**Computational Challenges in Risk Calculations for XVA (JL/SB)**

**Description**

Pricing of over-the-counter (OTC) derivatives has historically relied on the Black-Scholes’ risk neutral pricing framework, under the assumption of funding at the risk-free rate and the ability to perfectly replicate and hedge derivatives. This assumption however doesn’t hold anymore post financial crisis of 2008 and therefore the bank has to adjust the risk-neutral value to take into account all associated risk like counterparty credit risk when valuing OTC derivatives.

X-Value Adjustment (XVA) is the generic term referring collectively to a number of different adjustments made to the risk-neutral value of the derivatives contract held by the bank to take into account funding, credit risk and regulatory capital costs.

The purpose of XVA is twofold:

1. To hedge for possible losses due to counterparty default;
2. Determine (and hedge) the amount of capital required under Basel III.

Conventionally, XVA is performed using the “brute-force” approach. The “brute-force” approach is performed as follows:

1. Simulate all associated risk factor scenarios for the risk calculations through either Monte-Carlo simulation or historical collections;
2. Price every trade in the underlying portfolio based on the simulated risk factors and compute all associated sensitivities.

The conventional “brute-force” approach is computationally demanding, since multiple portfolios held by the bank must be re-valued. The main problem lies in the second step, since these calculations usually should be performed on a regularly basis, especially the sensitivities are expensive to compute.

To calculate the sensitivities, we need an algorithmic approximation method that is:

* fast;
* accurate;
* stable;
* low in memory use;
* flexible (in the sense that it can be used in different frameworks).

Popular methods developed for calculating sensitivities are Adjoint Algorithmic Differentiation (AAD) and Chebyshev Spectral Decomposition. An alternative for calculating sensitivities might be the Likelihood ratio method in cases the underlying probability density is known.

The goal of this project is to find an efficient approximation method to calculate sensitivities. For some specific choices of model dynamics (e.g., Hull-white) which are analytically tractable, probability density functions are known. In this case, especially the Likelihood ratio method seems interesting. However, high variance problems are expected, which might be tackled with variance reduction techniques.

In case time allows, also the other different algorithmic approximation methods can be investigated and modified to improve its performance.

**Objectives**

1. Review literature on XVA.
2. Implement the “brute-force” approach for Hull-White for the different valuation adjustments.
3. Study and implement different methods to approximate the sensitivities including the Likelihood ratio method.
4. Try to improve the Likelihood ratio. It is expected that variance reduction is required.
5. In case time allows, try to improve other methods.
6. Perform several numerical experiments to determine the best approach based on portfolio with vanilla and/or exotic interest rate products.

**Remark**

For this project it is a prerequisite that the student has affinity with programming as this is an important part of this project.

**References**

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