Traffic and Transportation Simulation

Looking Back and Looking Ahead: Celebrating 50 Years of Traffic Flow Theory, A Workshop

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Preface

In 1963, the TRB Committee on Traffic Flow Theory (TO-9) and the Committee on Characteristics of Traffic Flow (TO-12) were formed, with D. L. Gerlough and H. L. Michael as chairs, respectively. In 1971, the two committees merged to form the Committee on Traffic Flow Theory and Characteristics (AHB45). In recognition and celebration of these 50 years of traffic flow theory, several special events were organized in 2014, including a Sunday workshop held on January 12, 2014, at the Marriott Wardman Park Hotel in Washington, D.C.

With the accomplishments of the past 50 years in mind, the workshop focused on traffic and transportation simulation—looking back and looking ahead. Coincidentally, this theme was also consistent with the Transportation Research Board 93rd Annual Meeting theme, *Celebrating Our Legacy, Anticipating Our Future*. That year's theme was adopted because it was the year of the final TRB Annual Meeting at the Connecticut Avenue hotels, where it was held for nearly 60 years.

It was an appropriate time to recognize past accomplishments in the simulation field, reflect on the present state of the research community, and identify key future directions.

The committee invited top experts in the field to provide discussion papers on the history, current status, and future of traffic simulation. The audience was asked to provide input and frame a forward-looking discussion of future trends and research needs. We are very pleased to publish eight of these papers in this e-circular as part of our legacy.

The views expressed in the technical papers are those of the individual authors and do not necessarily represent the views of TRB or the National Research Council. The papers have not been subjected to the formal TRB peer review process.

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George List, *North Carolina State University*
Robert Lawrence Bertini, *Portland State University*

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Eighty years ago, Bruce Greenshields presented the first traffic flow model at the Annual Meeting of the Highway Research Board. Since then, many models and simulation tools have been developed. We show a model tree with four families of traffic flow models, all descending from Greenshields' model. The tree shows the historical development of traffic flow modeling and the relations between models. Based on the tree we discuss the main trends and future developments in traffic flow modeling and simulation.

INTRODUCTION

Traffic flow models have been applied for almost a century to describe, simulate, and predict traffic. The first model showed a relation between the distance between vehicles and their speed (*1*). Later, dynamics were included in the models and models were applied for predictions (*2*, *3*). Now, traffic flow simulation tools are used for long-term planning as well as for short-term predictions based on actual traffic data. In the future, the models and simulation tools may be developed further to (better) support, for example, adaptive cruise control, dynamic traffic management, and evacuation planning.

In this contribution, we give an overview of past developments in traffic flow modeling and simulation in the form of a model tree showing the genealogy of traffic flow models (Figure 1). It shows how four families of traffic flow models have developed from one common ancestor: the fundamental diagram by Greenshields (*1*). Each of the families, namely the fundamental diagram, microscopic models, mesoscopic models, and macroscopic models, will be discussed in separate sections below. Finally, using the model tree, we identify the main trends and give an outlook for future developments.

FUNDAMENTAL DIAGRAM

The fundamental diagram, as it was originally introduced at the 13th Annual Meeting of the Highway Research Board in 1934, relates the distance between two vehicles (spacing) to their speed (*1*) (Figure 2). However, the author, Bruce Greenshields, became famous for the fundamental he introduced 1 year later at the 14th Annual Meeting (*4*). This fundamental diagram relates the number of vehicles on one unit length of road (density) to their speed (Figure 3).

1
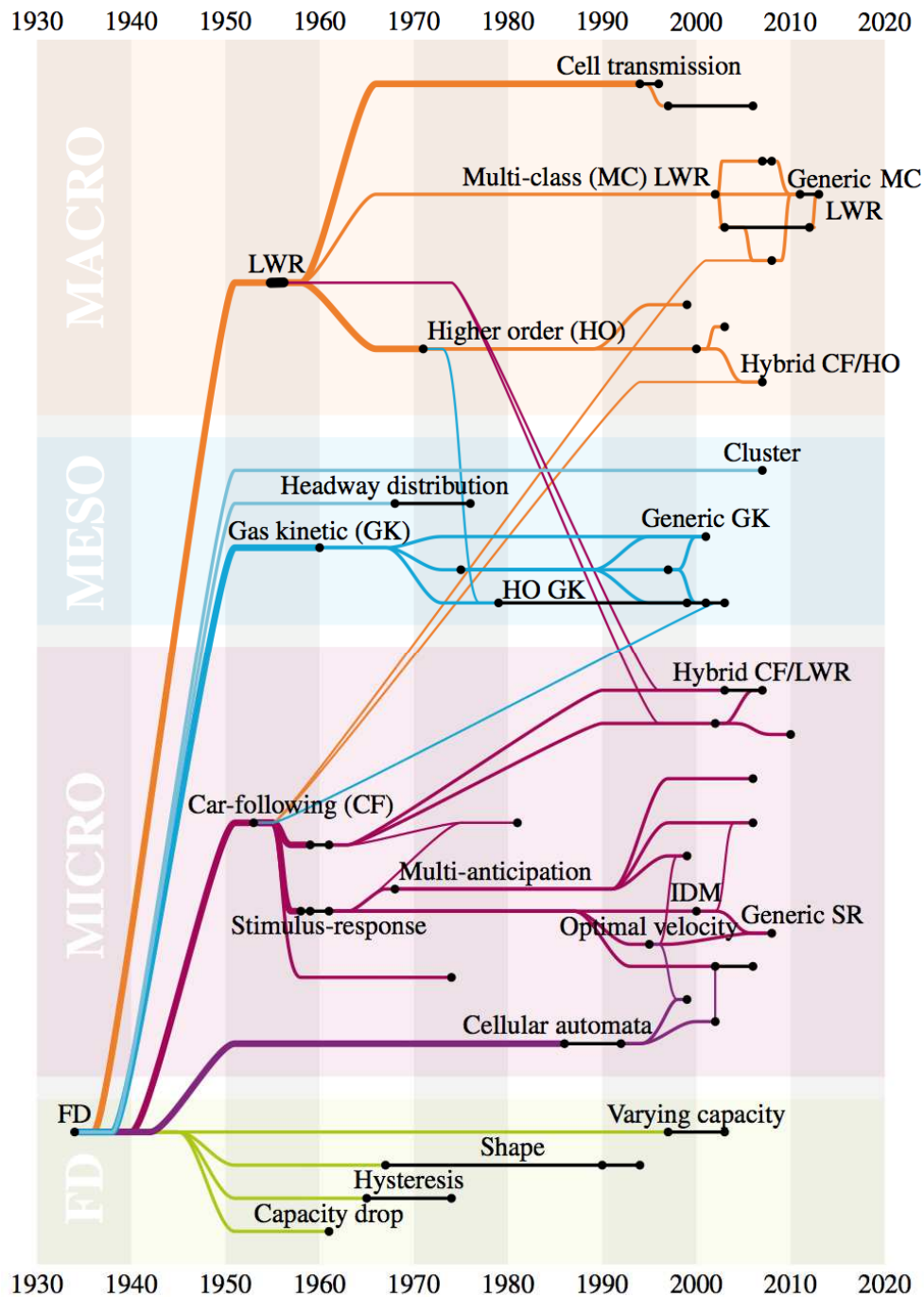
FIGURE 1 Genealogy of traffic flow models. Black dots indicate models, black lines between dots indicate that the same (or a very similar) model has been proposed multiple times, and colored lines indicate descent. A full (and much larger) version of the genealogy can be found in van Wageningen-Kessels (*63, 75*).
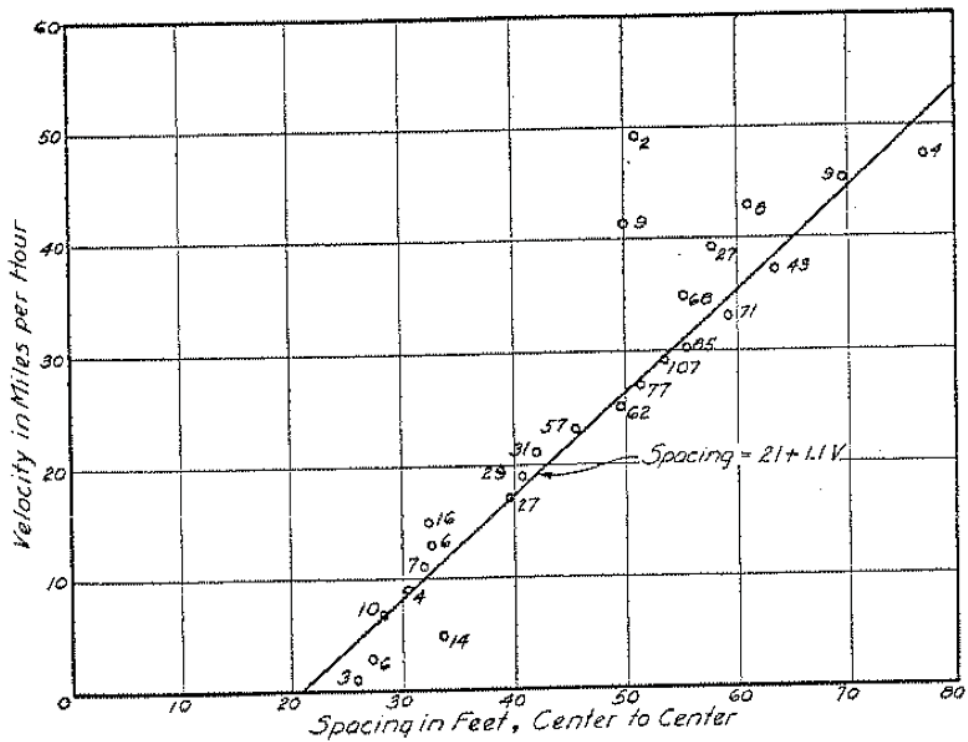
**FIGURE 2 Greenshields' original fundamental diagram (1934), showing a
linear relation between spacing and speed (*1*).**
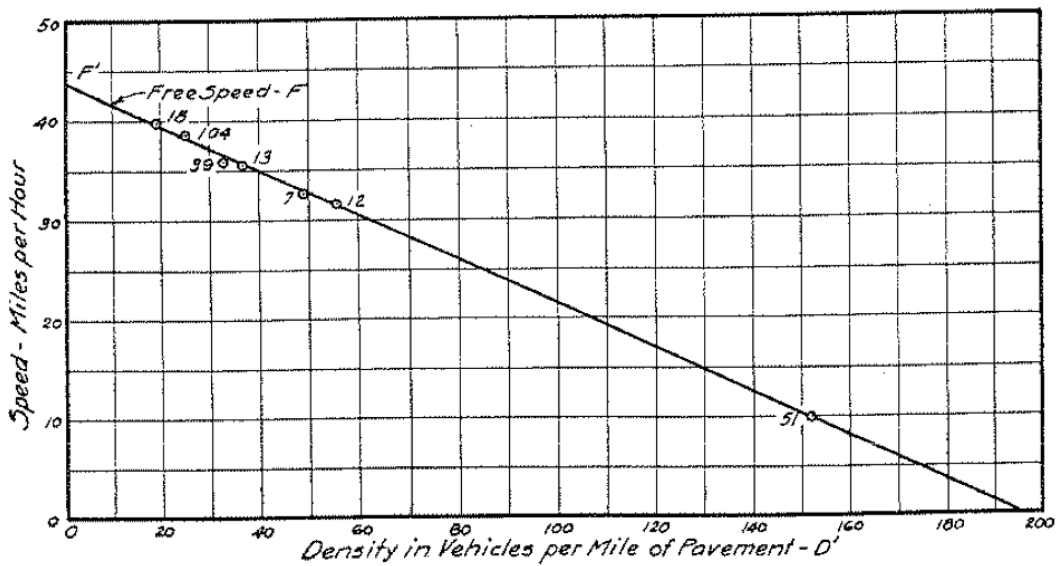
**FIGURE 3 Greenshields' fundamental diagram (1935), showing a
linear relation between density and speed (*4*).**

Shape of the Fundamental Diagram

Since its first introduction the shape of the fundamental diagram has been much debated. Table 1 shows some of the proposed shapes (*4–7*). It also shows an alternative representation of the fundamental diagram, relating the density to the flow: the number of vehicles per time unit. Del Castillo (*8*) recently introduced a set of requirements for the fundamental diagram. Of the fundamental diagrams in Table 2, the ones by Greenshields, Smulders and Daganzo, satisfy the criteria. However, it is argued that they do not represent scatter in observed density–flow (or density–speed) plots well enough.

Scatter in the Fundamental Diagram

Scatter in observed density–flow plots (Figure 4) is partly introduced by the measurement method and the aggregation of data. The remaining scatter is explained and modeled in different ways. In 1961, Edie (*9*) proposed a fundamental diagram with a capacity drop. The capacity drop models that the outflow out of a congested area is lower than the flow just before breakdown.

**TABLE 1 Different shapes of fundamental diagrams,
in density–flow and in density–speed plane.**
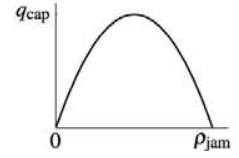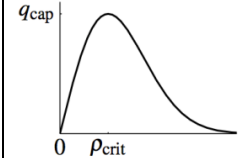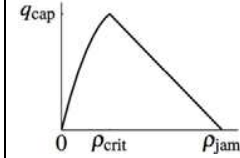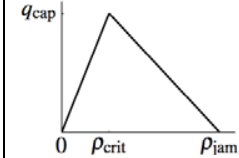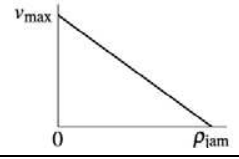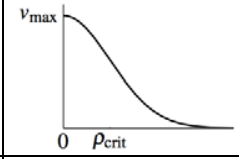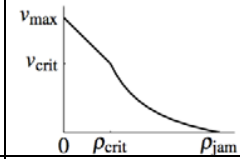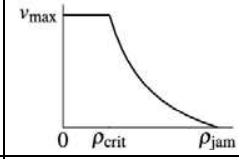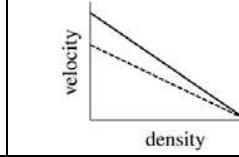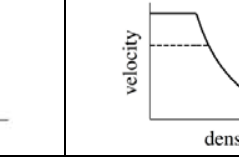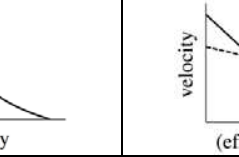
Density–flow				
Density–speed				
Shape	Parabolic	Bell	Parabolic–linear	Bilinear
Author	Greenshields	Drake	Smulders	Daganzo
Year	1934	1967	1990	1994
Reference	*4*	*5*	*6*	*7*

TABLE 2 Fundamental Diagrams of Multiclass Kinematic Wave Models

(Effective) Density–Speed			
Reference multiclass	Benzoni-Gavage and Colombo (*56*)	Logghe and Immers (*59*)	Chanut and Buisson (*13*); Van Wageningen-Kessels et al. (*61*)
Author mixed class	Greenshields (*4*)	Daganzo (*7*)	Smulders (*6*)

NOTE: Solid line = cars; broken line = trucks.
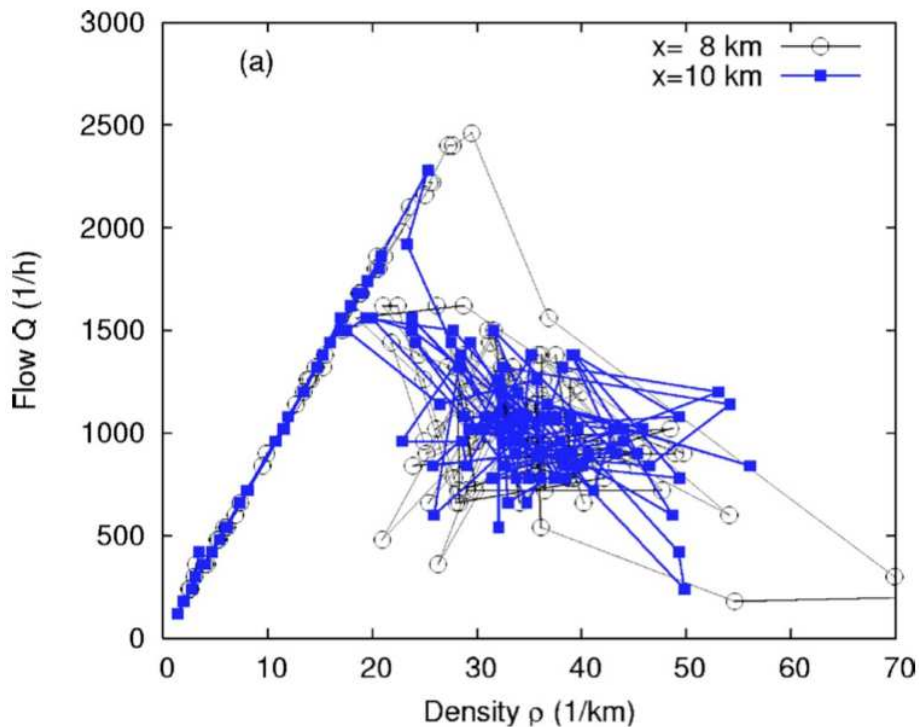
FIGURE 4 Scatter in an observed density–flow plot (*76*).

Using the graphs in Figure 5, Edie showed that a fundamental diagram with capacity drop better represents scattered data. A few years later, in 1965, Newell (*10*) introduced the concept of hysteresis: in congestion, when accelerating the density–speed relation is different from the relation when decelerating. Almost a decade later, Treiterer and Myers (*11*) showed that hysteresis could explain much of the observed scatter (Figure 6). In 1997, Kerner and Rehborn (*12*) take a different approach by proposing another nonunique relation between density and flow. They argue that in congestion, traffic may be in any state in the gray area in Figure 7. Finally, in 2003, Chanut and Buisson (*13*) propose a three-dimensional fundamental diagram. In this fundamental diagram the density of cars is taken into account separately from the density of trucks. Therefore, with the same total number of vehicles, a larger share of trucks leads to lower speeds (Figure 8).

MICROSCOPIC TRAFFIC FLOW MODELS

The three other families in the model tree include dynamics. They describe how traffic states evolve over time. The microscopic model family is the oldest of those families. Microscopic models describe and trace the behavior of individual vehicles and have evolved into car-following models, including three branches and one separate branch including cellular-automata models.
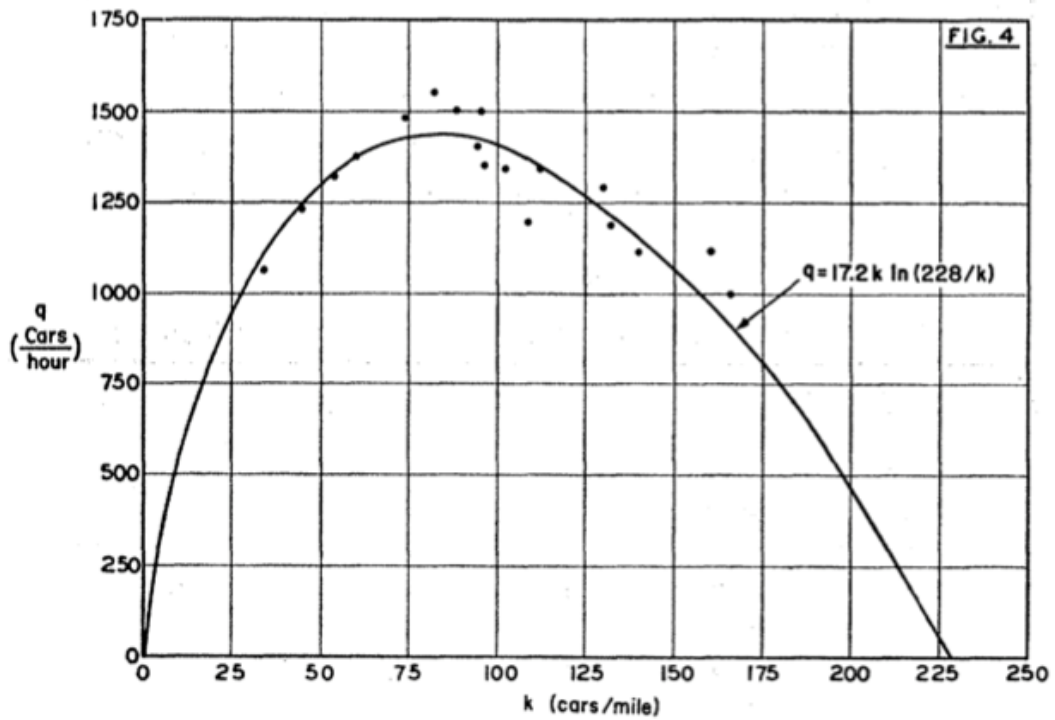
(a)
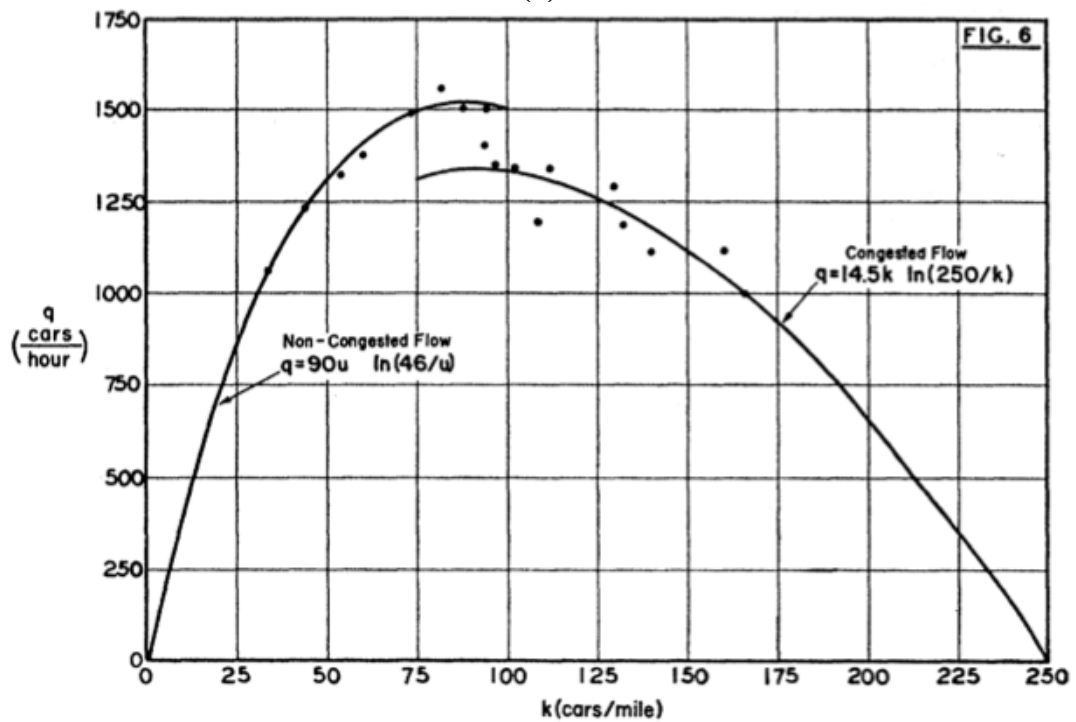
(b)

**FIGURE 5 Fundamental diagram (*a*) without and (*b*) with capacity drop. The graph shows
a better fit with the data of the fundamental diagram with capacity drop (*9*).**
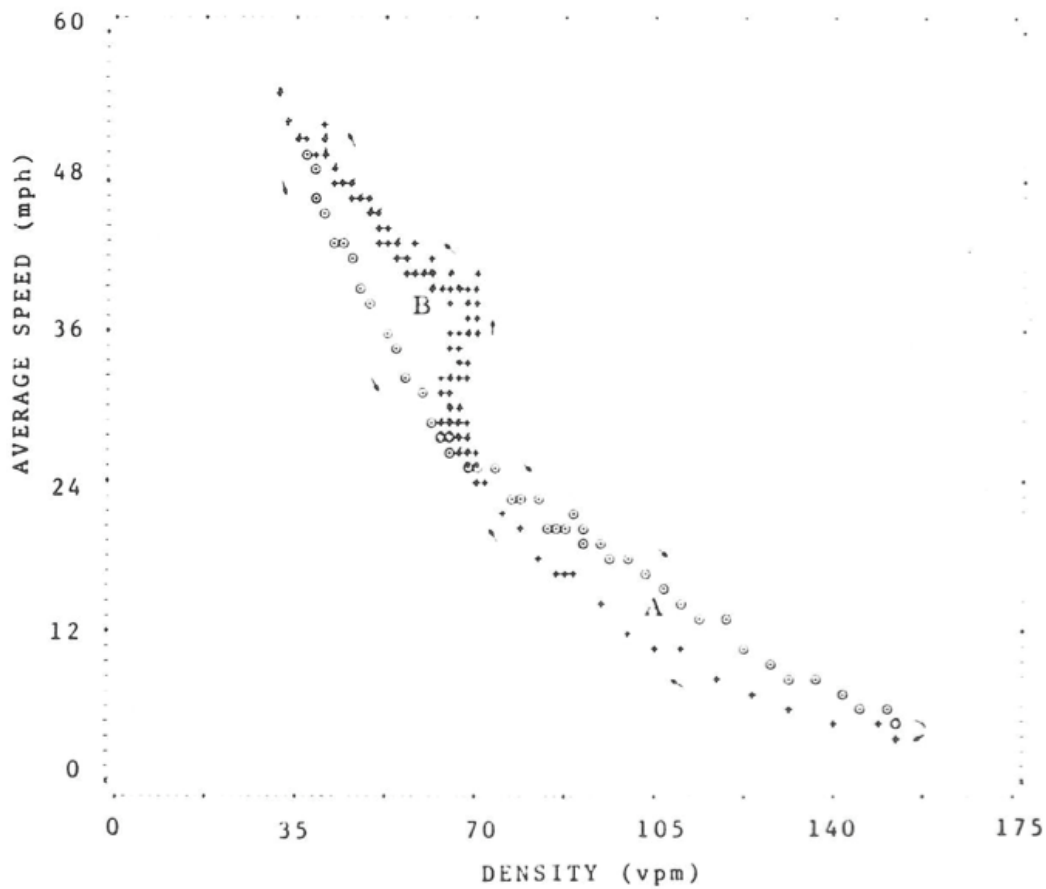
FIGURE 6 Observed densities and speeds showing the hysteresis phenomenon. At relatively low densities, speeds are higher when accelerating (diamonds) than when decelerating (circles), at relatively high densities, it is the other way around (*11*).
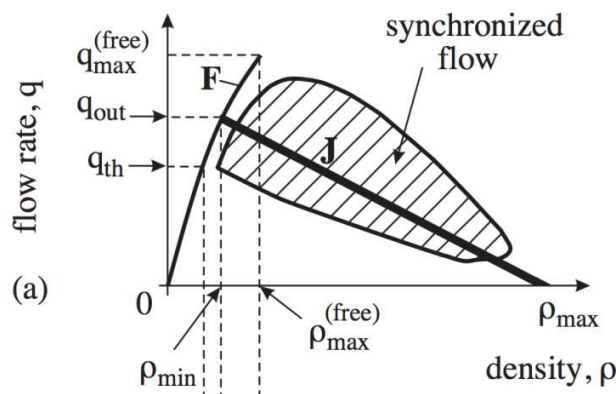
FIGURE 7 Fundamental diagram with infinitely many admissible states in the congestion branch (shaded area) (*77*). For detailed explanation of the labels, see Kerner (*77*).
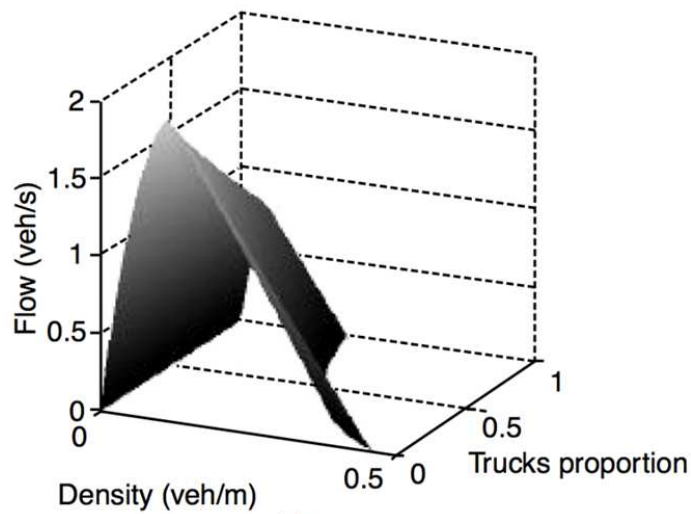
**FIGURE 8 Three-dimensional fundamental diagram where a high
trucks proportion leads to lower speeds and flows (*13*).**

Safe-Distance Models

The earliest car-following model was a safe-distance model and was introduced by Pipes in 1953
(*2*). In his model, vehicles adjust their speed according to a safe distance to their leader, as
illustrated in Figure 9. Safe-distance car-following models were refined by Gipps in 1981 by
introducing two regimes (*14*). In one regime the speed is limited by the vehicle or the (legal)
speed limit and in the other regime the speed is reduced because the drivers keeps a safe distance
to the leading vehicle.

A revival of safe-distance models took place in the last decade, starting by Newell with a
simplification of his 1961 car-following model (*15*, *16*). This simplified car-following model has
been shown to be equivalent to certain models in the cellular-automata branch and in the
kinematic wave branch (*17*, *18*). The equivalence is used to develop hybrid models combining
properties of microscopic and macroscopic models (*19*, *20*).

Stimulus–Response Models

The second branch of car-following models consists of stimulus–response models. The model
tree shows a rapid development of these models in around 1960 (*21–24*). The authors propose
that acceleration of drivers can be modeled as a reaction to three stimuli:

1. Own current speed,
2. Distance to leader, and
3. Relative speed with respect to leader.

A lot of effort has been put into calibrating and validating stimulus–response models.
However, in 1999 Brackstone and McDonald (*25*) concluded that the models were used less
frequently because of contradictory findings on parameter values. Nevertheless, new stimulus–
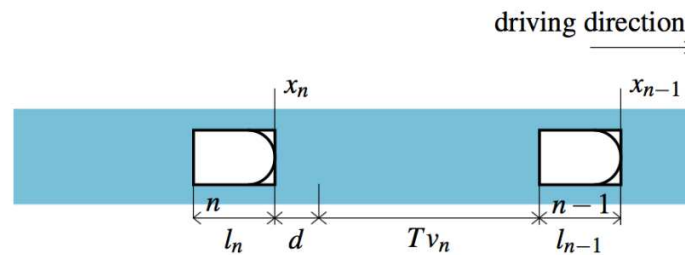
FIGURE 9 Variables and parameters in Pipes' safe-distance model. The position x_n of the nth vehicle is determined the position x_{n-1} of its leader and the safe distance between them. The safe distance is a constant distance determined by the distance at standstill d, the vehicle length l_{n-1} and a variable safe stopping distance Tv_n with T being safe time headway and v_n the speed.

response models have been developed since, including the optimal velocity model (*26*) and the intelligent driver model (*27*). Again, it is argued that it is often difficult, if at all possible, to find good parameter values (*28*). Wilson and Ward argue that researchers should focus on a small subset of stimulus–response models with good qualitative properties. Wilson also proposes a framework to assess the models with respect to qualitative properties (*29*).

Action Point Models

Action point models form the third, and last, branch of car-following models. They were first introduced by Wiedemann in 1974 (*30*). For these models, it is assumed that drivers only react if the change is large enough for them to be perceived. In contrast to other car-following models, this implies that driving behavior is only influenced by other vehicles if headways are small and if changes in relative velocity or headways are large enough to be perceived.

Cellular-Automata Models

Cellular-automata models are usually categorized in the microscopic model family, as a branch separate from the car-following models. In cellular-automata models, the movement of individual vehicles is described and traced, just like in other microscopic models. In contrast to car-following models, space and sometimes time is discretized as well. The first model in this branch stems from 1986 (*31*) but the model introduced in 1992 by Nagel and Schreckenberg (*32*) is regarded as the prototype cellular-automata model. The road is discretized into cells and in each time step each vehicle is advanced zero, one, or more cells, according to a certain algorithm. Some of the most popular cellular-automata models are compared in Knospe et al. (*33*).

MESOSCOPIC MODELS

Mesoscopic models fill the gap between microscopic models that model and trace the behavior of individual vehicles and macroscopic models that describe traffic as a continuum flow. Mesoscopic models describe vehicle movements in aggregate terms such as probability distributions. However, behavioral rules are defined for individual vehicles. The family of

mesoscopic models includes headway distribution models (*34, 35*) and cluster models (*36*). However, the oldest and most extended and popular branch within this family consists of gas–kinetic traffic flow models.

Gas–kinetic traffic flow models were first introduced in the early 1960s (*37, 38*). It describes traffic flow in a way similar to how gas is modeled in gas–kinetic models. The movements of vehicles (or molecules in a gas) are not modeled individually. Instead, distributions of density and speeds are used to calculate and lead to expected densities and speeds. A first revival of the branch took place in the mid- and late 1970s with an improved model (*39*) and a continuum gas–kinetic model (*40*). A second revival of gas–kinetic traffic flow models took place from the mid-1990s. The older models were extended and generalized (*41, 42*) and more continuum models were derived (*43–46*).

MACROSCOPIC MODELS

The fourth and last family in the model tree consists of macroscopic models. They describe traffic as if it were a continuum flow. Only aggregated variables such as (average) density, (average) flow, and (average) speed are considered. The family consists of two major branches: kinematic wave models and higher-order models. In order to include differences between types of vehicles (e.g., passenger cars and trucks), multiclass versions of both types of macroscopic models are developed as well.

Kinematic Wave Models

The prototype macroscopic model is a kinematic wave model introduced in the mid-1950s by Lighthill (*3*) and, independently, Richards (*47*). This model, also known as the LWR model, has received much attention and critique. The main critique is that vehicles are assumed to attain new speeds immediately after a change in the density. This implies infinite acceleration or deceleration. The issue has mainly been dealt with in higher-order macroscopic models (see next section), but also by relatively recent variants of the LWR model including bounded acceleration (*48, 49*). In the original LWR model, the transition from free-flow to congestion regime (breakdown) always happens at the same density and without capacity drop. This is considered as a second major drawback. It was addressed by introducing lane changes (*50, 51*) and by introducing breakdown probabilities (*52*).

LWR models are often used for simulations studies as they are relatively simple and computations can be done fast. Therefore, space and time are discretized into spatial cells of typically 200 m long and time steps of 0.5 s to several seconds. Densities in each cell are computed using the old densities and the flow into and out of the cell each time step. This approach is used in the cell transmission model introduced by Daganzo in 1994 (*7*) and the Godunov scheme (*53*). More advanced and accurate simulation methods have been introduced in the past few years (*18, 54*).

Furthermore, since 2001, many multiclass kinematic wave models have been proposed (*13, 55–62*). They address the issue of breakdown taking place at various densities place by introducing multiple vehicle classes. This model approach also allows for different speeds and other distinctive features for each class. As discussed in the section on fundamental diagrams, multiclass models can reproduce scattered fundamental diagrams. Multiclass models often

include different fundamental diagrams for different classes (Table 2). Furthermore, the speed does not depend on the total number of vehicles per time unit (density) but most models apply an "effective density" to which some classes contribute more than other classes. For example, trucks are supposed to have a higher impact on traffic flow than passenger cars. Therefore, relatively few trucks can create a breakdown, while many passenger cars are needed to do the same. Multiclass kinematic wave models were generalized in the Fastlane model (*61*, *63*), which can also be used to assess multiclass kinematic wave models (*63*, *64*).

Higher-Order Models

Higher-order models form the other main branch in the family of macroscopic traffic flow models. They were first introduced by Payne in 1971 (*65*). Higher order models include an equation to account for the acceleration and deceleration towards the equilibrium speed prescribed by the fundamental relation. This way, they address the issue of infinite acceleration–deceleration in the LWR model. However, also this type of models received much critique. In 1995, Daganzo initiated an ongoing discussion on whether or not higher order models are flawed because they are not anisotropic and on whether traffic flow models ought to be anisotropic (*66*, *67*). The most important implication of a traffic flow model that is not anisotropic is that, in the model and the simulation, vehicles do not only react on their leader but also on their follower which results in vehicles driving backward in certain situations. Since the start of this discussion, many anisotropic models have been developed (*68–70*), including a multiclass higher-order model (*71*).

　　　Other recent models in the higher-order branch include the generalized higher-order model by Lebacque et al. (*72*) and a hybrid model that couples a higher-order model with a microscopic version of it (*73*, *74*).

DISCUSSION AND OUTLOOK

The model tree is used to identify recent trends and provide outlooks for the future.

Trends

We identify four main trends in the model tree.

　　　1.　Certain branches converge to a generalized model. Del Castillo develops a framework that includes most fundamental diagrams (*8*); many car-following models are generalized in Wilson's model (*29*); Hoogendoorn and Bovy generalize gas–kinetic models (*42*); a generalized multiclass kinematic wave model is proposed by Van Wageningen-Kessels et al. (*61*, *63*); and a generalized higher-order model is proposed by Lebacque et al. (*72*).
　　　2.　The LWR model is extended and adapted to better reproduce observations. Zhang proposes a model that includes hysteresis (*68*); Lebacque includes bounded acceleration and deceleration (*48*); and multi-class models are introduced by Wong and Wong (*55*) and many other authors thereafter.
　　　3.　Multiclass versions of different types of models are introduced. Hoogendoorn introduces a multiclass gas–kinetic model (*44*); a multiclass higher order model is introduced by

Bagnerini and Rascle (*71*); and, again, multiclass kinematic wave models are introduced by Wong and Wong (*55*) and many other authors thereafter.

    4. Hybrid models are introduced to combine the advantages of both microscopic and macroscopic models. Bourrel and Lesort (*19*) and Leclercq (*20*) apply the LWR model for hybridization and a higher-order model is combined with a car-following model by Moutari and Rascle (*74*).

Outlook

From the model tree, we see that the cellular-automata and the mesoscopic model families do not receive much research attention recently. Cellular-automata models are used in simulations, but less often than microscopic and macroscopic models. Mesoscopic models are often hard to discretize and are therefore seldom applied in simulation tools. Therefore, we expect that future traffic flow modeling and simulation will focus on new and improved car-following and macroscopic models. Especially for the macroscopic models and simulation tools, good fundamental diagrams will be needed as well.

    Furthermore, we expect the other trends discussed above to set in. The development of generalized models as described in the first trend is valuable to assess models and to select qualitatively appropriate models. Future developments include even more generalized models and assessment of existing and new models. This way, it can be prevented that qualitatively inferior models, which inevitably lead to quantitatively poor results, are applied in simulations. Furthermore, it is prevented that resources are spend in quantitatively calibrating models that will give qualitatively undesirable results.

    Microscopic models and simulation tools predict traffic in more detail than macroscopic models. Therefore, they are well-suited for adaptive cruise control and similar applications where it is necessary to predict the behavior of individual vehicles. However, in many applications the details are less important and fast computations achieved with macroscopic models are necessary. This includes dynamic traffic management for large areas and evacuation optimization. For these applications more realistic macroscopic models as described in the second and third trend are valuable. Finally, some applications require on the one hand detail and accuracy in a small area and on the other hand fast computations to make predictions over a longer time horizon. These applications benefit from the fourth trend in which hybrid models are developed. Detailed predictions can be made, for example for a small urban area, and the less detailed prediction for the larger surrounding area allow for fast computations.

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Brief History of Traffic Simulation

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This paper presents a brief history of the development of traffic simulation from the 1950s to the present time, largely from an autobiographical perspective. Since I participated in some of the early developments and had personal exposure to those pioneers who preceded me, I am hopeful that this perspective can provide insights that a chronological literature survey, alone, could not provide. General-purpose digital computers became available in 1952. A small number of researchers at universities which had access to these early computers became interested in simulation technology and developed software applied to individual intersections and short freeway sections. Other researchers recognized the need to represent traffic flow in analytical terms and developed formulations which could be utilized by simulation modelers. In the 1960s and 1970s, as computers became more plentiful and of greater power, FHWA, NCHRP, and other national research agencies, supported the development of useful network simulation models. As a result, the technology's value in the field of traffic operations and control became apparent to an increasing number of practitioners. In the 1980s, simulation models that integrated traffic operations with traffic assignment were introduced; these attracted transportation planners who were seeking more effective, equilibrium-based tools. The continuing development of personal computer (PC) technology has fostered the development of regional simulation-based multimode models which are now routinely applied by practitioners worldwide. The pressing need to manage transportation systems to be increasingly productive and efficient in an environment of increasing demand will require simulation-based tools well into the future.

INTRODUCTION

Traffic simulation-based models describing all modes of transportation are now applied routinely by practitioners and researchers to analyze facilities ranging from individual intersections to extensive regional networks of some 50,000 links. The ability of simulation models to reliably detail the dynamic traffic environment and to be integrated with a range of other analytical models used by traffic engineers, transportation planners, and system designers, provides professionals with powerful tools. The widespread use of such models is reflected in the hundreds of papers presented at 93rd Annual Meeting of the Transportation Research Board in 2012 that discussed simulation models.

The emergence of traffic simulation parallels the emergence development of digital computers; the first developments in both technologies occurred some 60 years ago in the United States and in Europe; later developments originated in Asia. To provide an historical perspective, this paper presents a chronological account which identifies the "prime movers" and events, with an acknowledged bias to those who contributed in the United States. The development of simulation models did not take place in a vacuum: there were many relevant and supportive contributions by innovators in related fields who are likewise identified. Since the author participated in the simulation activity, several anecdotal tales derived from personal communication will be included to present a picture of the professional environment of each historical period.