Waveform Design for Wideband Beampattern and Beamforming

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Abstract—This paper considers wideband beamforming and waveform design for transmission from advanced multiple channel radar systems. This paper describes the initial designs of a wideband transmit beamformer for a multi-channel high resolution radar, applicable to airborne and spaceborne radars. As a case study for this paper, we use the EcoSAR instrument developed by NASA Goddard Space Flight Center for the measurement of science parameters. The wideband EcoSAR employs two multi-channel antennas and full transmit and receive digital beamforming to generate high resolution images of the ground features. Maintaining a consistent beampattern when the bandwidth is approximately 50% of the center frequency is a challenge. Breaking this nexus relies on optimum beam weight design and accompanying waveform design; not merely the former as is typically done. The approaches of this paper have shown how the beampatterns can be maintained while meeting beam requirements.

I. INTRODUCTION

Advanced radar systems rely on multi-channel, wideband operations. One of the most challenging functions is wideband transmit beamforming, because of the difficulty multiple waveforms with the correct beamsteering coefficients that vary with frequency. Conventional narrowband beamforming, where the percentage bandwidth is small compared to the carrier frequency, only require a unique set of invariant beamsteering coefficients per look angle.

Transmitter and receiver beamforming for wideband signals have been previously studied in the literature. The design of finite-impulse response (FIR) filters to directly optimize the resulting wideband array pattern is proposed in [1] and [2]. Frequency-dependent steering vectors are separated using a Bessel function and used to focus data vectors to a single frequency component in [3]. Subband decomposition methods with different filter banks for wideband beamforming are proposed in [4] and [5]. The linear least-squares based beampattern optimization has been studied in multiple papers.

Differing from these approaches and as described in this paper, we consider the problem from two perspectives: as a set of frequency-dependent array weights, and as a set of channel-dependent waveforms. Our aim is to design a multi-channel signal set that successfully performs wideband beamforming.

As example platforms with wideband multi-channel architectures, two airborne digital beamforming SAR systems, Ecosystem SAR (EcoSAR) and second generation Digital Beamforming SAR (DBSAR-2) [6]–[8], provide testbeds for the implementation and evaluation of our techniques. These instruments use solid-state power amplifiers at each of the radiating channels. Each channel of the multi-channel system has an independent FPGA-based arbitrary waveform generator. These systems are designed to measure science parameters, including forest biomass and forest canopy height.

For this paper, we use EcoSAR as a case study. The wideband EcoSAR employs two multi-channel antennas and full transmit and receive digital beamforming to generate high resolution images of the ground features. Multiple independent beams may be formed on either side of the track, as depicted in Figure 1. As known, the center frequency of the EcoSAR system is 435 MHz, with a frequency spread of ±100 MHz. Thus, the bandwidth (BW) of 200 MHz is a significant fraction of the carrier frequency. In general, 435 MHz is known to be in the P-band. This frequency was selected because its wavelength is known to penetrate deep into a forest’s canopy. Others have discovered that the P-band is excellent for monitoring forests [9], [10].

Fig. 1: The EcoSAR multi-channel SAR system can generate multiple beams for high resolution imaging.

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II. BeamPattern Definitions and Metrics

A. Narrowband Beampattern

The far-field, narrowband beampattern of a horizontal uniform linear array (ULA) is given by,

\[
G(\theta) = \left| \sum_{n=0}^{N-1} e^{-j2\pi n d \sin \theta / \lambda} \right|^2
\]

where \( N \) is the number of sensor elements, \( \theta \) is the azimuth angle \((-\pi/2 \leq \theta \leq \pi/2)\), \( d \) is the inter-element spacing, and \( \lambda \) is the operating wavelength. Figure 2 shows the narrowband beampattern for a sixteen element ULA with an inter-element spacing \( d = \lambda/2 \).

B. Wideband Beampattern

The bandpass signals whose complex envelopes satisfy the

\[
\text{BW } (N - 1) \frac{d}{c} \ll 1,
\]

is defined as narrowband signals [11]. The equation (1) is valid when the signal bandwidth is narrow enough to satisfy the assumption of constant wavelength. In wideband signals (i.e., where the bandwidth is a large fraction of the center frequency), this assumption is not satisfied. Note that, the left hand side of the equation (2) is computed 2.8037 using the EcoSAR design parameters, which does not satisfy the narrowband signal criteria. In this case, it is appropriate to represent the array pattern as a two-dimensional function of frequency and azimuth angle. Thus, the frequency-dependent wideband array pattern for a beam that is steered toward a desired azimuth angle \( \theta_i \) is given by

\[
G(\theta,f) = \left| \sum_{n=0}^{N-1} e^{-j2\pi f n d (\sin \theta - \sin \theta_i) / c} \right|^2
\]

where \( f \) is the frequency and \( c \) is the speed of light. Figure 3 depicts the two-dimensional wideband beampattern for a 16-element ULA when steered to 30°. The wideband beampattern has some interesting properties that point to the problem to be solved. In creating Figure 3, the antenna elements were spaced by half wavelengths \((d = 0.2802 \text{ m})\) as calculated at the smallest wavelength within the signal bandwidth (i.e., the highest frequency). The physical size of the array is fixed, but the electrical size varies with frequency. At the highest frequency (535 MHz), the array’s electrical size is at its largest; hence, the beampattern at this frequency has the narrowest mainlobe and the most nulls. In fact, a slice of the beampattern in Figure 3 taken at 535 MHz is equal to the pattern shown in Figure 2. As frequency decreases, however, the wavelength increases and the electrical length of the array becomes smaller. Therefore, the beampattern widens, and we observe the two-dimensional frequency-dependent pattern. From the perspective of a single azimuth angle, we see that the illumination pattern varies over the bandwidth of the waveform. For some azimuth angles, the transmit array has a null at some frequencies within the signal bandwidth, but not at other frequencies.

It will be useful to define a metric that reduces the two-dimensional wideband beampattern to a one-dimensional pattern that can be compared or designed to meet a desirable behavior. Two different metrics appear reasonable for this purpose. First, we could take the maximum of the wideband beam power pattern over all frequencies at a given angle, such that

\[
g(\theta) = \max_f \{ G(\theta, f) \}. \tag{4}
\]

Alternatively, we can use the total energy of the beampattern over all frequencies at a given angle, such that [12]–[14],

\[
g(\theta) = \int G(\theta, f) df. \tag{5}
\]

In this paper, we use the total energy definition, which seems to be more common in the literature. Figure 4 compares these two metrics for the 16-element wideband pattern shown in Figure 3. As seen from the figure, in both metrics the main beam is widened and deep nulls have disappeared due to the frequency dependent beampattern. In some radar applications, it may be preferred to have a frequency-independent (i.e., a pattern with aligned nulls and main beam) beampattern. In the next sections, we introduce optimization-based techniques to align the wideband beampattern.

![Fig. 2: Array beampattern.](image2)

![Fig. 3: Wideband beampattern angle vs frequency.](image3)

![Fig. 4: Wideband beampattern.](image4)
III. BeamPattern Optimization

For a given ULA, the array pattern can be controlled by phase shifters and amplifiers (we assume the amplifiers are operating in their linear region, not in compression). The frequency-dependent measured phase due to a source arriving from angle $\theta$ for a ULA can be written as

$$S_n(\theta, f) = e^{-j2\pi f n d \sin \theta/c}. \quad (6)$$

Therefore, the phase shifts needed to compensate and to steer the beam to target azimuth angle $\theta_t$ are given by

$$\phi_n(f) = e^{j2\pi f n d \sin \theta_t/c}. \quad (7)$$

Next, define a set of complex weights for each channel as $w_n$. The beampattern $y$ that results from steering the beam and applying the weights $w_n$ is

$$y = (\phi \circ w) S, \quad (8)$$

where $(\circ)$ is the Hadamard product, $\phi$ is a row vector obtained from evaluating (7) over the antenna elements for a particular frequency, $w$ is a row vector of the complex weights, and $S$ is a matrix that results from evaluating (6) over the $N$ elements of the array (in the rows of $S$) at $L$ different azimuth angles (in the columns of $S$).

We first propose to find the weight vector $w$ for a given, desired (template) beampattern $y_T$ by minimizing the cost function

$$\arg\min_w \| (\phi \circ w) S - y_T \|_2^2 \quad (9)$$

separately for each frequency, where the $l_2$-norm of an arbitrary vector $v$ is defined as $\|v\|_2 = \sum_k |v_k|^2$. This optimization will give a set of array weights that (approximately) produce the desired one-dimensional beampattern at each frequency. The inner part of the optimization can be written as

$$(\phi \circ w) S = \begin{bmatrix} \phi_1^T & \ldots & \phi_N^T \end{bmatrix} \begin{bmatrix} w_1^T \ldots w_N^T \end{bmatrix} = \begin{bmatrix} S_{1,1} & \ldots & S_{1,L} \\ \vdots & \ddots & \vdots \\ S_{N,1} & \ldots & S_{N,L} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} = \begin{bmatrix} S_{1,1} \phi_1 + \ldots + S_{N,1} \phi_N \\ \vdots \\ S_{1,L} \phi_1 + \ldots + S_{N,L} \phi_N \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} \quad (10)$$

where $L$ is the number of azimuth angles at which the desired template response $y$ is defined. Thus, (9) can be simplified to

$$\min_w \| Hw - y_T \|_2^2 \quad (11)$$

which is nothing more than a least squares problem that can be solved in closed form according to

$$w = (H^T H)^{-1} H^T y_T. \quad (12)$$

In setting up the problem, the inter-element spacing for the ULA should be selected as $d = c/(2 f_H)$ where $f_H = (f_c + BW/2)$ is the highest in-band frequency [14]. Furthermore, the template beampattern should be defined such that it is achievable at the lowest in-band frequency $f_L = (f_c - BW/2)$ [12]. The reason for this last requirement is that the array can always be tapered at higher frequencies to produce a wide pattern; however, if the template pattern is defined at the highest frequency, it may be impossible for the array to achieve the desired gain at lower frequencies. Using the unweighted beampattern at the lowest in-band frequency and half-element spacing at the highest in-band frequency, the template beampattern can be written as

$$y_T(\theta) = \sum_{n=0}^{N-1} e^{-j2\pi f_L n d \sin \theta/\sin \theta_t} \quad (13)$$

Note that this approach (i.e., by solving 9) effectively defines the transmit waveforms by obtaining a set of frequency-dependent weights. In other words, the resulting set of frequency-dependent weights for a particular antenna element gives the frequency-domain specification of the waveform for that element. Unfortunately, because the optimizations are performed in isolation for each frequency, this approach may yield waveforms that are not suitable for transmission by real hardware and may not be appropriate for some radar applications. We address this issue in Section IV by taking a more waveform-centric approach to the design problem.

Sidelobe Reduction (SR)

It is possible to reduce the sidelobes of the beampattern by constraining the sidelobes of the template pattern. For example, one such definition is

$$y_{SR}(\theta) = \begin{cases} \varepsilon \quad y_T(\theta) < T \\ y_T(\theta) \quad \text{otherwise} \end{cases} \quad (14)$$

where $T$ is the threshold value and $\varepsilon$ is a very small value comparing to peak power. One again, this reduced-sidelobe template pattern should be achievable at the lowest in-band frequency; otherwise, there is no potential for a good solution.

IV. BeamPattern Optimization for a Given Signal Set

Assume that $x(t)$ is a desired transmit signal and $X(f)$ is its Fourier domain representation. To steer the main beam through the azimuth angle $\theta_t$, we define a true-time-delayed signal set as

$$x_n(t) = x(t + n \Delta \tau) \quad (15)$$

where $\Delta \tau = d \sin \theta_t/c$. The wideband beampattern for this $N$-dimensional signal set $x_n(t)$ is given by

$$G(\theta, f) = \sum_{n=0}^{N-1} X_n(f) e^{-j2\pi f n d \sin \theta/\sin \theta_t} \quad (16)$$
where \( X_n(f) = X(f)e^{2\pi fn d \sin \theta_i/c} \) is the Fourier domain representation of the signal set \( x_n(t) \). To compute the weightings, we solve the minimization problem

\[
\arg \min_W \| (X \circ w) S - y_T \|^2_2 \tag{17}
\]

where \( X = [X_0, X_1, \ldots, X_{N-1}] \). Similar to (9), after mathematical manipulations the cost function in (17) can be solved as least squares problem. The optimized signal set can be achieved by taking the inverse Fourier transform \( \hat{x}_n(t) = \mathcal{F}^{-1} \{ X_n(f) w_n(f) \} \).

Figure 5 shows the optimized beampattern for a 16-element ULA. A linear frequency modulated (LFM) chirp waveform is used as transmit signal \( x(t) \). Transmit signal \( x(t) \) is time tapered with a Hanning window to smooth the start and end portion of the waveforms and is used to create the signal set \( x_n(t) \). The bandwidth of the waveform is selected as 200 MHz and carrier frequency \( f_c \) is set to 435 MHz. Figure 6 shows the optimized beampattern for the SR processing, where the threshold value for sidelobe reduction is selected 13 dB lower than the peak power. Figure 7 depicts the averaged beampattern of original wideband, narrowband, optimized wideband and optimized wideband with sidelobe reduction waveforms. As seen from the figure, the optimized wideband pattern without SR processing, has same null structure with the beampattern of narrowband waveform. Optimized waveform with SR processing has slightly off first peak location but has a dense power inside the main peak.

The first eight waveforms of the optimized signal set (for the beampattern in Figure 5) are shown in Figure 8. The summation of all sixteen channel has same structure of selected transmit signal \( x(t) \). The instantaneous frequency

\[
f_n(t) = \frac{1}{2\pi} \frac{d}{dt} \varphi_n(t) \tag{18}
\]

(where \( \varphi_n \) is the phase of \( \hat{x}_n \) ) of each individual signal covers the bandwidth of interest which is highlighted with green dashed lines in Figure 8. The instantaneous frequency of each signal behaves unexpectedly in the beginning and end of the signal however this effect can be negligible due to the low amplitude. Keep in mind that this is a positive effect of Hamming tapering in time-domain transmit waveform.

V. Conclusion

Maintaining a consistent beampattern when the bandwidth is approximately 50% of the center frequency is a challenge. Breaking this nexus relies on optimum beam weight design and accompanying waveform design; not merely the former as typically done. The approaches of this paper have shown how the beampatterns can be maintained while ensuring low sidelobes. To motivate our theoretical formulations, this paper has studied a uniform linear array as a precursor to the beamformer and waveform design of our EcoSAR radar system. Some of the important metrics such as power efficiency, pulse compression ratio are still in progress and will finalized after extensive laboratory tests.

REFERENCES

Fig. 8: Instantaneous frequencies (left) plots and Real \{ \hat{x}(t) \} signal (right) plots of the first eight optimized transmit signals.