as the nature of the soil, differentiating between *natural* and *controlled soil conditions*.

Vertical-Centric Loading:

The mean and coefficient of variations (*COV*) of the biases are calculated for the 172 total vertical centric loading cases. Figure 3 summarizes the mean of bias grouped by test soil conditions and footing widths. The mean bias for the footings in *natural soil conditions* is found to be around 1.0 irrespective of the footing sizes (the largest footing tested being of 3.0m width). In contrast, for the footings in *controlled soil conditions* the mean bias value is about 1.7 to 2.2. For footings in controlled soil conditions, there is less variation in the biases for small footings ($B \le 0.1$m) as compared to the larger footings, even though the tests are from a larger number of sites. Compared to the tests in controlled soil conditions, the biases for those in natural soil conditions have higher variation even when the number of sites is comparable. The higher mean bias reflect conservatism (under-prediction) in the theoretical prediction of the BC factor N_γ by Vesić (1973), which was found to represent the lower bound of the back-calculated values from the load tests especially in the range of soil friction angles between 42° and 46°. The bias of N_γ (back-calculated from tests over Vesić) is found to increase from 0.95 to 2.18 for an increase in the friction angle from 42° to 46°. In both the natural and controlled soil conditions, it was found that there is a trend of increase in the bias with an increase in the footing size. It may be hence logically stated that the evaluation of uncertainties for small scale model tests to failure are valid for prototype large footings too. This inference is of importance especially for the cases under combined loadings, for which the testing on large scale models are limited by economical and practical reasons.

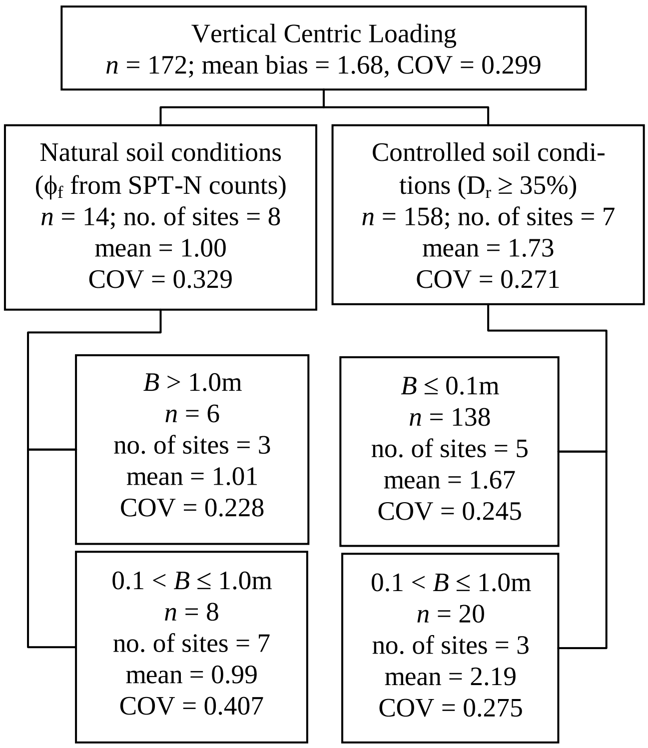

Figure 3. Summary statistics of the bias (measured over calculated BC) for footings under vertical-centric loadings.

Other Loadings:

For the 42 cases of vertical-eccentric loadings, the bias is found to have a mean of 1.81 and *COV* of 0.349. For the 39 cases of inclined-centric loadings, the bias is found to have a mean of 1.43 and *COV* of 0.295. There are 8 cases of positive or reversible moment for inclined-eccentric loadings and 6 cases of negative moment; the mean and *COV* for the former is found to be 1.41 and 0.278, respectively, and for the latter 2.03 and 0.094, respectively.

3 RELIABILITY ANALYSES

3.1 *Typical bridge foundation loading and load factors*

The loading condition has been taken as that used by Paikowsky et al. (2004) in establishing the LRFD for deep foundations. The load combination defined as Strength I in AASHTO is applied as follows in its primary form.

$$Z = R - D - LL \tag{10}$$

where R is the resistance or bearing capacity of shallow foundation, D is the dead load and LL is vehicular live loads. The statistical characteristics of the random variables D and LL are assumed to be as those used in NCHRP Report 368 (Nowak, 1999). The load factors, γ_L for live load and γ_D for dead load from AASHTO (2007) (Tables 3.4.1-1 and 3.4.1-2), and the statistical characteristics used are as given below.

$\gamma_L = 1.75$ $\lambda_{QL} = 1.15$ $COV_{QL} = 0.2$

$\gamma_D = 1.25$ $\lambda_{QD} = 1.05$ $COV_{QD} = 0.1$

Further, Paikowsky et al. (2004) examined the influence of the dead load to live load ratios demonstrating very little sensitivity of the resistance factors to that ratio, with overall decrease of the resistance factors with the increase in the dead load to live load ratio. The large dead-to-live load ratios represent conditions of bridge construction, typically associated with very long bridge spans. The relatively small influence of the dead-to-live-load ratio on the resistance factor lead Paikowsky et al. (2004), thereby, to use a typical ratio of 2.0 knowing that the obtained factors are by and large applicable for long span bridges being on the conservative side. This ratio was adopted, therefore, for the present study calibrations as well.

Equation 10 above does not include the effects of the horizontal earth pressure, hence, the resistance factors developed and presented in this paper do not include the uncertainties due to the horizontal earth pressure. Except for the footings under vertical-centric loading, the resistance factors would likely to be different especially for the cases under the inclined-eccentric loadings when the lateral loading due to horizontal earth pressure is also considered.

3.2 *Resistance factor calibration*

The preliminary resistance factors based on the evaluation of biases in the ultimate limit estimation of shallow foundations in/on granular soils, presented in the previous section, have been calibrated for a target reliability β_T of 3.0 or target exceedance probability of 0.135%, assuming lognormal distributions for loads and resistance.

The resistance factors obtained from the FOSM (original AASHTO calibration procedure, Barker et al. 1991) and from the Monte Carlo simulation using 500,000 simulations, along with the recommended factors, for shallow foundations under the different loading types are presented in Table 2.

4 SUMMARY AND CONCLUSION

The NCHRP 24-31 research project aims to develop LRFD procedures and to modify the current AASHTO design specifications for the Ultimate Limit State (ULS) design of bridge shallow foundations. The research study utilizes a comprehensive database of

Table 2. Statistical details of the biases of the bearing capacities of shallow foundations in/on granular soils and resistance factors under different loadings.

Loading type		Underlying soil conditions	No. of cases	No. of sites	Mean bias λ	COV_λ	Resistance factor ϕ		
							MCS	FOSM	Recommended
Vertical-centric		Controlled	158	7	1.73	0.271	0.937	0.793	0.90
		Natural	14	8	1.00	0.329	0.457	0.396	0.45
Vertical-eccentric		Controlled	42	4	1.81	0.349	0.779	0.680	0.75
Inclined-centric		Controlled	39	3	1.43	0.295	0.722	0.617	0.70
Inclined-eccentric	Positive or reversible moment	Controlled	8	1	1.41	0.278	0.748	0.635	0.70
	Negative moment	Controlled	6	1	2.03	0.094	1.773	1.318	1.00

Note: λ = bias = measured over predicted COV_λ = coefficient of variation of the bias
MCS= Monte Carlo Simulation using 100,000 iterations
FOSM = First Order Second Moment

load tests to establish the uncertainty in bearing capacity (BC) calculation. The failure loads in the model tests have been determined by different failure criteria, among which the minimum slope criterion proposed by Vesić (1963) and was employed for the database load tests interpretation. This criterion was established as the most appropriate one to identify the failure load. The uncertainties in the design method were expressed by the bias defined as the ratio of measured over calculated BCs. This lumped value includes all sources of uncertainties in the BC prediction originating from the model (e.g. BC factors, scale effects), variation in soil properties, etc.

The bearing capacity analysis of shallow foundations on controlled soil conditions was found to systematically under-predict the measured capacity for all examined load combinations, namely; vertical-centric, vertical-eccentric, inclined-centric and inclined eccentric. This under-prediction results with the bias varying between 1.4 and 2.0 with *COV* values of approximately 0.3 (excluding the limited cases of inclined-eccentric loading under negative moment).

Investigation of the bearing capacity factor N_γ suggests that a similar bias exists in this factor assuming all other factors are known and measured. The bias in N_γ was found to be related to the soil internal friction angle, where the bias increases (i.e. BC under-prediction increases) with the increase in the internal friction angle. The controlled soil conditions of the examined cases suggest that the bias in the BC factor N_γ may explain in a large part the uncertainty in the BC calculations. This statement and its validity is currently further evaluated.

The findings related to the foundations tested in/on natural soils is more difficult at it introduces larger variability in the soil type and strength parameters interpretation. It is noticeable however that the internal friction angles associated with all of the tested cases on natural soils resulted with internal friction angles lower than those for controlled soil and hence match the area for which the BC parameter N_γ did not exhibit a bias. Though qualitatively these observations match the data analyzed, the implementation of the indicated findings for natural soil condition is thus incomplete at this stage.

In both the natural and controlled soil conditions, however, there was a trend of increase in the bias with an increase in the footing size. It may be hence logically stated that the evaluation of uncertainties for small scale model tests to failure are valid for prototype large footings too. This inference is of importance especially for the cases under combined loadings, for which the testing on large scale models are limited by economical and practical reasons.

The initial investigation leads to the conclusion that one single resistance factor for the bearing capacity is not sufficient as different soil and/or loading conditions result in different level of uncertainties. Hence, different resistance factors were established based on probabilistic analyses (FOSM and Monte Carlo simulations), each for vertical-centric, vertical-eccentric, inclined-centric and inclined-eccentric loading conditions.

DISCLAIMER AND ACKNOWLEDGEMENTS

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