Characteristic shear strength values for EC7: Guidelines based on a statistical framework

Valeurs caractéristiques de la résistance de cisaillement pour le EC7: Les directives basés sur un cadre statistique

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ABSTRACT

Geotechnical performance of a structure is generally governed by spatial average soil properties, such as the average shear strength along a potential slip surface in a slope. In Eurocode the characteristic soil value is defined as "a cautious estimate of the value affecting the occurrence of the limit state". In addition it is stated that the selection of this value should be based on, among other factors, "the extent of the zone of ground governing the behavior of the geotechnical structure at the limit state being considered". A statistical framework is presented to rationally assess the characteristic values according to Eurocode. Three statistical properties are required to quantify the characteristic shear strength values along a failure surface: the arithmetic mean, the variance and the scale of fluctuation. Simplified formulae for the determination of characteristic values are presented along with guidelines of typical input parameters. Stiff soils such as overconsolidated clays and clay shales exhibit brittle stress-strain behavior and are prone to progressive failures due to strain-softening. The shearing resistance first increases and then decreases with increasing displacements, and, as a result, the peak shear strengths of the materials at all points along a slip surface cannot be mobilized simultaneously. For these complex conditions, a pragmatic approach to estimate the characteristic values has been given. In addition, the apparently different definitions of characteristic values for construction materials such as concrete and soils have been presented and explained in a unified approach.

RÉSUMÉ

La performance géotechnique d'une structure est régnée par la moyenne spatiale des propriétés du sol, comme par exemple la moyenne de la résistance de cisaillement le long d'une surface glissante dans un versant. Dans l'Eurocode les valeurs caractéristiques du sol sont définies comme "une estimation prudent de la valeur, qui concerne l'apparition de l'état limité". En plus c'est spécifié que la sélection de cette valeur devrait être base, entre autres facteurs, sur "l'implication de l'extension de la zone du sol régné du comportement de la structure géotechnique dans l'état limité." Un cadre statistique est ainsi présenté pour évaluer logiquement les valeurs caractéristiques suivant l'Eurocode. Pour pouvoir quantifier complètement les caractéristiques de la résistance de cisaillement le long d'une surface défectueuse, il y en a besoin de trois propriétés statistiques: la valeur moyenne arithmétique, la variance et l'échelle de fluctuation. Les formules simplifiées pour la détermination des valeurs caractéristiques avec les directives des paramètres entrées sont présentées ci-joint. Sol raide comme par exemple terre glaise trop consolidée et argilardoise montre un comportement de tension et extension (stress-strain behaviour) friable et il sont prédisposés à une défaillance progressive à cause du ramollissement de la tension. La résistance à l'effort tranchant en premier lieu augmente et puis baisse avec une tension qui augmente, et comme conséquence le pic de la résistance de cisaillement des matériaux de touts les points le long d'une surface glissante ne peut pas être mobilisé simultané. Pour ces conditions très complexes, on a donné une approche pragmatique pour estimer les valeurs caractéristiques. Par ailleurs, les définitions qui sont apparemment différentes pour les valeurs caractéristiques pour les matériaux de construction comme béton et sols ont été présentés et expliqués dans une approche unifiée.

Keywords: Eurocode, characteristic values, shear strength, autocorrelation, uncertainties, variance reduction, spatial variability

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1 INTRODUCTION

The geotechnical profession has traditionally used conservative, mostly subjective estimates of design soil properties along with global factors of safety of 1.5 up to 5 or even higher.

The design philosophy of Eurocode has changed this design procedure and requires that the uncertainties in the design process should be considered more explicitly and in a consistent manner.

The safety chain of geotechnical design can roughly be divided into 4 components, each one possessing uncertainty:

- Loads or actions
- Soil properties
- Calculation models
- Appropriate safety level

In this paper only the soil properties; in particular the fundamental "characteristic value" is treated. It is the intention to provide guidelines for practicing engineers how to assess the characteristic shear strength to be used in Eurocode.

2 DEFINITION AND MEANING OF THE CHARACTERISTIC VALUE

2.1 The head Eurocode 1990 §4.2(3) "Basis of Design" states:

"The characteristic value should be defined as the 5% fractile value". According to [1] this definition works well for man-made materials, such as steel and concrete, but fails to account for the remarkable variability of geomaterials.

2.2 Eurocode EC 7"Geotechnical Design": Characteristic values of geotechnical parameters (§ 2.4.5.2)

(2) P: The characteristic value of a geotechnical parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state.

(4) P: The selection of characteristic values for geotechnical parameters shall take account of the following:

- Geological and other background information, such as data from previous projects
- The variability of the measured property values and other relevant information, e.g. from existing knowledge
- The extent of the field and laboratory investigation
- The type and number of samples
- The extent of the zone of ground governing the behavior of the geotechnical structure at the limit state being considered
- The ability of the geotechnical structure to transfer loads from weak to strong zones in the ground

(7) P: The zone of ground governing the behavior of a geotechnical structure at a limit state is usually much larger than a test sample or the zone of ground affected in an in situ test. Consequently the value of the governing parameter is often the mean of a range of values covering a large surface or volume of the ground. The characteristic value should be a cautious estimate of this mean value.

(9) P: When selecting the zone of ground governing the behavior of a geotechnical structure at a limit state, it should be considered that this limit state may depend on the behavior of the supported structure. For instance, when considering a bearing resistance ultimate limit state for a building resting on several footings, the governing parameter should be the mean strength over each individual zone of ground under a footing, if the building is unable to resist a local failure. If, however, the building is stiff and strong enough, the governing parameter should be the mean of these mean values over the entire zone or part of the zone of ground under the building.

(11) P: If statistical methods are used, the characteristic value should be derived such that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%.

In summary it can be stated that the characteristic value should:

- be a 5%-fractile value for any material
- be a cautious estimate of the value affecting the occurrence of the limit state
- Take account of experience as well as lab and field test results
- Depend on the zone or extent of ground governing the behavior of the structure at a limit state

From the above definitions it is difficult to understand the meaning of the characteristic value. Eurocode does not provide more details or rules how to apply the above definitions or how to determine the characteristic values.

The goal of this contribution is to provide some guidance to determine the characteristic shear strength values. It is attempted to fully comply with the above definitions and still present a procedure easily applicable to routine applications.

3 VARIABILITY AND UNCERTAINTIES IN SOIL PROPERTIES

Geotechnical analyses according to Eurocode EC 7 are carried out with design parameters x_d, determined for each homogeneously treated soil layer.

$$x_d = \frac{x_k}{\gamma_M} \quad (1)$$

Partial (safety) factors γ_M are nationally determined and fixed for a soil property such as cohesion or angle of internal friction, independent of x_k and how it has been determined. They ensure a nationally specified safety level or reliability of geotechnical structures.

The characteristic value x_k on the other hand represents the "unknown" mean or average soil value, averaged over the zone of the governing limit state.

It is of fundamental importance to realize that it is the characteristic value x_k only which completely describes the behavior of the soil for a given design situation. The combination of any field and laboratory test results as well as any previous experience (including corrections for uncertainties) produces one single deterministic value, namely the characteristic value x_k.

3.1 *Definition of characteristic value x_k in terms of statistics*

$$x_k = N \cdot x_m \cdot (1 \pm k \cdot COV_{tot}) \quad (2)$$

x_k = characteristic value; x_m = mean value estimated or measured/derived from laboratory or field tests; N = modifier to correct a possible bias in the measured mean; k = factor to account for the 5%-fractile and the corresponding probability distribution; e.g. k = 1.645 for a normally distributed value, COV_{tot} = total coefficient of variation to account for the combined overall variability and uncertainties as well as the spatial extent of the governing limit state under consideration.

$$COV_{tot} = \frac{\text{Standard deviation}_{tot}}{\text{mean}_{tot}} = \frac{S_{x\,tot}}{x_{m\,tot}} \quad (3)$$

3.2 *Sources of uncertainties and variability*

Typically, field or laboratory investigations are carried out on a very small portion of the soil volume influencing the geotechnical structure. The measured laboratory or field data therefore represent the soil properties of a small testing volume. Those measured properties are not necessarily the ones governing the performance at the site. In fact, the pertinent soil property controlling the performance of a geotechnical structure often involves a much larger volume of soil. Hence the domain-average property is needed instead of the property of discrete soil specimens.

[2] summarizes: *Geotechnical performances are governed by spatial average soil properties, such as the average shear strength along a potential slip surface in slope stability analysis or the average compressibility of a volume of soil beneath a footing in settlement calculation. Even in the case where performance may be governed by local properties such as progressive failure of slope or piping failure, it still involves the average soil property over a small domain.*

In any case, the characteristic value x_k is the average soil property over an appropriate spatial domain, large or small; explicitly taking into account the soil variability and uncertainties.

Excellent contributions dealing with soil variability and uncertainties are found in [1], [2], [3], [4], [5], [6], [7], [8] and [11] among others and show the following primary sources of geotechnical uncertainty to be relevant for the assessment of soil properties:

- **Inherent soil variability $COV_{inherent}$**
- **Measurement error $COV_{measurement}$**
- **Statistical error $COV_{statistical}$**
- **Transformation uncertainty resulting from empirical correlations $COV_{transformation}$**

4 CONCEPTUAL FRAMEWORK TO ASSESS CHARACTERISTIC VALUES

Based on the proposed additive models by [2] and [7] the characteristic value is derived to take account of the combined effects of aleatory and epistemic uncertainties as presented in [9], [10] and [13]. The resulting equation (4) is valid for any construction material to determine the characteristic value x_k according to Eurocode. It is derived from equation (2):

$$x_k = N \cdot x_m \cdot \left(1 \pm k \cdot (\Gamma^2 \cdot COV^2_{inherent} + COV^2_{measurement} + COV^2_{transformation} + COV^2_{statistical})^{1/2}\right) \qquad (4)$$

Although there exist several approaches to mathematically describe the variance reduction function Γ^2, [3], [4] and [7] propose to use the following simple form for practical applications:

$$\Gamma^2 = \frac{\delta}{L} \qquad \text{for} \qquad L > \delta \qquad \text{and}$$

$$\Gamma^2 = 1 \qquad \text{for} \qquad L \leq \delta \qquad (5)$$

In which: δ = scale of fluctuation (in m), L = governing failure length/surface or zone governing the limit state (in m). Extensions to 3-D or 2-D [3], [4] situations are straightforward as follows:

$$\Gamma^2_{volume} = \Gamma^2_x \cdot \Gamma^2_y \cdot \Gamma^2_z = \frac{\delta_x}{L_x} \cdot \frac{\delta_y}{L_y} \cdot \frac{\delta_z}{L_z} \qquad (6)$$

with the range of validity as in equation (5) in each direction.

5 SIMPLIFICATION FOR DESIGN PRACTICE

The general equation (4) is valid for any construction material. However, for practical application the equation is not handy and should be simplified considerably without sacrificing much accuracy. Setting N = 1, k = 1.65 (approximately for 5%-fractile of the Normal distribution), $COV_{transformation} = 0$ for negligible transformation model uncertainty and $COV_{statistical} = 0 = COV_{inherent}/\sqrt{n}$, assuming a hypothetically unlimited number of samples n or previous knowledge/experience corresponding to test results from many samples. Field and laboratory tests performed according to standards such as Eurocodes should be accurate enough to set $COV_{measurement} = 0$. Equation (4) for 2-D and Normal distribution simplifies to:

$$x_k \cong x_m \cdot [1 - 1.65 \cdot \Gamma_x \cdot \Gamma_z \cdot COV_{inherent}] \qquad (7)$$

Equation (7) - as a convenient by-product - reduces smoothly to the equation given in Eurocode for the determination of the characteristic value of concrete, because L < δ in the majority of applications with δ ≅ 1m for concrete structures [9].

For soil properties lognormally distributed or for $COV_{inherent} > 0.3$ equation (7) becomes:

$$x_k \cong x_m \cdot \frac{0.2^{\sqrt{\ln\left(1+\Gamma^2_x \cdot \Gamma^2_z \cdot COV^2_{inherent}\right)}}}{\sqrt{1+\Gamma^2_x \cdot \Gamma^2_z \cdot COV^2_{inherent}}} \qquad (8)$$

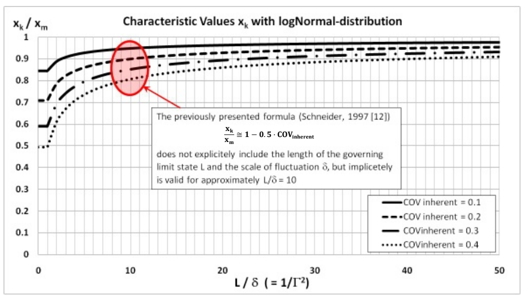

Figure 1: Graphical determination of characteristic soil property for logNormal distribution eq. (8) and $\Gamma_x = \Gamma_y = \Gamma$.

5.1 Application to failure surface

The variance reduction of a failure surface generally depends on geometry and orientation relative to the layering of the natural soil in addition to failure surface length and scales of fluctuation.

An approximate approach to consider geometry and orientation by converting anisotropic variability into an equivalent isotropic scale of fluctuation δ_{yz} is presented by [16]:

$$\delta_{yz} \cong \sqrt{\delta_y \delta_z} \quad (9)$$

The corresponding variance reduction for the failure surface therefore is:

$$\Gamma_x^2 \cdot \Gamma_{yz}^2 \cong \frac{\delta_x}{L_x} \cdot \frac{\delta_{yz}}{L_{yz}} \quad (10)$$

Application to slope stability

Considering a horizontally layered slope and typical spatial variability parameter as given in Figure 2, leads to an equivalent isotropic scale of fluctuation of 10 m based on eq. (9). The total variance reduction on the failure surface calculated with eq. (10) becomes 0.4, based on the limiting condition (5) that each Γ cannot be larger than 1. The normalized characteristic value x_k/x_m can be calculated using eq. (8), replacing $\Gamma_x^2 \cdot \Gamma_z^2$ by $\Gamma_x^2 \cdot \Gamma_{yz}^2$ according to eq. (10), which finally leads to $x_k/x_m = 0.73$.

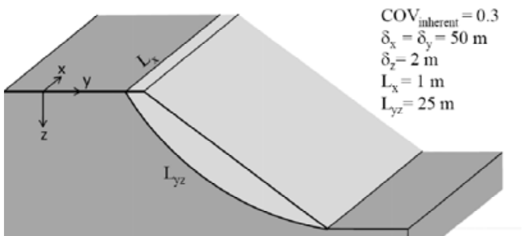

Figure 2: Example for failure surface in slope stability.

5.2 Guidelines for shear strength parameters

Three statistical properties are required to quantify the characteristic shear strength values along a failure surface:

- the arithmetic mean x_m
- the coefficient of variation COV
- the scale of fluctuation δ

The mean and the variance (or equivalently the standard deviation or the coefficient of variation) is known to most geotechnical engineers and should – according to Eurocode – be provided as derived values of each soil property, based on comparable and/or subjective experience as well as supplemented by field- and/or laboratory investigation results.

The scale of fluctuation is not generally known and may be interpreted as distance within which soil properties are largely correlated, whereas for greater distances the soil properties are statistically largely uncorrelated. The determination of the scale of fluctuation is complex and outside the scope of routine soil investigation campaigns. However there is considerable literature on the subject, allowing values of scales of fluctuation to be estimated reasonably well for practical application (see Table 2).

Based on the derived values of shear strength and geometry of the governing limit state, the characteristic value x_k can be assessed with eqs. (7) or (8).

5.2.1 Estimates of x_m and $COV_{inherent}$

Many well documented sources on a worldwide basis show that the coefficient of variation COV, as a measure of scatter in data, is quite stable for any soil property. Thus, even without any test results, a wealth of information is available on x_{m1} and $COV_{inherent1}$ from documented sources in addition to subjective experience and personal judgment. Those values can be supplemented by local test results and updated to obtain improved estimates of x_m and $COV_{inherent}$ of soil properties [12] and [14].

a) first guesses of mean x_{m1} and $COV_{inherent1}$

As a first guess the following procedure based on experience can be used to estimate the values of the mean x_m and the coefficient of variation $COV_{inherent}$ based on estimated minimum a, mean b and maximum value c:

$$x_{m1} \cong \frac{a+4 \cdot b+c}{6} \quad (11)$$

$$COV_{inherent\,1} = \frac{s_x}{x_m} \cong \frac{c-a}{4 \cdot x_{m1}} \quad (12)$$

Alternatively, the typical recommended values of $COV_{inherent\ 1}$ presented in Table 1 could be used.

Table 1: Typical values of $COV_{inherent1}$

Soil Property	Range of typical values of $COV_{inherent1}$	Recommended values $COV_{inherent1}$
Densities	0.01 - 0.10	0
Angle of internal friction	0.05 - 0.15	0.1
Cohesion / undrained shear strength	0.3 - 0.5	0.4

b) Values of x_{m2} and $COV_{inherent2}$ from test results

From n local soil testing results, if available, determine the mean x_{m2} and $COV_{inherent\ 2}$ based on elementary statistics:

$$x_{m2} = \frac{\sum x_i}{n} \tag{13}$$

$$S_{x2} = \sqrt{\frac{\sum(x_i - x_{m2})^2}{n-1}} \tag{14}$$

$$COV_{inherent\ 2} = \frac{S_{x2}}{x_{m2}} \tag{15}$$

c) Updated x_{m3} and COV_{inh3} with Bayes` theorem [2] and [12]

$$x_{m3} = \frac{x_{m2} + \frac{x_{m1}}{n} \cdot \left(\frac{COV_{inh2}}{COV_{inh1}}\right)^2}{1 + \frac{1}{n} \cdot \left(\frac{COV_{inh2}}{COV_{inh1}}\right)^2} \tag{16}$$

$$COV_{inh3} = COV_{inh2} \cdot \sqrt{\frac{1}{n + \left(\frac{COV_2}{COV_1}\right)^2}} \tag{17}$$

5.2.2 *Scale of fluctuation δ from [7]*

Table 2: Typical values of scale of fluctuations δ

	Vertical direction δ_z in m	Horizontal direction $\delta_x = \delta_y$ in m
Range of all soil properties	0.3 – 5.7	44.5 – 50.7
Recommended design values	2	50

5.3 *Characteristic shear strength for stiff soils*

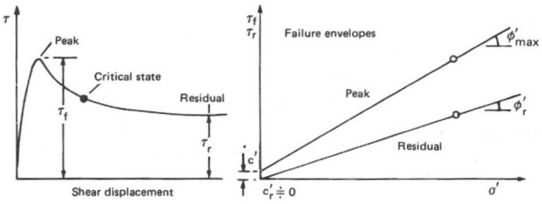

Figure 3: Shear strength of stiff soil (from [15])

To assess the shear strength of stiff soils such as overconsolidated clays or clay shales for design purposes is complex. Figure 4 shows typical shear strength vs. displacement curve of an initially stiff soil sample, reaching the peak strength after some displacements and degrading afterwards until eventually the fully softened condition is reached. In order to assess the characteristic shear strength values for the governing limit state, the displacements of e.g. a slope or retaining wall during the design life should be estimated, which is a very difficult to almost impossible task, because the displacements are caused by several different and combined processes such as

- Decrease of negative pore pressures due to precipitation
- Weathering of the stiff soils
- Stress concentrations along failure surfaces in a slope
- Softening along discontinuities

To estimate the characteristic values of cohesion and angle of internal friction for strain-softening soils, the following pragmatic strategies could be employed:

- For the longtime condition and possibility of progressive failure due to uncontrolled displacements: use residual angle of internal friction and no cohesion along the entire failure surface.
- In case displacements are actively controlled and kept below peak displacements (e.g. by anchors): use peak angle of internal friction and cohesion only, if its availability is proven for the soil mass and the expected climatic conditions during the design life of the structure.

6 SUMMARY AND CONCLUSION

Geotechnical performance of a structure is usually governed by spatial average soil properties, such as the average shear strength along a potential slip surface in a slope.

In EC 7 the characteristic value x_k is the fundamental soil value. Despite its important safety relevance, the definition and determination of the characteristic value is far from clear.

An attempt is made to develop a statistical framework to determine the characteristic value. It is based on the mean value, the standard deviation (or coefficient of variation) and the scale of fluctuation. A simple form of a variance reduction function is presented, which models the reduction of scatter as a result of averaging along the zone of influence of the governing limit state.

It becomes clear that the characteristic value cannot be quantified by a field- and laboratory investigation alone, but is dependent on the spatial extent and the governing failure mechanism to be designed for.

Simplified formulae for the determination of characteristic values are presented along with guidelines of typical input parameters. In addition, the apparently different definitions of characteristic values for construction materials such as concrete and soils have been presented and explained in a unified approach.

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