# Contour Detection in <br> Multi-Angle Time-Lapse <br> Images of Growing Plants 

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## Quantifying growth

- Six potato varieties
- 2 Climate rooms
- Dry and wet sections

Problem


table

moving camera


table


## Overview



## Traditional Snakes

- Active contours to detect outline in image



## Traditional Snakes

- Energy functional of a traditional snake parameterized by $\mathbf{q}(s)=[u(s), v(s)]$

$$
\begin{aligned}
E_{\text {snake }}^{*} & =\int_{0}^{1} E_{\text {snake }}(\mathbf{q}(s)) \mathrm{d} s \\
& =\int_{0}^{1} E_{\text {int }}(\mathbf{q}(s))+E_{\text {ext }}(\mathbf{q}(s)) \mathrm{d} s
\end{aligned}
$$

- $\quad \boldsymbol{a}$ and $\beta$ control parameters for amount of stretch and curvature

$$
E[\mathbf{q}]=\int_{0}^{1} \frac{1}{2}\left(\alpha\left\|\mathbf{q}^{\prime}(s)\right\|^{2}+\beta\left\|\mathbf{q}^{\prime \prime}(s)\right\|^{2}\right)+E_{\mathrm{ext}}(\mathbf{q}(s)) \mathrm{d} s
$$

- Assume local minimum of $E$ in $\mathbf{q}$ to derive the Euler-Lagrange equation

$$
\alpha \mathbf{q}^{\prime \prime}-\beta \mathbf{q}^{\prime \prime \prime \prime}-\nabla E_{\mathrm{ext}}(\mathbf{q})=0
$$

## Traditional Snakes

- Typical external energy $E_{\text {ext }}(\mathbf{q})=-|\nabla I(\mathbf{q})|^{2}$ which points towards regions of interest in image $I$



## Gradient Vector Flow

- New external force field $\quad \mathbf{w}(u, v)=[\phi(u, v), \psi(u, v)]$
which minimizes the functional
$\mathcal{E}[\mathbf{w}]=\iint \mu\left(\phi_{u}^{2}+\phi_{v}^{2}+\psi_{u}^{2}+\psi_{v}^{2}\right)+|\nabla f|^{2}|\mathbf{w}-\nabla f|^{2} \mathrm{~d} u \mathrm{~d} v$ $\mu_{\text {smoothing }}$ term and edge map $f$ derived from image
- GVF Snake
$\alpha \mathbf{q}^{\prime \prime}-\beta \mathbf{q}^{\prime \prime \prime \prime}+\mathbf{w}=0$


## Gradient Vector Flow


$\mu=0$

$\mu=1 \mathrm{e}-3$


## GVF-Snakes

- Solve for $\frac{\partial \mathbf{q}}{\partial t}=\alpha \frac{\partial^{2} \mathbf{q}}{\partial s^{2}}-\beta \frac{\partial^{4} \mathbf{q}}{\partial s^{4}}+\mathbf{w}(\mathbf{q})$
with time-integration method
- Initial contour



## Results


(a) $t=0$

(b) $t=1$

(c) $t=5$

(d) $t=10$

(e) $t=50$

## Results


(a) Angle-1

(b) Angle 0

(c) Angle +1




## GVF-Snakes

- Algorithm to detect outline
- Success ensured by initial contour and preprocessing steps


## Pinhole camera model



## Pinhole camera model

- Point in camera reference frame:
- Projection equations:

$$
\begin{aligned}
& f \frac{x^{\prime}}{z^{\prime}}=u-u_{0} \\
& f \frac{y^{\prime}}{z^{\prime}}=v-v_{0}
\end{aligned}
$$

$\left(u_{0}, v_{0}\right)$ center of image plane


$$
\mathbf{r}^{\prime}=\left[x^{\prime}, y^{\prime}, z^{\prime}\right]
$$



## Pinhole camera model

- Point in camera reference frame:

$$
\mathbf{r}^{\prime}=\left[x^{\prime}, y^{\prime}, z^{\prime}\right]
$$

- Normalized image coordinates:

$$
\frac{1}{z^{\prime}} \mathbf{r}^{\prime}=\left[\begin{array}{l}
x^{\prime} / z^{\prime} \\
y^{\prime} / z^{\prime} \\
z^{\prime} / z^{\prime}
\end{array}\right]=\mathbf{y}
$$



3D Contour


## Two-view geometry

- Two camera's aiming at the same point $\mathbf{P}$

$$
\begin{aligned}
& \mathbf{r}^{\prime}=\left[x^{\prime}, y^{\prime}, z^{\prime}\right] \\
& \mathbf{r}^{\prime \prime}=\left[x^{\prime \prime}, y^{\prime \prime}, z^{\prime \prime}\right] \\
& \mathbf{t}=\left[t_{1}, t_{2}, t_{3}\right] \\
& \mathbf{r}^{\prime \prime}=R \mathbf{r}^{\prime}+\mathbf{t}
\end{aligned}
$$



## Essential Matrix

$$
\begin{aligned}
\mathbf{r}^{\prime \prime} & =R \mathbf{r}^{\prime}+\mathbf{t} \\
z^{\prime \prime} \mathbf{y}^{\prime \prime} & =z^{\prime} R \mathbf{y}^{\prime}+\mathbf{t} \\
0 & =\left(\mathbf{y}^{\prime \prime}\right)^{T} E \mathbf{y}^{\prime}
\end{aligned}
$$

## Essential Matrix

$$
\begin{aligned}
\mathbf{r}^{\prime \prime} & =R \mathbf{r}^{\prime}+\mathbf{t} \\
z^{\prime \prime} \mathbf{y}^{\prime \prime} & =z^{\prime} R \mathbf{y}^{\prime}+\mathbf{t} \\
0 & =\left(\mathbf{y}^{\prime \prime}\right)^{T} E \mathbf{y}^{\prime}
\end{aligned}
$$

- Where $E$ is a $3 \times 3$ matrix called the Essential Matrix defined by $E=[\mathbf{t}]_{\times} R$

$$
\mathbf{t} \times R=[\mathbf{t}]_{\times} R, \quad[\mathbf{t}]_{\times}=\left[\begin{array}{ccc}
0 & -t_{3} & t_{2} \\
t_{3} & 0 & -t_{1} \\
-t_{2} & t_{1} & 0
\end{array}\right]
$$

## Essential Matrix

- Determine $E$ from $\left(\mathbf{y}^{\prime \prime}\right)^{T} E \mathbf{y}^{\prime}=0$
- $Y \mathbf{e}=\mathbf{0}, \mathrm{n} \times 9$ matrix $Y$
- Minimum of 8 point correspondences
- Find nullspace of $Y$ using Singular Value Decomposition
$U, D, V^{T}=\operatorname{svd}(Y)$ in column of $V$


## Point matching


(a) Image and contour 1

(b) Image and contour 2

3D Contour

## Point matching

- Rough alignment
- Choose 8 random points on contour and find the corresponding closest points



## 3D Contour reconstruction

- Obtain rotation and translation via another

Singular Value Decomposition of $E$

- Retrieve depth coordinate
$\mathbf{y}^{\prime \prime}=\frac{1}{z^{\prime \prime}} \mathbf{r}^{\prime \prime}=\frac{R \mathbf{r}^{\prime}+\mathbf{t}}{\mathbf{e}_{3}^{T}\left(R \mathbf{r}^{\prime}+\mathbf{t}\right)}$
$\mathbf{y}^{\prime \prime}=\frac{R z^{\prime} \mathbf{y}^{\prime}+\mathbf{t}}{\mathbf{e}_{3}^{T}\left(R z^{\prime} \mathbf{y}^{\prime}+\mathbf{t}\right)}$
$z^{\prime}=-\frac{\mathbf{y}^{\prime T} R^{T}\left(\mathbf{y}^{\prime \prime} \mathbf{e}_{3}^{T}-I\right)^{T}\left(\mathbf{y}^{\prime \prime} \mathbf{e}_{3}^{T}-I\right) \mathbf{t}}{\mathbf{y}^{\prime T} R^{T}\left(\mathbf{y}^{\prime \prime} \mathbf{e}_{3}^{T}-I\right)^{T}\left(\mathbf{y}^{\prime \prime} \mathbf{e}_{3}^{T}-I\right) R \mathbf{y}^{\prime}}$


## 3D Contour reconstruction



# 3D Contour 

 ReconstructionO

## 3D Contour reconstruction





## 3D Contour reconstruction




## Observation of growth



3D Contour
Reconstruction
$\bigcirc$

## Misalignment

- $\mathbf{r}^{\prime}\left(\tau_{1}\right)=a R \mathbf{r}^{\prime}\left(\tau_{2}\right)+\mathbf{t}$
- Alignment based solely on corner points
$\approx \leftarrow \rightarrow \pm Q \ddagger \omega$ 回



## Observation of growth



- 3D reconstruction up to scale
- In camera reference frame
- SVD
- Alignment of plant's container


## Double Snakes

- Combine previous problems
- Evolve both snakes simultaneously and match points on contour
- Minimize the functional

$$
\begin{aligned}
& \Phi\left[\mathbf{y}_{1}, \mathbf{y}_{2}\right]=\int_{0}^{1}\left[\frac{\alpha}{2}\left(\left\|\mathbf{y}_{1}^{\prime}\right\|^{2}+\left\|\mathbf{y}_{2}^{\prime}\right\|^{2}\right)+\frac{\beta}{2}\left(\left\|\mathbf{y}_{1}^{\prime \prime}\right\|^{2}+\left\|\mathbf{y}_{2}^{\prime \prime}\right\|^{2}\right)+\frac{1}{2}\left(\mathbf{y}_{2}^{T} E \mathbf{y}_{1}\right)^{2}\right] \mathrm{d} s \\
& \frac{\partial \mathbf{y}_{1}}{\partial t}=\alpha \frac{\partial^{2} \mathbf{y}_{1}}{\partial s^{2}}-\beta \frac{\partial^{4} \mathbf{y}_{1}}{\partial s^{4}}-E^{T} \mathbf{y}_{2} \mathbf{y}_{2}^{T} E \mathbf{y}_{1}+\mathbf{F}\left(\mathbf{y}_{1}\right), \\
& \frac{\partial \mathbf{y}_{2}}{\partial t}=\alpha \frac{\partial^{2} \mathbf{y}_{2}}{\partial s^{2}}-\beta \frac{\partial^{4} \mathbf{y}_{2}}{\partial s^{4}}-E \mathbf{y}_{1} \mathbf{y}_{1}^{T} E^{T} \mathbf{y}_{2}+\mathbf{F}\left(\mathbf{y}_{2}\right) .
\end{aligned}
$$

## Convexity

- When does $\left(\mathbf{y}^{\prime \prime}\right)^{T} E \mathbf{y}^{\prime}=0$ hold?
- $\left(\mathbf{y}^{\prime \prime}\right)^{T} E=\mathbf{0}$

$$
E \mathbf{y}^{\prime}=\mathbf{0}
$$

The left and right nullspace of $E$

## Double Snakes

- Simultaneously updating both contours
- Convexity of the essential matrix


## Conclusion

- GVF-Snakes algorithm to detect contour
- 3D contour reconstruction up to scale
- Observation of growth due to alignment of plant's container
- Introduction of Double Snakes



## Recommendations

- Stabilizing the estimation of the essential matrix
- More advanced camera model
- Improve alignment of plant's container
- Explore convexity of the coplanarity constraint


## Questions?

