Slope stability analysis based on autocorrelated shear strength parameters

La stabilité d'un versant basé sur l'autocorrélation de la résistance de cisaillement

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ABSTRACT

The stability of a slope is governed by the spatial average of the shear strength over the extent of the failure surface. In Eurocode the average soil properties are taken into account by defining the characteristic soil parameter as being "a cautious estimate of the value affecting the occurrence of the limit state" and further stating that this value should be based on, among other factors, "the extent of the zone of ground governing the behaviour of the geotechnical structure at the limit state being considered".

To completely quantify the characteristic shear strength along a failure surface, three statistical values are required: the arithmetic mean, the variance and the spatial correlation. The mean soil properties and to a lesser degree the variance (or equivalently the standard deviation or the coefficient of variation) are known and used by most geotechnical engineers for the selection of characteristic soil properties. The spatial correlation, however, is not generally used. It is a measure of the scale of fluctuation, i.e. the range within which soil properties are correlated and beyond which they are statistically uncorrelated.

This paper investigates the influence of the variability of shear strength on the reliability of slopes based on simulated autocorrelated random fields created by the turning bands method. In particular, the influence of the length of the failure surface on the characteristic value is investigated. The numerical Monte Carlo analyses verify the validity of a simplified practical approach presented to determine the characteristic soil properties according to Eurocode.

RÉSUMÉ

La stabilité d'un versant est régnée par la moyenne spatiale de la résistance de cisaillement sur l'extension de la surface coulissante. Dans l'Eurocode une moyenne des propriétés du sol est tenue compte, par définir le paramètre caractéristique du sol comme "une estimation prudent de la valeur, qui concerne l'apparition de l'état limité" et puis, que cette valeur devrait, entre autres facteurs, être basée sur "l'implication de l'extension de la zone du sol régné du comportement de la structure géotechnique dans l'état limité". Pour pouvoir quantifier complètement les caractéristiques de la résistance de cisaillement le long de la surface coulissante, il y en a besoin de trois valeurs: la valeur moyenne arithmétique, la variance et la corrélation spatiale. La valeur moyenne des propriétés du sol, et dans une moindre mesure la variance (ou bien aussi l'écart normal ou le coefficient de variation) sont connus et utilisés d'une majorité des ingénieurs en géotechnique pour sélectionner des propriétés caractéristiques du sol. La corrélation spatiale n'est pas beaucoup utilisée en général. C'est une mesure pour l'échelle de fluctuation, par exemple la marge entre les propriété du sol qui sont en corrélation et au-delà quelles ne sont statistiquement pas en corrélation.

Cette mémoire recherche l'influence de la variabilité de la résistance de cisaillement sur la fiabilité d'un versant, basé sur des prélèvements simulés en autocorrélation avec la *turning bands method.* L'influence de la longueur de la surface coulissant sur la valeur caractéristique est en particulier recherchée. L'analyse numérique de Monte Carlo vérifie la validité d'une approche plus simple et plus pratique, présenté pour déterminer les caractéristiques des propriétés du sol selon l'Eurocode.

Keywords: Slope Stability, Characteristic Value, Eurocode, Spatial Variability, Monte Carlo Analysis, Random Fields

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1 INTRODUCTION

The assessment of slope stability involves the essential task of selecting geotechnical parameters represented by their characteristic values according to Eurocode. No guidance is offered in Eurocode to quantitatively determine characteristic values for practical applications. Therefore considerable difficulties exist in selecting characteristic values by taking account of both, data and model uncertainty.

Data uncertainty relates to the basic availability of data, transformation uncertainty, measurement errors and statistical errors derived from the measured values, to the statistical interdependence of soil parameters (correlation) and to the spatial variability (e.g. [1], [2]). Model uncertainty relates to the assumed conditions used in the different calculation methods and is neglected here.

The characteristic values according to Eurocode EN 1997-1 among other factors are dependent on the extent of the failure surface, which in turn influences the variance of soil properties.

In this paper the influence of the spatial variability of soils on estimating the characteristic value and its implications on slope stability is investigated. Numerical Monte Carlo analyses are employed to verify the validity of a simplified practical approach presented to determine the characteristic soil properties according to Eurocode.

2 MATERIAL AND METHODS

2.1 *Description of Cases*

In order to assess the different methods to estimate the characteristic soil properties, two typical slope stability examples are introduced. The method of slices [3] is employed with a constant unit weight γ as well as a constant slope angle and dimensions.

The first example represents a saturated clay slope with a mean undrained shear strength of c_u = 40kPa (friction angle $\phi = 0°$). The slope angle is 26.57° and the depth of the failure surface is limited to 20 m as shown in Figure 1.

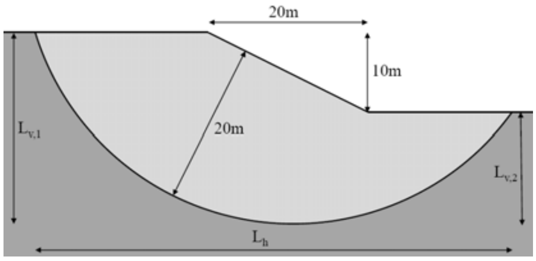

Figure 1: Example 1, undrained clay slope (F_s = 1.171).

The second example represents a slope with an angle of 36.87°, consisting of a clayey sand with a mean friction angle of ϕ=30° and a mean cohesion of c=8kPa.

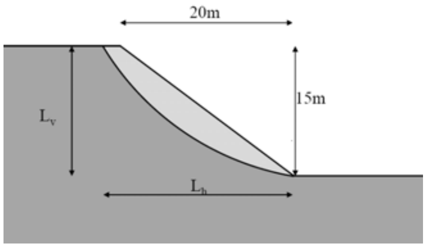

Figure 2: Example 2, slope in clayey sand (F_s = 1.155).

The statistical parameters of the inherent soil variability, mean μ, coefficient of variation CV_w (=σ/μ), vertical correlation length ρ_v, horizontal correlation length ρ_h, anisotropy ratio (i.e. ρ_h/ρ_v) for the two examples are shown in Table 1 (cohesion c) and Table 2 (friction angle ϕ) and are based on [2].

Table 1: Statistical parameters for cohesion

Cohesion	Example 1	Example 2
μ	40 kPa	8 kPa
CV_w	0 - 0.45	0 - 0.45
ρ_v	0.5 - ∞ m	0.5 - ∞ m
δ_v	1 - ∞ m	1 - ∞ m
$\rho_h/\rho_v = \delta_h/\delta_v$	10	10

Typically, $CV_{w,c}$ is 0.3 and $\rho_{v,c}$ is 0.5 - 1m and $\delta_{v,c}$ is 1 - 2m, respectively.

Table 2: Statistical parameters of the friction angle

Friction angle [°]	Example 1	Example 2
μ	0	30°
CV_w	0	0 - 0.2
ρ_v	-	0.5 - ∞ m
δ_v	-	1 - ∞ m
$\rho_h/\rho_v = \delta_h/\delta_v$	-	10

Typically, $CV_{w,\phi}$ is 0.1 and $\rho_{v,\phi}$ is 0.5 - 1m and $\delta_{v,c}$ is 1 - 2m, respectively.

2.2 Calculation of the factor of safety

The computation of the factor of safety F_S of a slope is based on:

- Estimate of the *characteristic values* x_k of cohesion and friction angle as the 5% confidence limit for estimating the average of *n* measurements (*t* is the 5% percentile of Student's *t*-distribution with *n* degrees of freedom) (e.g. [4], [5], [6]):

$$x_k = \mu_x - \frac{t}{\sqrt{n}}\sigma_x = \mu_x\left(1 - \frac{t}{\sqrt{n}}CV_w\right) \tag{1}$$

- Calculate the *factor of safety* F_S using:

$$F_S = \frac{cL + \tan(\varphi)\cdot\gamma\int H\cos(\alpha)\,dx}{\gamma\int H\sin(\alpha)\,dx} \tag{2}$$

- Find the minimum factor of safety among all possible slip surfaces

$$F_{s,min} = min\{F_s\} \tag{3}$$

A simplified equation for determining the characteristic value according to equation (1), which gives good values in practical terms (implicitly valid for about L/δ = 8-10 was found by [4]. Thereby, the characteristic value could be calculated as:

$$\mathrm{x_k} = \mu_\mathrm{x} - 0.5\sigma_\mathrm{x} = \mu_\mathrm{x}(1 - 0.5\mathrm{CV_w}) \tag{4}$$

2.3 Spatial Variability

Spatial variability describes the variation of geotechnical parameters in one, two, or three spatial dimensions. It is assumed that measurements are correlated if the distance between them is small, and are statistically independent if the distance between them is large. A mathematical function that describes the (auto)correlation *r(h)* is given by:

$$r(h) = \exp\,(-h/\rho) \tag{5}$$

This exponential autocorrelation function is a function of the length h and the correlation length ρ. In geotechnical applications the *scale of fluctuation* [7] is mostly used which is defined as $\delta = 2\rho$ and leads to

$$r(h) = \exp\,(-2h/\delta) \tag{6}$$

Geotechnical parameters show a dependency of the scale of fluctuation on direction, namely the vertical and the horizontal distance due to the soils deposition and loading history. This produces two exponential autocorrelation functions; one with the horizontal and one with the vertical correlation length. The ratio of the horizontal and the vertical correlation length ρ_h/ρ_v is called the anisotropy ratio.

2.4 Simplified formula regarding spatial variability

A major difficulty in assessing the characteristic value according to EN 1997-1 is to account for "the zone of ground affecting the limit state".

In a zone larger than the scale of fluctuation the spatially variable properties tend to "average out", whereas within a distance smaller than the scale of fluctuation the spatial average varies considerably, in an extreme case as much as the variance of the samples [8]. This averaging occurs because of an increasing probability that high property values are balanced on low property values on other points [9], if the correlation (i.e. the scale of fluctuation) decreases and/or the size (length, area, volume) of the failure mechanism increases. This effect is known as variance reduction due to spatial averaging.

Thus, the *effect* of the scale of fluctuation depends on the size of the investigated area, or in this context on the length of the failure surface *L*.

Taking into account the spatial variability in a simplified way, the characteristic value according to Eurocode can be estimated by equation (7)

$$x_k = \mu_x\left(1 - 1.645\sqrt{\frac{\delta}{L}}CV_w\right) \tag{7}$$

where δ is the scale of fluctuation, L is the length of the governing failure mechanism, and 1.645 is the 5% fractile of the normal probability distribution. Eqn. (7) is valid for $L \geq \delta$. For the case of $L \leq \delta$, $\frac{\delta}{L} = 1$ [7]. Note that eqn. (7) only accounts for the inherent variability, whereas (1) accounts for the statistical uncertainty. The two uncertainties can be linked as follows:

$$x_k = \mu_x \left(1 - 1.645 \sqrt{\frac{\delta}{L} + \frac{1}{n}} CV_w\right) \quad (8)$$

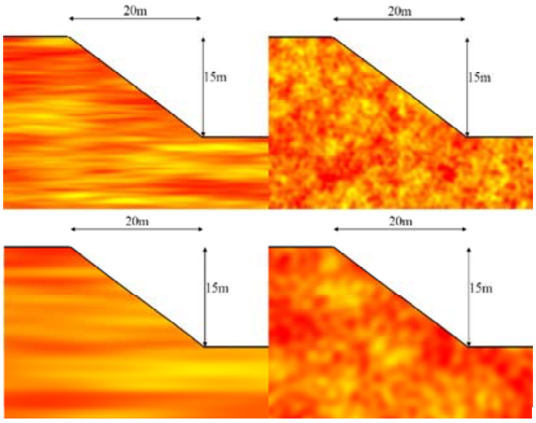

Figure 3: Autocorrelated random fields in Example 2: dark zones with weak cohesion, light zones with strong cohesion. Upper part with correlation length 1m, lower part with correlation length 3m, right slopes isotropic, left slopes with anisotropy ratio 10.

With the focus of this paper on the quantification of the effect of spatial variability on slope stability, eqn. (7) is used and the statistical measurement error neglected.

In case the lognormal distribution should be used (e.g. when the coefficient of variation CV_w is larger than about 0.3) to estimate the characteristic values:

$$x_k = x_m \cdot \frac{0.193^{\sqrt{ln\left(1+\frac{\delta}{L} \cdot CV_w^2\right)}}}{\sqrt{1+\frac{\delta}{L} \cdot CV_w^2}} \quad (9)$$

With these characteristic values the factor of safety at 5% probability of failure is obtained.

2.5 *Conditions of spatial averaging*

Spatial averaging is only possible if the governing failure mechanism is capable of redistributing forces or stresses along failure surfaces.

True cohesion is often not ductile, i.e. brittle (is lost after small strains) and is mobilized on the failure surface depending on stress history and stress level. Cohesion can therefore not generally be averaged because the strains acting on the failure surface generally vary along the failure surface. Many practitioners are well aware of this fact and will consequently neglect cohesion in most slope stability calculations. Despite this fact, in the calculations here it is assumed for generality that the cohesion is redistributed on the failure surface.

For the frictional resistance, the same general remarks as for cohesion apply. For more details refer to the companion paper (cf. Schneider & Fitze, 2011).

3 METHODOLOGY AND RESULTS

3.1 *Anisotropic spatially variable slip surfaces*

The variance reduction factor Γ^2 along the slip surface of an anisotropic soil can be described as:

$$\Gamma^2 = \frac{\delta_v}{L_v} \quad \text{for } \frac{\delta_v}{L_v} < \frac{\delta_h}{L_h}, \text{otherwise } \Gamma^2 = \frac{\delta_h}{L_h} \quad (10)$$

where $L_v = L_{v1} + L_{v2}$. See Figure 1 and 2 for the definitions of L.

3.2 *Calculation methodology*

Equations (1), (4), (7) and (9) are used to estimate the characteristic values in order to determine the 5%-fractile for the factor of safety. Additionally, Monte Carlo (MC) analyses with the soil described by autocorrelated random fields soils [10; 11] are performed to compute the 5%-fractile for the factor of safety as well.

Figure 4 to Figure 7 show the results of three different MC-methods to obtain the 5%-fractile of the factor of safety (F_s) as well as the results obtained from the 3 simplified eqn. (1), (4), (7) and (9) for comparison:

1. **Method 1**: MC analysis with a spatially homogeneous soil, but accounts for spatial variability because the reduced variance (δ/L) is used. This situation can be characterized by an infinite correlation length ($\rho = \infty$). For both examples a fixed midpoint and radius of

the failure surface was determined with average values of c and ϕ.

2. **Method 2:** MC analysis with (auto)correlated random fields. Each MC run creates a spatially variable random soil with correlation length ρ. Method 2 uses the fixed midpoint and radius of the failure surface determined in Method 1, although the spatial variability could imply a different failure surface in each MC run.
3. **Method 3:** MC analysis with correlated random fields and search of the critical failure surface according to eqn. (3) in each MC run. Method 3 selects the circle as failure surface for which the generated soil shows the most unfavourable strength.
4. **Equation (1)** neglects spatial variability and assumes n to be large.
5. **Equation (4)** – as a rule of thumb – accounts for both, statistical error and the spatial variability. But the uncertainty is only roughly accounted for, using the CV_w, but neglecting the correlation structure (δ) and the size (L) of the governing failure mechanism.
6. **Equation (7) and (9)** account explicitly for the spatial variability and the size of the governing failure mechanism. They calculate the variance reduction simplified by dividing the scale of the fluctuation (δ) and the size (L) of the governing failure mechanism.

Because all other methods or equations can be derived from Method 3 by means of simplification, this method is used as the reference method for the comparison.

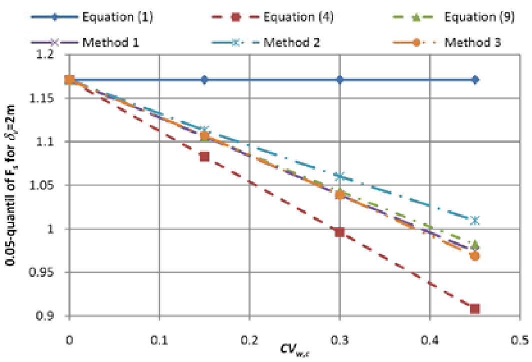

Figure 4. Example 1: 0.05-fractile of F_s as a function of $CV_{w,c}$ (δ_v=2m).

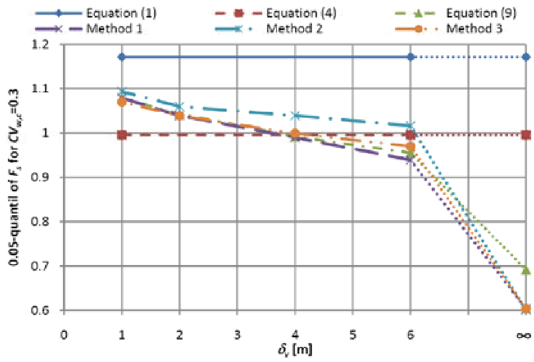

Figure 5. Example 1: 0.05- fractile of F_s as a function of δ_v (CV_w=0.3).

3.3 *Comparative Results*

For both examples a sensitivity analysis is presented. Figure 4 (Example 1) and Figure 6 (Example 2) show the sensitivity of the calculation methods to the coefficient of variation, fixing the vertical scale of fluctuation (δ_v=2m). Figure 5 (Example 1) and Figure 7 (Example 2) show the sensitivity of the calculation methods to vertical scale of fluctuation, fixing the coefficient of variation.

Equation (1) overestimates the factor of safety F_s and can be unsafe, especially if CV_w is large and/or the scale of fluctuation is large (see Figures 4 and 5).

Equation (4) shows F_s to be slightly lower than the reference (Method 3) and thus is slightly conservative for a typical scale of fluctuation (2m) and for the governing failure mechanism assumed here. However for a large correlation length and/or a small governing failure mechanism, Equation (4) might overestimate F_s and therefore might be unsafe (Figure 5 or 7).

Equation (7) and (9) yield very good results. The simple variance reduction used for the circular slip surface (eqn. (10)) is slightly conservative. The results show that there is a small difference between eqn. (7) or (9) and the reference Method 3.

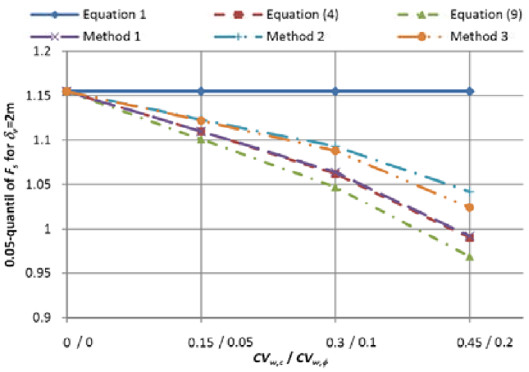

Figure 6. Example 2: 0.05- fractile of F_s as a function of the CV_w (δ_v=2m).

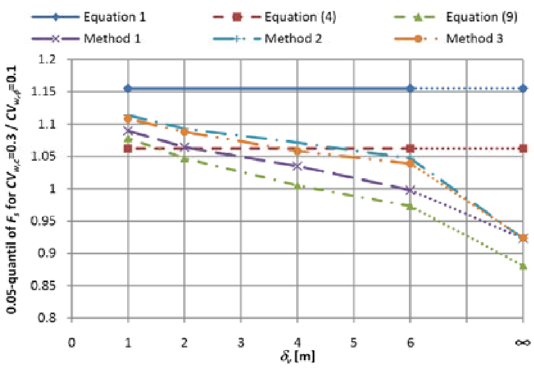

Figure 7. Example 2: 0.05- fractile of F_s as a function of δ_v ($CV_{w,c}$=0.3 and $CV_{w,\phi}$=0.1).

Results of example 1 and 2 are very similar. In example 2 both, ϕ and c, are spatially variable and are uncorrelated. Thus the critical slip surfaces determined in each MC run in Method 3 are not much different from the one critical slip surface used in Method 2.

4 CONCLUSIONS

The numerical investigations show that the risk of slope failure also depends on the scale of the fluctuation of slope properties.

The variance of cohesion and friction angle is reduced due to spatial averaging, if δ/L decreases. A higher variance reduction due to spatial averaging leads to higher characteristic values and larger factor of safety respectively and vice versa. Equations (7) and (9) explicitly account for spatial variability with the scale of fluctuation (i.e. the autocorrelation) and the size of the governing failure mechanism. The validity of Equation (7) and (9) has been proven by independent MC-analyses.

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REFERENCES

[1] J.T. Christian, C.C. Ladd, G.B. Baecher, Reliability Applied to Slope Stability Analysis, *J Geotech Eng-Asce* **120** (1994), 2180-2207.

[2] K.K. Phoon, F.H. Kulhawy, Characterization of geotechnical variability, *Can Geotech J* **36** (1999), 612-624.

[3] W. Fellenius, *Calculation of the stability of earth dams*, Transactions of the 2nd Congress on Large Dams, Washington, D.C., 1936, pp. 445-462.

[4] H.R. Schneider, *Definition and determination of characteristic soil properties*, 12th Int. Conf. Soil Mech. & Fdn Engng, Balkema, Hamburg, 1997.

[5] T.L.L. Orr, D. Breysse, *Eurocode 7 and reliability-based design*, in: K.-K. Phoon, (Ed.), Reliability-based design in geotechnical engineering: computations and applications, Taylor & Francis, London, 2008, pp. 298-343.

[6] A. Bond, A. Harris, *Decoding Eurocode 7*, Taylor & Francis, London, 2008.

[7] E. Vanmarcke, *Random fields, analysis and synthesis*, MIT Press, Cambridge, Mass., 1983.

[8] E.H. Vanmarcke, Probabilistic Modeling of Soil Profiles, *J Geotech Eng-Asce* **103** (1977), 1227-1246.

[9] N.R.C., *Probabilistic methods in geotechnical engineering*, National Academy Press, Washington, D.C, 1995.

[10] O. Tietje, O. Richter, Stochastic modeling of the unsaturated water flow using auto-correlated spatially variable hydraulic parameters, *Modeling Geo-Biosphere processes* **1** (1992), 163-183.

[11] A. Mantoglou, Digital simulation of multivariate two- and three-dimensional stochastic processes with a spectral turning bands method, *Mathematical Geology* **19** (1987), 129-149.

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