

# Modeling three phases of steel

## Austenite, ferrite and cementite

Thijs Verbeek  
19 April 2018

# Introduction

- 1 Physics of phase transformation
- 2 Mathematical model
- 3 Discretization
- 4 Evaluating results

# Introduction

- 1 Physics of phase transformation
- 2 Mathematical model
- 3 Discretization
- 4 Evaluating results

# Introduction

- 1 Physics of phase transformation
- 2 Mathematical model
- 3 Discretization
- 4 Evaluating results

# Introduction

- 1 Physics of phase transformation
- 2 Mathematical model
- 3 Discretization
- 4 Evaluating results

# Phase Transformation Ice to Water

Ice to water phase transformation.

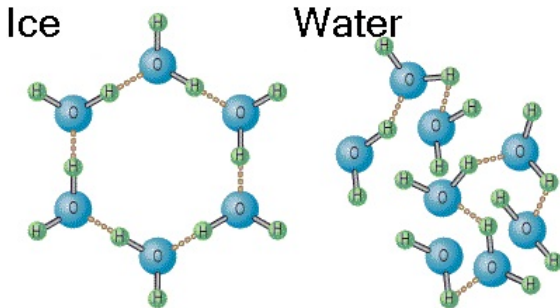


Figure: <https://kids.britannica.com/students/assembly/view/54121>

# Phase Transformation Ice

Ice to ice phase transformation.

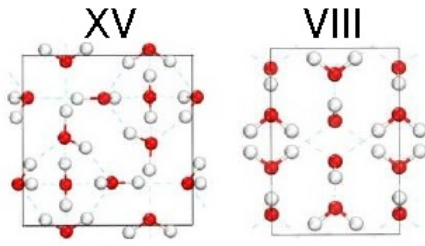
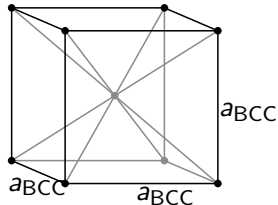
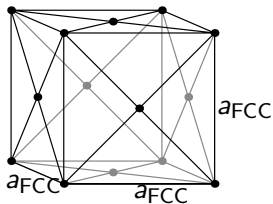


Figure: *The Journal of chemical physics*. V 139. 154702.  
10.1063/1.4824481.

# Phase Structures Steel



(a) Austenite ( $\gamma$ ) crystal structure Face-Center-Cubic  
(b) Ferrite ( $\alpha$ ) crystal structure Body-Center-Cubic

Cementite has a fixed ratio of  $Fe_3C$ .



# Phase Diagram Steel

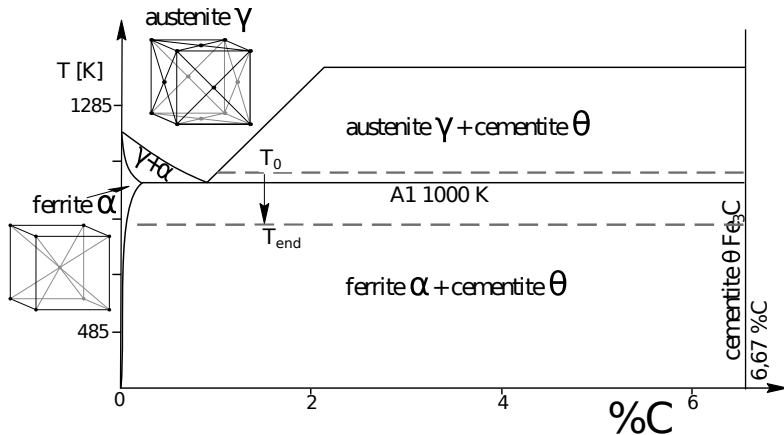
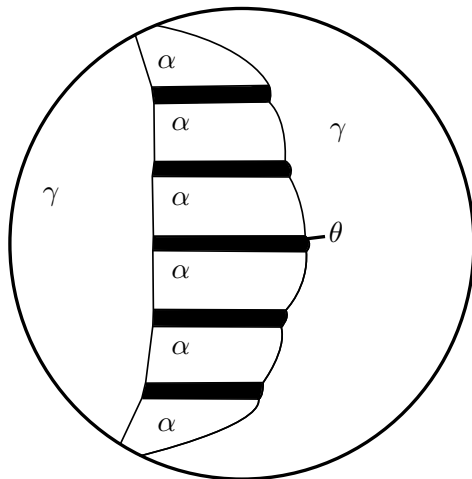


Figure: Part of the steel phase diagram

# Phase Transformation in Steel



# Result Mathematical Model

# Result Mathematical Model

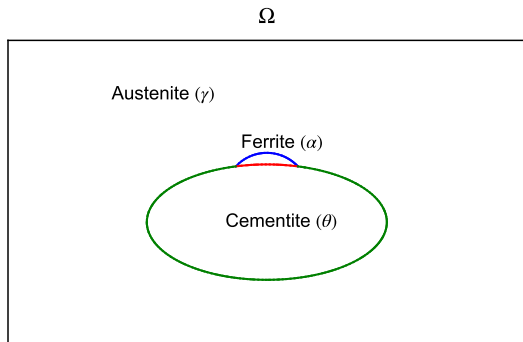
# Stefan Problem

$$\left\{ \begin{array}{l} \frac{\partial c_\gamma}{\partial t} = \nabla \cdot (D_\gamma \nabla c_\gamma) \quad , \mathbf{x} \in \gamma, \\ \frac{\partial c}{\partial n} = 0 \quad , \mathbf{x} \in \partial\Omega, \\ \text{BC} \quad \Gamma^{\gamma\alpha}, \Gamma^{\gamma\theta}, \\ \text{Initial conditions for } c_\gamma, c, \Gamma^{\gamma\alpha}, \Gamma^{\gamma\theta}. \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\partial c_\alpha}{\partial t} = \nabla \cdot (D_\alpha \nabla c_\alpha) \quad , \mathbf{x} \in \alpha, \\ \frac{\partial c}{\partial n} = 0 \quad , \mathbf{x} \in \partial\Omega, \\ \text{BC} \quad \Gamma^{\gamma\alpha}, \Gamma^{\alpha\theta}, \\ \text{Initial conditions for } c_\alpha, \Gamma^{\gamma\alpha}, \Gamma^{\alpha\theta}. \end{array} \right.$$

Cementite constant concentration  $c_\theta$

# Stefan Problem: Domain



# Boundary Condition $\Gamma^{\gamma\theta}$

On  $\Gamma^{\gamma\theta}$  in  $\gamma$  there are two unknowns  $c_\gamma$  and  $v_n^{\gamma\theta}$ , so two boundary conditions are needed:

- Mass conservation,
- First-order reaction in flux over  $\Gamma^{\gamma\theta}$ .

This gives boundary condition and interface velocity

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\theta}}{c_\theta} (c_\theta - c_\gamma) (c_{\gamma\theta} - c_\gamma),$$
$$v_n^{\gamma\theta} = \frac{K^\gamma}{c_\theta} (c_{\gamma\theta} - c_\gamma).$$

# Boundary Condition $\Gamma^{\gamma\theta}$

On  $\Gamma^{\gamma\theta}$  in  $\gamma$  there are two unknowns  $c_\gamma$  and  $v_n^{\gamma\theta}$ , so two boundary conditions are needed:

- Mass conservation,
- First-order reaction in flux over  $\Gamma^{\gamma\theta}$ .

This gives boundary condition and interface velocity

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\theta}}{c_\theta} (c_\theta - c_\gamma) (c_{\gamma\theta} - c_\gamma),$$
$$v_n^{\gamma\theta} = \frac{K^\gamma}{c_\theta} (c_{\gamma\theta} - c_\gamma).$$



# Boundary Condition $\Gamma^{\gamma\theta}$

On  $\Gamma^{\gamma\theta}$  in  $\gamma$  there are two unknowns  $c_\gamma$  and  $v_n^{\gamma\theta}$ , so two boundary conditions are needed:

- Mass conservation,
- First-order reaction in flux over  $\Gamma^{\gamma\theta}$ .

This gives boundary condition and interface velocity

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\theta}}{c_\theta} (c_\theta - c_\gamma) (c_{\gamma\theta} - c_\gamma),$$
$$v_n^{\gamma\theta} = \frac{K^\gamma}{c_\theta} (c_{\gamma\theta} - c_\gamma).$$

# Boundary Condition $\Gamma^{\gamma\theta}$

On  $\Gamma^{\gamma\theta}$  in  $\gamma$  there are two unknowns  $c_\gamma$  and  $v_n^{\gamma\theta}$ , so two boundary conditions are needed:

- Mass conservation,
- First-order reaction in flux over  $\Gamma^{\gamma\theta}$ .

This gives boundary condition and interface velocity

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\theta}}{c_\theta} (c_\theta - c_\gamma)(c_{\gamma\theta} - c_\gamma),$$
$$v_n^{\gamma\theta} = \frac{K^\gamma}{c_\theta} (c_{\gamma\theta} - c_\gamma).$$

# Boundary Condition $\Gamma^{\gamma\theta}$

On  $\Gamma^{\gamma\theta}$  in  $\gamma$  there are two unknowns  $c_\gamma$  and  $v_n^{\gamma\theta}$ , so two boundary conditions are needed:

- Mass conservation,
- First-order reaction in flux over  $\Gamma^{\gamma\theta}$ .

This gives boundary condition and interface velocity

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\theta}}{c_\theta} (c_\theta - c_\gamma) (c_{\gamma\theta} - c_\gamma),$$
$$v_n^{\gamma\theta} = \frac{K^\gamma}{c_\theta} (c_{\gamma\theta} - c_\gamma).$$

# Boundary Condition $\Gamma^{\alpha\theta}$

Similarly boundary condition and interface velocity on  $\Gamma^{\alpha\theta}$

$$D_{\alpha} \frac{\partial c_{\alpha}}{\partial n} = \frac{K^{\alpha\theta}}{c_{\theta}} (c_{\theta} - c_{\alpha}) (c_{\alpha\theta} - c_{\alpha}),$$
$$v_n^{\alpha\theta} = \frac{K^{\alpha}}{c_{\theta}} (c_{\alpha\theta} - c_{\alpha}).$$

# Boundary Condition $\Gamma^{\gamma\alpha}$

On  $\Gamma^{\gamma\alpha}$  there are three unknowns  $c_\gamma$ ,  $c_\alpha$  and  $v_n^{\gamma\alpha}$ , so three boundary conditions are needed:

- Mass conservation,
- First-order reaction in flux over  $\Gamma^{\gamma\alpha}$ ,
- $c_\alpha$  at equilibrium concentration  $c_{\alpha\gamma}$  in  $\alpha$ .

This gives boundary condition and interface velocity

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\alpha}}{c_{\alpha\gamma}} (c_{\gamma\alpha} - c_\gamma) (c_{\alpha\gamma} - c_\gamma) + \frac{D_\alpha}{c_{\alpha\gamma}} \frac{\partial c_\alpha}{\partial n} c_\gamma,$$
$$v_n^{\gamma\alpha} = \frac{K^{\gamma\alpha}}{c_{\alpha\gamma}} (c_{\gamma\alpha} - c_\gamma) - \frac{D_\alpha}{c_{\alpha\gamma}^{\text{sol}}} \frac{\partial c_\alpha}{\partial n}.$$

# Boundary Condition $\Gamma^{\gamma\alpha}$

On  $\Gamma^{\gamma\alpha}$  there are three unknowns  $c_\gamma$ ,  $c_\alpha$  and  $v_n^{\gamma\alpha}$ , so three boundary conditions are needed:

- Mass conservation,
- First-order reaction in flux over  $\Gamma^{\gamma\alpha}$ ,
- $c_\alpha$  at equilibrium concentration  $c_{\alpha\gamma}$  in  $\alpha$ .

This gives boundary condition and interface velocity

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\alpha}}{c_{\alpha\gamma}} (c_{\gamma\alpha} - c_\gamma) (c_{\alpha\gamma} - c_\gamma) + \frac{D_\alpha}{c_{\alpha\gamma}} \frac{\partial c_\alpha}{\partial n} c_\gamma,$$
$$v_n^{\gamma\alpha} = \frac{K^{\gamma\alpha}}{c_{\alpha\gamma}} (c_{\gamma\alpha} - c_\gamma) - \frac{D_\alpha}{c_{\alpha\gamma}^{\text{sol}}} \frac{\partial c_\alpha}{\partial n}.$$

# Boundary Condition $\Gamma^{\gamma\alpha}$

On  $\Gamma^{\gamma\alpha}$  there are three unknowns  $c_\gamma$ ,  $c_\alpha$  and  $v_n^{\gamma\alpha}$ , so three boundary conditions are needed:

- Mass conservation,
- First-order reaction in flux over  $\Gamma^{\gamma\alpha}$ ,
- $c_\alpha$  at equilibrium concentration  $c_{\alpha\gamma}$  in  $\alpha$ .

This gives boundary condition and interface velocity

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\alpha}}{c_{\alpha\gamma}} (c_{\gamma\alpha} - c_\gamma) (c_{\alpha\gamma} - c_\gamma) + \frac{D_\alpha}{c_{\alpha\gamma}} \frac{\partial c_\alpha}{\partial n} c_\gamma,$$
$$v_n^{\gamma\alpha} = \frac{K^{\gamma\alpha}}{c_{\alpha\gamma}} (c_{\gamma\alpha} - c_\gamma) - \frac{D_\alpha}{c_{\alpha\gamma}^{\text{sol}}} \frac{\partial c_\alpha}{\partial n}.$$

# Domain Tracking: Level-Set Method

Level-set method uses signed-distance function  $\phi(\mathbf{x}, t)$

$$\phi^{kl} = \begin{cases} + \min_{\mathbf{y} \in \Gamma^{kl}(t)} \|\mathbf{y} - \mathbf{x}\|_2 & , \text{ if } \mathbf{x} \in k, \\ 0 & , \text{ if } \mathbf{x} \in \Gamma^{kl}(t), \\ - \min_{\mathbf{y} \in \Gamma^{kl}(t)} \|\mathbf{y} - \mathbf{x}\|_2 & , \text{ if } \mathbf{x} \in l. \end{cases}$$



# Domain Tracking: Level-Set Method

$\phi^{kl}(\mathbf{x}, t)$  is updated by convection equation

$$\frac{\partial \phi^{kl}}{\partial t} + v_n^{\text{ex},kl} \|\nabla \phi^{kl}\|_2 = 0,$$

with convection coefficient  $v_n^{\text{ex},kl}(\mathbf{x}, t)$  obtained from

$$\left\{ \begin{array}{ll} \Delta v_n^{\text{ex},kl} = 0 & , \mathbf{x} \in \Omega_k \setminus \Gamma^{kl}, \\ v_n^{\text{ex},kl} = v_n^{kl} & , \mathbf{x} \in \Gamma^{kl}, \\ \frac{\partial v_n^{\text{ex},kl}}{\partial n} = 0 & , \mathbf{x} \in \partial\Omega. \end{array} \right.$$

# Domain Tracking: Level-Set Method

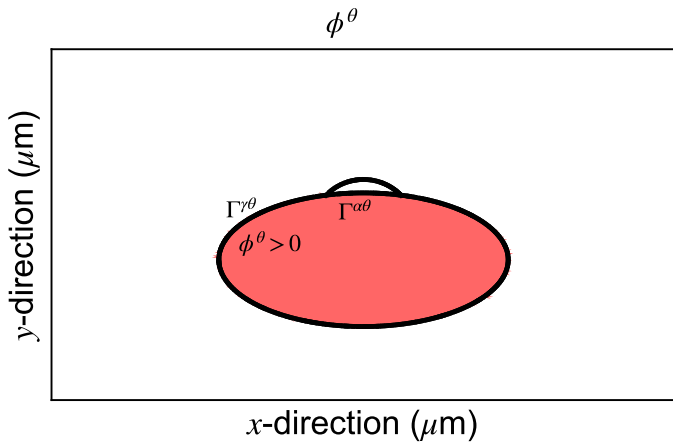
$\phi^{kl}(\mathbf{x}, t)$  is updated by convection equation

$$\frac{\partial \phi^{kl}}{\partial t} + v_n^{\text{ex},kl} \|\nabla \phi^{kl}\|_2 = 0,$$

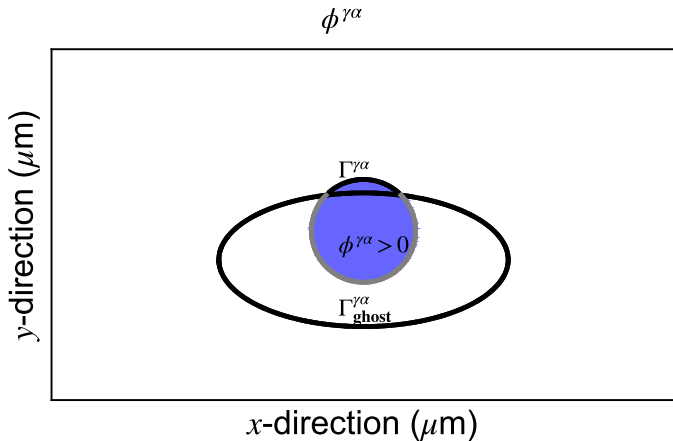
with convection coefficient  $v_n^{\text{ex},kl}(\mathbf{x}, t)$  obtained from

$$\left\{ \begin{array}{ll} \Delta v_n^{\text{ex},kl} = 0 & , \mathbf{x} \in \Omega_k \setminus \Gamma^{kl}, \\ v_n^{\text{ex},kl} = v_n^{kl} & , \mathbf{x} \in \Gamma^{kl}, \\ \frac{\partial v_n^{\text{ex},kl}}{\partial n} = 0 & , \mathbf{x} \in \partial\Omega. \end{array} \right.$$

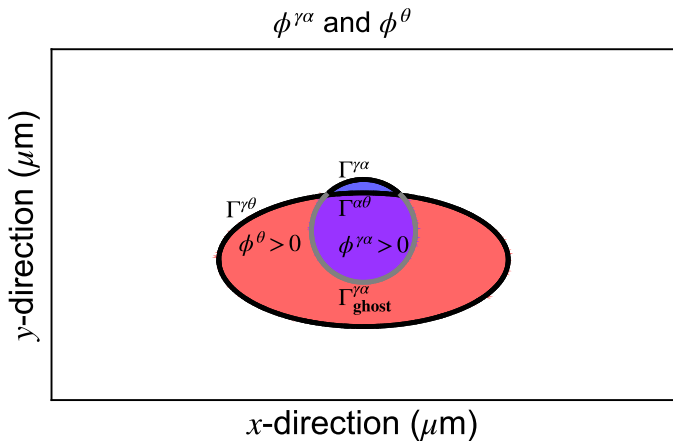
# Domain Tracking: Multi Level-Set Method



# Domain Tracking: Multi Level-Set Method



# Domain Tracking: Multi Level-Set Method



# Space Discretization and Time Integration

## Space discretization

- Galerkins Finite Element Method.

## Time integration

- implicit Euler for the Stefan problem,
- Total Variation Diminishing Runge-Kutta-Third-Order (TVD-RK3) for the convection equation.

# Space Discretization and Time Integration

Space discretization

- Galerkins Finite Element Method.

Time integration

- implicit Euler for the Stefan problem.
- Total Variation Diminishing Runge-Kutta-Third-Order (TVD RK3) for the convection equation.

# Space Discretization and Time Integration

## Space discretization

- Galerkins Finite Element Method.

## Time integration

- implicit Euler for the Stefan problem,
- Total Variation Diminishing Runge-Kutta-Third-Order (TVD RK3) for the convection equation.



# Space Discretization and Time Integration

Space discretization

- Galerkins Finite Element Method.

Time integration

- implicit Euler for the Stefan problem,
- Total Variation Diminishing Runge-Kutta-Third-Order (TVD RK3) for the convection equation.

# Space Discretization

Define fixed background mesh  $T$ , on  $T$  is defined

- $\phi^{\gamma\alpha}$ ,
- $\phi^{\theta}$ .

From  $\phi^{\gamma\alpha}, \phi^{\theta}$  we can find the interface points. Adding interface points to  $T$  gives enriched mesh  $T^E$ . On  $T^E$  is defined

- $c_{\gamma}$  where  $\phi^{\gamma\alpha} \leq 0$  and  $\phi^{\theta} \leq 0$ ,
- $c_{\alpha}$  where  $\phi^{\gamma\alpha} \geq 0$  and  $\phi^{\theta} \leq 0$ .

# Space Discretization

Define fixed background mesh  $T$ , on  $T$  is defined

- $\phi^{\gamma\alpha}$ ,
- $\phi^{\theta}$ .

From  $\phi^{\gamma\alpha}, \phi^{\theta}$  we can find the interface points. Adding interface points to  $T$  gives enriched mesh  $T^E$ . On  $T^E$  is defined

- $c_{\gamma}$  where  $\phi^{\gamma\alpha} \leq 0$  and  $\phi^{\theta} \leq 0$ ,
- $c_{\alpha}$  where  $\phi^{\gamma\alpha} \geq 0$  and  $\phi^{\theta} \leq 0$ .

# Space Discretization

Define fixed background mesh  $T$ , on  $T$  is defined

- $\phi^{\gamma\alpha}$ ,
- $\phi^{\theta}$ .

From  $\phi^{\gamma\alpha}, \phi^{\theta}$  we can find the interface points. Adding interface points to  $T$  gives enriched mesh  $T^E$ . On  $T^E$  is defined

- $c_{\gamma}$  where  $\phi^{\gamma\alpha} \leq 0$  and  $\phi^{\theta} \leq 0$ ,
- $c_{\alpha}$  where  $\phi^{\gamma\alpha} \geq 0$  and  $\phi^{\theta} \leq 0$ .

# Space Discretization

Define fixed background mesh  $T$ , on  $T$  is defined

- $\phi^{\gamma\alpha}$ ,
- $\phi^{\theta}$ .

From  $\phi^{\gamma\alpha}, \phi^{\theta}$  we can find the interface points. Adding interface points to  $T$  gives enriched mesh  $T^E$ . On  $T^E$  is defined

- $c_{\gamma}$  where  $\phi^{\gamma\alpha} \leq 0$  and  $\phi^{\theta} \leq 0$ ,
- $c_{\alpha}$  where  $\phi^{\gamma\alpha} \geq 0$  and  $\phi^{\theta} \leq 0$ .

# Discretization: Finding Interface

Level-set function is piece-wise linear on  $T$ . Interface point  $\mathbf{x}^e$  can be found over an edge  $\mathbf{e} = (\mathbf{x}_0, \mathbf{x}_1)$  by:

$$\begin{aligned}\mathbf{x}^e &= (1 - \tau^e) \mathbf{x}_0 + \tau^e \mathbf{x}_1, \\ \tau^e &= -\frac{\phi(\mathbf{x}_0)}{\phi(\mathbf{x}_1) - \phi(\mathbf{x}_0)}.\end{aligned}$$

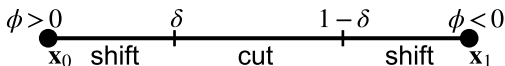
Depending on position of  $\mathbf{x}^e$  either shift node from  $T$  or add to  $T^E$ .

# Discretization: Finding Interface

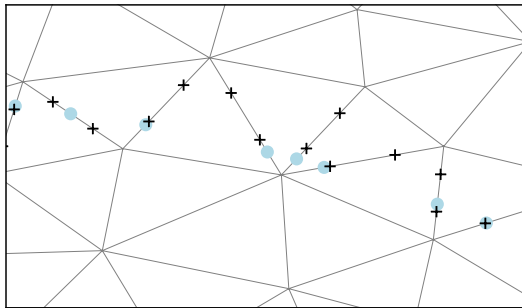
Level-set function is piece-wise linear on  $T$ . Interface point  $\mathbf{x}^e$  can be found over an edge  $\mathbf{e} = (\mathbf{x}_0, \mathbf{x}_1)$  by:

$$\begin{aligned}\mathbf{x}^e &= (1 - \tau^e) \mathbf{x}_0 + \tau^e \mathbf{x}_1, \\ \tau^e &= -\frac{\phi(\mathbf{x}_0)}{\phi(\mathbf{x}_1) - \phi(\mathbf{x}_0)}.\end{aligned}$$

Depending on position of  $\mathbf{x}^e$  either shift node from  $T$  or add to  $T^E$ .



# Discretization: Mesh Creation



**Figure:** Position of interface points (light blue) on background mesh  $T$ . + show shift/cut regions per edge.



# Discretization: Mesh Creation

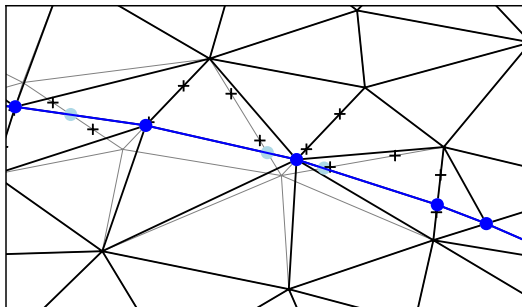
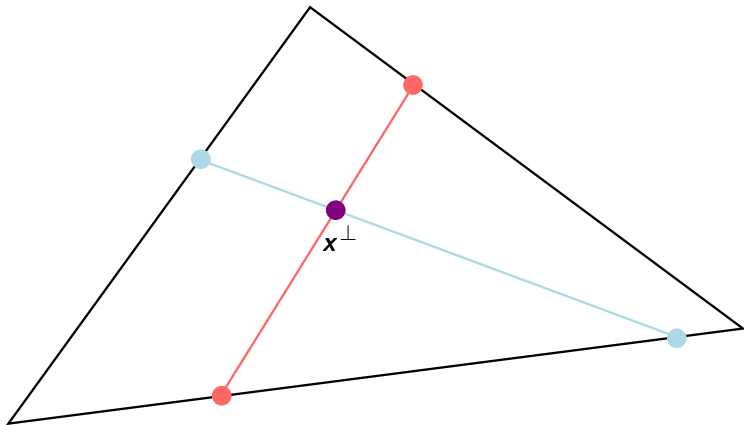


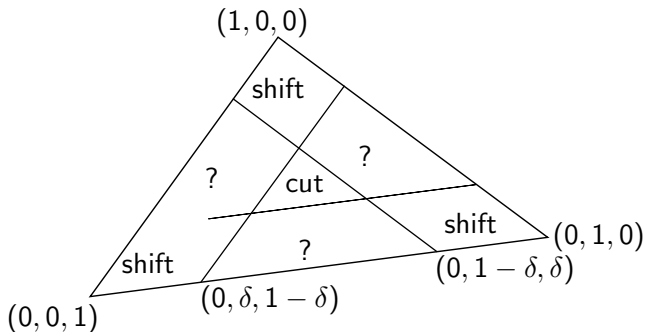
Figure: Enriched mesh  $T^E$  with blue interface points.

# Discretization: Triple Points $x^\perp$



# Discretization: Triple Points

Using barycentric coordinates we can find the position of  $\mathbf{x}^\perp$  in the triangle.



# Discretization: Shift Triple Point

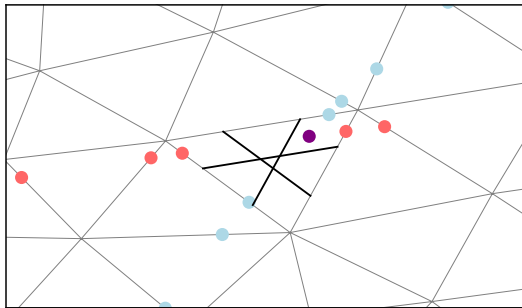


Figure: Triple point in background mesh  $T$  with interface points (blue from  $\phi^{\gamma\alpha}$  and red from  $\phi^{\theta}$ ).

# Discretization: Shift Triple Point

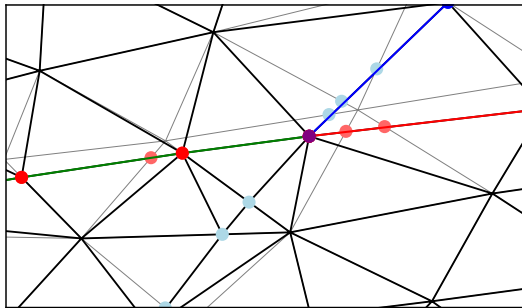


Figure: Shifted to triple point (purple) in enriched mesh  $T^E$  with interface points (blue from  $\phi^{\gamma\alpha}$  and red from  $\phi^{\theta}$ ).

# Discretization: Bend Triple Point

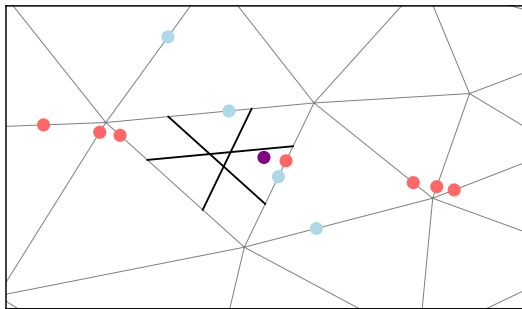


Figure: Triple point in background mesh  $T$  with interface points (blue from  $\phi^{\gamma\alpha}$  and red from  $\phi^{\theta}$ ).

# Discretization: Bend Triple Point

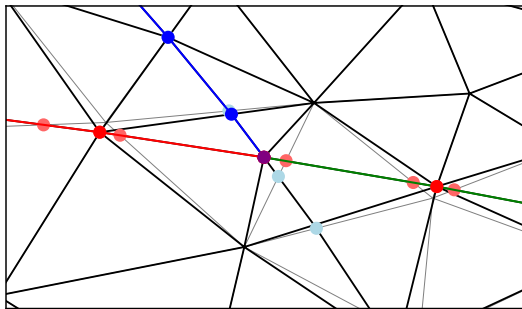


Figure: Bend edge to triple point (purple) in enriched mesh  $T^E$  with interface points (blue from  $\phi^{\gamma\alpha}$  and red from  $\phi^\theta$ ).

# Discretization: Meshing Algorithm

In order:

- Shift triple points;
- Shift vertices to interfaces;
- Shift/cut/bend triple points;
- Cut points;



# Back to Results

# Triple Point Boundary Condition

$$v_n^{\gamma\theta} = \frac{K^{\gamma\theta}}{c_\theta} (c_{\gamma\theta} - c_\gamma), \quad v_n^{\alpha\theta} = \frac{K^{\alpha\theta}}{c_\theta} (c_{\alpha\theta} - c_\alpha).$$

In  $x^\perp$  these velocities have to be equal and  $c_\alpha = c_{\alpha\gamma}$ , this gives

$$c_\gamma = c_{\gamma\theta} - \frac{K^{\alpha\theta}}{K^{\gamma\theta}} (c_{\alpha\theta} - c_{\alpha\gamma}).$$

Only reaction speeds  $K^{\gamma\theta}$  and  $K^{\alpha\theta}$  have some freedom, as

$$K^{\gamma\theta} = k_{\gamma\theta} \frac{D_\gamma}{a_\gamma}, \quad K^{\alpha\theta} = k_{\alpha\theta} \frac{D_\alpha}{a_\alpha},$$
$$k^{\alpha\theta} = k_{\gamma\theta} \frac{D_\gamma}{D_\alpha} \frac{a_\alpha}{a_\gamma}.$$

# Triple Point Boundary Condition

$$v_n^{\gamma\theta} = \frac{K^{\gamma\theta}}{c_\theta} (c_{\gamma\theta} - c_\gamma), \quad v_n^{\alpha\theta} = \frac{K^{\alpha\theta}}{c_\theta} (c_{\alpha\theta} - c_\alpha).$$

In  $x^\perp$  these velocities have to be equal and  $c_\alpha = c_{\alpha\gamma}$ , this gives

$$c_\gamma = c_{\gamma\theta} - \frac{K^{\alpha\theta}}{K^{\gamma\theta}} (c_{\alpha\theta} - c_{\alpha\gamma}).$$

Only reaction speeds  $K^{\gamma\theta}$  and  $K^{\alpha\theta}$  have some freedom, as

$$K^{\gamma\theta} = k_{\gamma\theta} \frac{D_\gamma}{a_\gamma}, \quad K^{\alpha\theta} = k_{\alpha\theta} \frac{D_\alpha}{a_\alpha},$$
$$k^{\alpha\theta} = k_{\gamma\theta} \frac{D_\gamma}{D_\alpha} \frac{a_\alpha}{a_\gamma}.$$

# Triple Point Boundary Condition

$$v_n^{\gamma\theta} = \frac{K^{\gamma\theta}}{c_\theta} (c_{\gamma\theta} - c_\gamma), \quad v_n^{\alpha\theta} = \frac{K^{\alpha\theta}}{c_\theta} (c_{\alpha\theta} - c_\alpha).$$

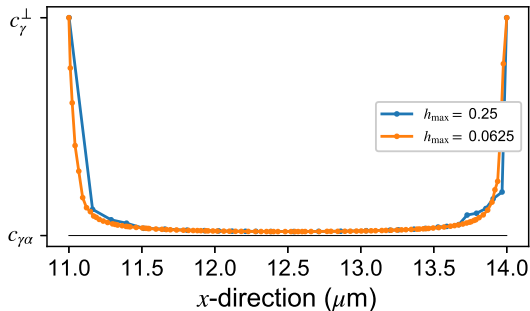
In  $x^\perp$  these velocities have to be equal and  $c_\alpha = c_{\alpha\gamma}$ , this gives

$$c_\gamma = c_{\gamma\theta} - \frac{K^{\alpha\theta}}{K^{\gamma\theta}} (c_{\alpha\theta} - c_{\alpha\gamma}).$$

Only reaction speeds  $K^{\gamma\theta}$  and  $K^{\alpha\theta}$  have some freedom, as

$$K^{\gamma\theta} = k_{\gamma\theta} \frac{D_\gamma}{a_\gamma}, \quad K^{\alpha\theta} = k_{\alpha\theta} \frac{D_\alpha}{a_\alpha},$$
$$k^{\alpha\theta} = k_{\gamma\theta} \frac{D_\gamma}{D_\alpha} \frac{a_\alpha}{a_\gamma}.$$

# Austenite Carbon Concentration Triple Points



**Figure:** Austenite carbon concentration over austenite-ferrite interface for two mesh sizes.

# Austenite Carbon Concentration Triple Points

Remember that on  $\Gamma^{\gamma\alpha}$  in  $\gamma$ :

$$D_\gamma \frac{\partial c_\gamma}{\partial n} = \frac{K^{\gamma\alpha}}{c_{\alpha\gamma}^{\text{sol}}} \left( c_{\gamma\alpha}^{\text{sol}} - c_\gamma \right) \left( c_{\alpha\gamma}^{\text{sol}} - c_\gamma \right) + \frac{D_\alpha}{c_{\alpha\gamma}^{\text{sol}}} \frac{\partial c_\alpha}{\partial n} c_\gamma.$$

The gradient  $\frac{\partial c_\alpha}{\partial n}$  has to be recovered over  $\alpha$ .

# Ferrite Carbon Concentration Gradient

$\frac{\partial c_\alpha}{\partial n}$  is recovered by fitting a second order polynomial  $c_\alpha^{h^2}$  to  $c_\alpha$  over a patch of mesh points and calculating  $\nabla c_\alpha^{h^2} \cdot n$ .

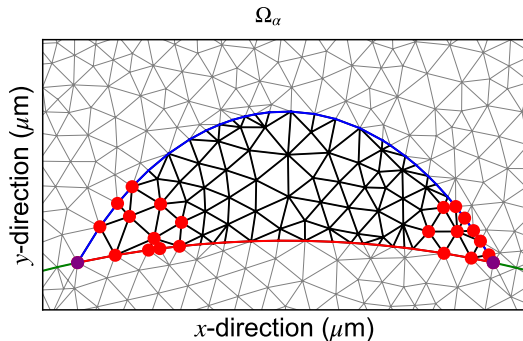
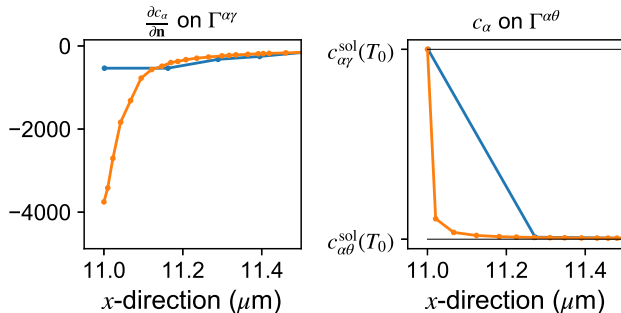


Figure: Patch for left and right triple point.

# Ferrite Carbon Concentration Gradient



**Figure:** Ferrite carbon concentration gradient over austenite-ferrite interface (left) and ferrite carbon concentration over the ferrite-cementite interface (right) for two mesh sizes.



$$\frac{\partial c_\alpha}{\partial n}$$

and

$$v_n^{\gamma\alpha}$$

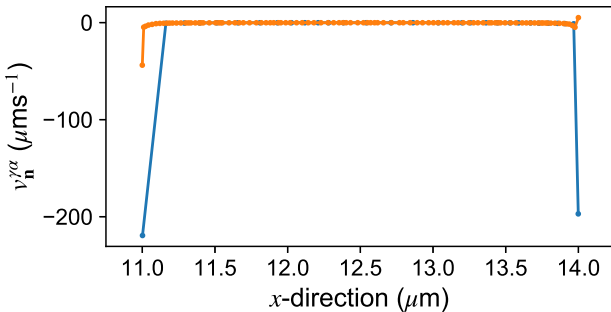


Figure: Austenite-ferrite interface normal velocity for two mesh sizes.

# Conclusion Three-Phase Growth

So difference in results of the model and realistic three-phase behavior is due:

- Unknown carbon concentration for austenite in triple points. 'Chosen' concentration value resulted in unrealistic results.
- Resolution issue for the mesh size used. Finer mesh sizes were too computationally heavy.

However the rest of the simulation show results coinciding with the 1D model of the literature study.

# Conclusion Three-Phase Growth

So difference in results of the model and realistic three-phase behavior is due:

- Unknown carbon concentration for austenite in triple points. 'Chosen' concentration value resulted in unrealistic results.
- Resolution issue for the mesh size used. Finer mesh sizes were too computationally heavy.

However the rest of the simulation show results coinciding with the 1D model of the literature study.

# Conclusion Three-Phase Growth

So difference in results of the model and realistic three-phase behavior is due:

- Unknown carbon concentration for austenite in triple points. 'Chosen' concentration value resulted in unrealistic results.
- Resolution issue for the mesh size used. Finer mesh sizes were too computationally heavy.

However the rest of the simulation show results coinciding with the 1D model of the literature study.

# Conclusion Three-Phase Growth

So difference in results of the model and realistic three-phase behavior is due:

- Unknown carbon concentration for austenite in triple points. 'Chosen' concentration value resulted in unrealistic results.
- Resolution issue for the mesh size used. Finer mesh sizes were too computationally heavy.

However the rest of the simulation show results coinciding with the 1D model of the literature study.

# Conclusion Three-Phase Growth

So difference in results of the model and realistic three-phase behavior is due:

- Unknown carbon concentration for austenite in triple points. 'Chosen' concentration value resulted in unrealistic results.
- Resolution issue for the mesh size used. Finer mesh sizes were too computationally heavy.

However the rest of the simulation show results coinciding with the 1D model of the literature study.

# Thanks for your attention!