DELFT UNIVERSITY OF TECHNOLOGY

FACULTY OF ELECTRICAL ENGINEERING, MATHEMATICS AND COMPUTER SCIENCE

TEST SCIENTIFIC COMPUTING (wi4201)
Wednesday January 6 2016, 13:30-16:30

1. (a) Give the definition of the 2-norm of a vector x and a matrix A.

   (b) A is an SPD matrix. Show that $\|A\|_2$ is given by:

   $$\|A\|_2 = \max\{\lambda_j | \lambda_j \in \sigma(A)\}.$$

   (c) Give an upperbound for $\|A\|_2$, where A is given by:

   $$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 6 \end{pmatrix}.$$

   (d) Given the linear system $Au = b$ and the perturbed system $A(u + \Delta u) = b + \Delta b$. Derive an upperbound for $\frac{\|\Delta u\|}{\|u\|}$ where $\|.\|$ is an arbitrary vector norm.

2. In this exercise we have to solve a linear system $Au = b$, where A is an $n \times n$ SPD matrix.

   (a) Take $u_1 = \alpha b$. Derive an expression for α such that $\|u - u_1\|_2$ is minimal.

   (b) Give the definition of a Krylov subspace of dimension k, matrix A and starting vector b.

   (c) Give the optimisation property of the Conjugate Gradient method. Motivate why $u_n = u$ (without rounding errors).

   (d) Given the algorithm at the top of page 101. Determine the minimal amount of memory and flops per iteration (+ motivation).

3. (a) Give the three most important properties of the Conjugate Gradient method.

   (b) Give three classes of Krylov type methods for the solution of $Au = b$, where A is a non-singular matrix. Summarise the advantages and disadvantages for each class of methods.

   (c) Suppose that A can be written as: $A = XDX^{-1}$. Show that

   $$A + 3A^2 - A^3 = X(D + 3D^2 - D^3)X^{-1}$$

   (d) Given figure 7.9 in the lecture notes. Motivate when it is beneficial to use Bi-CGSTAB, GMRESR or GMRES.