INTRODUCTION

Advanced radar systems rely on multichannel, wideband operation. NASA’s Ecosystem SAR (EcoSAR) and second-generation digital beamforming SAR [1]–[3] use active array architectures, where an active array radar system contains independent transmit and receive chains with unique amplifiers for each antenna element in the multichannel system. These radar systems have an independent arbitrary waveform generator for each channel and are designed to address wideband transmit beamforming. Unlike conventional narrowband beamforming, which only require a unique set of beamsteering coefficients per look angle, wideband beamforming requires beamsteering coefficients that vary with both look angle and frequency.

Transmitter and receiver beamforming for wideband signals has been previously studied in the literature. In receiver processing, frequency-dependent steering vectors are separated using a Bessel function expansion and used to focus data vectors to a single frequency component in [4]. Subband decomposition methods with different filter banks for wideband beamforming are proposed in [5], [6], [7]. Although frequency-dependent beampatterns can be focused through receiver postprocessing, the actual echoes reflected from the target area have an inherent element of destructive interference due to the frequency-dependent nature of the transmitted waveforms. Thus, intelligent beamforming of the transmitted wideband waveforms is necessary to minimize undesired effects.

The direct optimization of the resulting wideband array pattern through the design of finite-impulse response (FIR) filters for use on the transmitted waveform is proposed in [8], [9], [10]. Scholnik and Coleman were able to achieve low sidelobes in the processed beampattern while preserving constant power over the mainbeam; however, their examples show that the optimized design produces high sidelobes (or grating lobes) outside the bandwidth of interest [8]. Neinhuis and Solbach experimentally verified a FIR filter-based beamforming network, achieving a nearly frequency-independent radiation pattern; however, the maximum sidelobe level was reported as −10 dB [10]. Optimization of the cross-spectral density matrix to generate a desired spatial beampattern, subject to transmitter power constraints, was first introduced in [11]. It was proposed by [11] that even though it is possible to achieve a desired beampattern through optimization, it may be challenging to synthesize desirable waveforms sets that also maintain very low peak-to-average power ratios. To address this issue, He et al. introduced an iterative technique for wideband beampattern formation that constrains the peak-to-average power ratio to achieve waveforms that can be efficiently generated with current hardware [12]. The resulting synthesized waveforms satisfy unit-modulus or low peak-to-average power ratio constraints, but they do not clearly address the formation of a frequency-invariant beampattern. The linear least-squares-based beampattern optimization has been studied in multiple papers.

Differing from previous methods, the approach in this article proposes to design and synthesize a multichannel waveform set that successfully performs wideband beamforming, while preserving a frequency-invariant beampattern, producing low beampattern sidelobes, and creating an effective waveform with a constant modulus main beam power envelope over the target area. In addition to the design study, trade-offs of the resulting optimized waveform are addressed, including power efficiency and pulse compression ratio.

BACKGROUND AND TERMINOLOGY

NARROWBAND BEAMPATTERN

The far-field narrowband beampattern of a horizontal uniform linear array (ULA) is given by
where $N$ is the number of sensor elements, $\theta$ is the azimuth angle $(-\pi/2 \leq \theta \leq \pi/2)$, $d$ is the interelement spacing, and $\lambda$ is the operating wavelength.

**WIDEBAND BEAMPATTERN**

Bandpass signals whose complex envelopes satisfy

$$B (N-1) \frac{d}{c} \ll 1$$

are defined as narrowband signals, where $B$ refers to signal bandwidth and $c$ is the speed of light [13]. Equation (1) is valid when a signal’s bandwidth is narrow enough to satisfy the assumption of constant wavelength. Depending upon the interelement spacing $d$, this assumption may not be valid in wideband signals. In this case, it is appropriate to represent the array pattern as a two-dimensional function of frequency and azimuth angle. Thus, the frequency-dependent wideband array pattern for a beam that is steered toward a desired azimuth angle $\theta_t$ is given by

$$G(f, \theta) = \sum_{k=0}^{N-1} e^{j2\pi f d (\sin \theta - \sin \theta_t)/c}$$

where $f$ is the frequency.

Figure 1 depicts the two-dimensional wideband beampattern for an eight-element ULA (for $\theta_t = 0^\circ$), displaying some interesting properties that highlight the problem to be solved. In this example, the bandwidth is 200 MHz centered at 435 MHz. The antenna elements are spaced by half wavelengths, as calculated at the smallest wavelength within the signal bandwidth (i.e., the highest frequency $f_H$) [14]. The physical size of the array is fixed, but the electrical size varies with frequency. At the highest frequency, the array’s electrical size is at its largest; hence, the beampattern at this frequency has the narrowest mainlobe and the most nulls. However, as frequency decreases, the wavelength increases, and the electrical length of the array becomes smaller. Therefore, the beampattern widens, and we observe the necessity of the two-dimensional frequency-dependent pattern. From the perspective of a single azimuth angle, we see that the illumination pattern varies over the bandwidth of the waveform. For some azimuth angles, the transmit array has a null at some frequencies within the signal bandwidth but not at other frequencies.

It will be useful to define a metric that reduces the two-dimensional wideband beampattern to a one-dimensional pattern that can be used as a reference or designed to meet a desirable behavior. Two different metrics appear reasonable for this purpose. First, we could take the maximum of the wideband beam power pattern over all frequencies at a given angle [14], such that

$$g(\theta) = \max \{G(f, \theta)\}.$$  

Alternatively, we can use the total energy of the beampattern over all frequencies at a given angle [11], [14], [15], such that

$$g(\theta) = \int G(f, \theta) df.$$  

In this paper, we use the total energy definition, which seems to be the more commonly used metric in the literature. Figure 2 compares these two metrics, along with the narrowband pattern resulting from the highest frequency present, for the eight-element wideband pattern shown in Figure 1. As seen in the figure, both metrics result in the deep nulls disappearing relative to any nar-
Waveform Weighting for a Frequency-Invariant Transmit Beampattern

Rowband beampattern within the same frequency range due to the frequency-dependent nature of the wideband beampattern. In some radar applications, such as target detection [4], it may be preferred for the signal to have a frequency-independent beampattern (i.e., a pattern with aligned nulls and main beam). In the next sections, we introduce optimization-based techniques to find complex waveform-weighting matrices to achieve frequency-independent wideband beampatterns.

**BEAMPATTERN WEIGHTING**

For a given ULA with an active array architecture, the array pattern can be controlled directly by altering the waveforms generated in each transmit chain’s digital-to-analog converter. Although amplifiers exhibit nonlinear behavior across their input power range and experience compression at high input powers, radar systems are usually operated in compression, as this results in the maximum power-added efficiency. There exist some approaches for linearly reproducing the desired waveform, including digital predistortion (as described in the “Digital Predistortion” section), which allows the system to linearly reproduce the desired waveform while still operating in the amplifier’s compression region, but in this section, we assume all amplifiers in the system are operating in their linear region and are not in compression. Amplification is assumed to be linear in this section so that the design of the waveform weights can be calculated assuming ideal unweighted waveforms. This assumption is also valid in a physical system with digital predistortion implemented, as each channel uses its individual weighted ideal waveform as the predistortion input signal, thus leading to accurate physical reproductions of the modeled waveforms.

Under these assumptions, it is possible to find the frequency-dependent \([N \times F]\) complex weighting matrix \(W\) for a \([L \times F]\) given the desired (template) beampattern matrix \(Y\) at target azimuth angle \(\theta\). Note that \(F\) is the number of frequency bins, \(N\) is the number of array elements, and \(L\) is the number of beampattern azimuth angles \(\theta\) to be evaluated. Each \([N \times 1]\) column of the complex weighting matrix \(W\) can be represented by the vector \(w_r\). Similarly, each \([L \times 1]\) column of the template beampattern matrix \(Y\) can be represented by the vector \(y\). Both \(w_r\) and \(y\) represent individual frequencies within \(W\) and \(Y\), respectively. The full complex weighting matrix \(W\) can be assembled one column (frequency) at a time by minimizing the cost function

\[
\arg \min_{w_r} [S w_r - y]_+, \tag{6}
\]

where the \([L \times N]\) array steering matrix \(S\) is calculated for each frequency \(f\). The \(r\)th column (array element) of \(S\) is calculated as

\[
S_r(\theta, f) = e^{-j2\pi f u \sin \theta / c}. \tag{7}
\]

This optimization will result in the set of array weights \(w_r\) that most closely reproduce the desired one-dimensional beampattern \(y\) at each frequency.

To accurately produce the required frequency range with the narrowest mainlobe beamwidth and minimum number of elements (to minimize hardware costs), the interelement spacing for the ULA should be selected as \(d = c / (2f_\text{max})\), where \(f_\text{max} = (f_c + B/2)\) is the highest in-band frequency and \(f_c\) is the carrier (center) frequency of the desired signal set [11]. This ensures that no grating lobes appear in the beampattern. In this paper, the primary goal is to obtain a set of waveforms that results in a beampattern that is frequency invariant for the intended platform EcoSAR [16]. The secondary goal is to minimize the mainlobe beamwidth of the beampattern. A template beampattern must be defined such that these goals are physically achievable. For a given ULA antenna spacing, the lowest in-band frequency \(f_\text{min} = (f_c - B/2)\) results in the widest mainlobe [14]. Therefore the frequency-invariant template beampattern \(Y\) should have a mainlobe equivalent to the lowest in-band frequency case to minimize the mainlobe beamwidth, while still allowing a frequency-invariant solution. The array excitation can always be tapered at higher frequencies to produce a wide pattern, but if the template pattern is defined at the highest frequency in the bandwidth, it may be impossible for the array to achieve the desired beamwidth at lower frequencies. Using the unweighted beampattern at the lowest in-band frequency and half element spacing at the highest in-band frequency, the template beampattern can be written as [17]

\[
y(\theta) = \sum_{n=0}^{N-1} e^{-j2\pi f_c n \lambda \sin \theta / c} \tag{8}
\]

To reduce the computational burden and strictly limit the weighting optimization to the frequency bins of interest, we propose to only compute the weights inside the desired bandwidth and to hard set the weights corresponding with frequency bins outside the desired bandwidth. This approach is shown as

\[
w_r = \begin{cases} S^* S \begin{bmatrix} y \\ f_c - B/2 < f < f_c + B/2 \end{bmatrix} & \text{otherwise} \\ \varepsilon & \end{cases} \tag{9}
\]

where \(\varepsilon\) is a very small value in relation to the calculated weights.

Note that this approach effectively defines the transmit waveforms by obtaining a set of frequency-dependent weights. In other words, the resulting set of frequency-dependent weights for a particular antenna element \(w_r\) gives the frequency-domain

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**Figure 2.**

Narrowband beampattern compared with one-dimensional wideband beampattern visualization metrics.
specification of the waveform for that element. A time domain signal set can be achieved by taking the inverse Fourier transform of the weights. Unfortunately, it has been observed that the inverse Fourier transform of the weights suffers from the Gibbs phenomenon, which creates high ripples in the time domain because the resulting weights are strictly band limited, and there is no transition band in the computed weights (the template beampattern is also strictly band limited). This approach can yield waveforms that may not be appropriate for some radar applications. However, these weights can be used to correct the frequency dependency of a selected signal set. Thus, a time domain frequency-independent (weighted) signal set can be achieved by taking the inverse Fourier transform, where $x_n$ is the Fourier domain representation of the desired time domain signal set. The proposed approach to achieve a time domain signal set that will result in the creation of a frequency-independent beampattern is highlighted in Table 1.

For demonstration purposes, a 20-μs windowed linear frequency-modulated (LFM) chirp waveform is used as the desired transmit signal. The window used is equivalent to a Hanning window convolved with a rectangular window to smoothly transition the beginning and ending portions of the waveform, while preserving the central portion of the waveform. The bandwidth of the waveform is selected as 200 MHz, the carrier frequency $f_c$ is set to 435 MHz, and the number of frequency bins is selected as $2^{15}$. Figure 3 shows the resulting weighted beampattern for an eight-element ULA. Note that the waveform, system, and antenna parameters were chosen due to their similarity or applicability to the NASA EcoSAR platform. Note the similarity in general appearance of Figures 1 and 3, except the latter is nearly uniform across the frequency domain, thus demonstrating the frequency-independent nature of the optimization. Figure 4 depicts the weighted beampattern for the same system frequency parameters when the number of array elements is increased to 16, and it can be seen that the frequency-independent nature of the weighted beampattern is further improved as the number of antenna elements is increased. Notice that due to the proposed in-band optimization approach shown in Equation 9 the achieved beampattern is limited to only in-band frequencies. Unlike the other approaches mentioned in the “Introduction” section, no unwanted lobes outside the desired bandwidth are produced.

### Sidelobe Reduction (SR)

In addition to the goal of obtaining a waveform set that results in a frequency-invariant beampattern with a minimal mainlobe beamwidth, the additional goal of minimizing the peak sidelobe level can also be implemented. It is possible to reduce the sidelobes of the designed beampattern by constraining the sidelobes of the template pattern [17]. For example, one such definition is

$$y_{SR}(\theta) = \begin{cases} \varepsilon & y(\theta) < T \\ y(\theta) & \text{otherwise} \end{cases}$$

(10)

where $T$ is the power threshold value and $\varepsilon$ is a very small value compared to the peak power. The new SR template pattern $y_{SR}$ is then used in place of $y$ in Equation 9. The idea behind the thresh-
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The existing approach is to reduce the sidelobes, while using the same unconstrained optimization scheme. This approach can be replaced with a constrained optimization or min-max design (not covered in this paper) to achieve the exact peak-to-sidelobe ratios (PSLR).

For an eight-element ULA, the beampatterns of the original wideband waveform case, the weighted wideband waveform case, and the weighted wideband waveform with the SR case are shown in Figure 5, using the total energy metric described in the “Wideband Beampattern” section, where the beam power is adjusted to the same level for comparison purposes. The power threshold value $T$ in Equation (10) for SR is selected as 11 dB lower than the peak power to preserve the main beam, while suppressing the sidelobes. The parameter $\epsilon$ is set to a value of $2^{-52}$ (IEEE Standard 754 for double precision). Note that the weighted wideband beampattern without SR processing has the same null structure as the beampattern of a narrowband waveform at the lowest in-band frequency [17], while the weighted waveform with SR processing has a slightly widened first peak, with more of the beampattern’s overall power inside the main peak. In Figure 5, the PSLR of the averaged beampatterns of the original wideband waveform, the weighted wideband waveform (referred to as SR-Off), and the weighted wideband waveform with beampattern SR (referred to as SR-On) are computed as 14.31 dB, 12.64 dB, and 23.54 dB respectively.

DIGITAL PREDISTORTION

As mentioned in the beginning of this section, some approaches exist for linearly reproducing desired waveforms in hardware exhibiting nonlinear output characteristics. The sets of wideband waveforms formed through weighting both with and without SR exhibit nonconstant envelopes and require high-fidelity generation in both amplitude and phase, making the wideband waveforms ideal candidates for digital predistortion as presented in [18]. The main purpose behind digital predistortion is to make possible high-quality synthesis of desired waveforms at high amplifier output powers. This is accomplished by carefully remapping given input signals to different input signals, so when altered by the nonlinear distortion of the high power amplifier (HPA), the desired ideal output signals result. This also allows the system to minimize spectral regrowth, while still operating in the compression region of the amplifier, thus maximizing power added efficiency. In practice, this is accomplished through collecting a set of calibration data for the transmit chain that captures the behavior of the system over the amplitude and frequency range to be used. Using a memory polynomial (MP) model with predetermined order of memory and nonlinearity to represent the wideband predistortion function, the calibration data is used as a set of training data to solve for the set of coefficients satisfying the least-squares solution to the MP model. These coefficients are then used by the MP model to predistort the desired beamforming waveforms, thus maximizing power added efficiency and minimizing spectral spreading upon amplification. A high-level flowchart of the digital predistortion methodology is presented in Figure 6. Further explanation of the wideband digital predistortion method used can be found in [18], [19].

DESIGN TRADE-OFFS

EFFICIENCY

Figure 7 depicts the power consumption of the system when weighted waveforms are used. The power consumption of the system is defined as 100% for original waveforms, and the weighted waveforms are all linearly scaled so that the maximum output voltage out of all of the weighted waveforms equals the constant output voltage of the original waveforms. This rescaling results in a decrease in system power utilization for any set of weighted waveforms, where the output power is not constant. As seen from the figure, there is a significant transmit power drop associated with the weighted waveforms. The maximum power drops are computed as 7.14 dB (at ±12°) and 20.87 dB (at ±30°) for weighted waveforms (SR-Off) and weighted waveforms with SR (SR-On), respectively. At some angles, such as ±20°, the power difference between SR-Off and SR-On waveforms is 1.28 dB, while the PSLR difference between SR-Off and SR-On waveforms is 11.48 dB. Therefore, around 0° and ±20° SR-On waveforms may be preferred over SR-Off waveforms.

EFFECTIVE WAVEFORM AND MATCHED FILTER RESPONSE

In radar systems using active array architectures, the transmit array elements usually operate in a synchronized manner. If each ele-
ment transmits a waveform \( x_n \) at the same time, these waveforms will arrive at the (nonbroadside) target area at different times. Summation of these waveforms in the target area creates an effective waveform that reflects from the target. The effective waveform \( x_e(t) \) in the far field can be defined as

\[
x_e(t) = \sum_{n=1}^{N} x_n(t - t_n),
\]

where \( t_n = n d \sin \theta/c \). In the receiver, the collected data can have the template waveform applied as the matched filter to obtain the maximum signal-to-noise ratio. The matched filter response \( g(t) \) of the effective waveform with the template waveform is given as

\[
g(t) = \int_{-\infty}^{\infty} x_e(u) x^*(T-u)du.
\]

It is observed that even though none of the individual transmitted waveforms match the template waveform, the effective waveform has similar envelop and phase characteristics as those of the given template waveform (see Figures 13 and 14).

**Low Sidelobe Pulse Compression Waveforms**

Once the effective waveform has been formed, the pulse compression characteristics of the received waveform can be analyzed. The benefit of waveforms exhibiting low-pulse compression sidelobe behavior is now realized. The standard LFM chirp PSLR (about \(-13 \text{ dB}\)) may be too large to be useful in some radar applications. The LFM chirp’s PSLR may be increased at the expense of a small amount of transmitted power through the application of a windowing function. The PSLR may be further increased while maintaining the transmitted power and resolution by using a coded waveform as the template waveform (such as a Frank or P4 code [20]) or by using a continuous nonlinear FM waveform as the template waveform, as in [21], [22], [23].

**EXPERIMENTAL RESULTS**

An experimental testbed, shown in Figure 8, was assembled to verify the weighted waveforms. The testbed consisted of a Tektronix AWG7122C Arbitrary Waveform Generator with a 12-GHz sample generation rate, a Tektronix DPO70604 Digital Phosphor Oscilloscope, with a 25-GHz sampling rate and 6-GHz instantaneous bandwidth, a Mini-Circuits ZX60-V82-S+ wideband amplifier used as a preamplifier, and a Specwave OBH-7-4012 high power amplifier (HPA).\(^1\) All proposed waveforms were created in MATLAB before being uploaded to the waveform generator, amplified by the wideband amplifier, again amplified by the HPA, and then sampled by the oscilloscope. For all measurements, the measured signal for each channel was appropriately delayed in the digital domain to simulate the effective waveform at target angle \( \theta \). The summation of all eight measured and delayed channels ultimately resulted in the same structure as the selected transmit signal \( x(t) \).

\( ^1 \) The actual EcoSAR hardware is different from the testbed shown here; for more details about the current EcoSAR system hardware, please see [16].

\( ^2 \) Note that the maximum allowable steering angle in EcoSAR is \( \pm 35^\circ \) off broadside.
Waveform Weighting for a Frequency-Invariant Transmit Beampattern

The beampattern resulting from the measured signal set using the LFM waveform transmit signal was created, and it was seen that the mainbeam, sidelobes, and nulls were well aligned in frequency. Therefore, the transmit gain pattern resulting from the weighted LFM waveform transmit signal set is, for all practical purposes, frequency independent. The same experiment was repeated for the weighted waveforms without and with the SR technique implemented. The resulting beampatterns are depicted in Figures 9 and 10, respectively. The power inside the mainbeam is preserved and experiences minimal changes across the frequency domain (see Figure 11), while the sidelobes are suppressed. Although all of the nulls are well aligned in the SR-Off weighted beampattern, SR-On processing results in a beampattern in which the first three nulls are aligned well and the other nulls are not well aligned in the angle domain due to the low sidelobe constraint. Figure 12 illustrates the comparison of the measured weighted waveforms both with and without SR implemented, and it can be clearly seen that while the PSLR is increased by SR processing, the peak power is reduced. The effective waveform in the far field and its matched filter response are computed, as described in the “Effective Waveform and Matched Filter Response” section. As seen from the figure, effective waveform is in the shape of intended LFM signal. The real part of the effective waveform together with estimated envelop are depicted in Figure 13. Figure 14 shows the matched filter results.
The decision to implement the weighted wideband beamforming system is much lower than that of a similar nonweighted system. Beamforming optimization methods, the power utilization of the technique at the expense of a small amount of peak power. For both was shown to be further reduced through introduction of the SR junction with the measured LFM waveform results, the measured outputs are practically identical in shape, with the only difference being the decreased magnitude in the SR-On case due to decreased power transmission, as addressed in the “Efficiency” section.

The beampattern of the measured signal set was calculated using the P4 polyphase-coded waveform transmit signal. The sidelobes were suppressed and the mainbeam, sidelobes, and nulls were well aligned in the frequency domain. However, there was noticeable roll-off in the mainbeam and sidelobe intensity as the instantaneous frequency departed from the carrier frequency. The P4 waveform was also weighted with the SR technique implemented, and the resulting beampattern is depicted in Figure 15. The mainbeam and sidelobe intensity experiences slight roll-off as the instantaneous frequency departs from the carrier frequency. These results demonstrate that weighted beampatterns resulting from coded waveforms, while not totally frequency independent, have significantly improved characteristics. The matched filter result of the effective waveform resulting from the measured weighted P4 waveform set with SR-On is shown versus the ideal P4-matched filter result in Figure 16. The resolution degradation is extremely small and the sidelobes, with a maximum of only 0.67 dB higher than the ideal P4-matched filter case, are much lower than those seen in the LFM waveform cases, shown in Figure 14. In conjunction with the measured LFM waveform results, the measured weighted P4 waveform results demonstrate that the primary trade-off in SR implementation is the reduction of power in exchange for increased PSLR.

CONCLUSION

The presented method for wideband beamforming has been shown, both in simulated and measured results, to create a set of optimally weighted transmit waveforms that results in a constant modulus beampattern in the target area that is, for all practical purposes, frequency invariant with low sidelobes at every other position. This proposed frequency weighting method results in a beampattern that has low sidelobes and constant power across the mainbeam, and it also does not exhibit grating lobes outside the bandwidth of interest and varies from very frequency invariant to completely frequency invariant, depending upon the initial waveform. The PSLR was shown to be further reduced through introduction of the SR technique at the expense of a small amount of peak power. For both beamforming optimization methods, the power utilization of the system is much lower than that of a similar nonweighted system. The decision to implement the weighted wideband beamforming in a system must be made on a system-by-system basis, taking into account the implications of the trade-off between system power utilization and a frequency-invariant constant modulus waveform. In combination with digital predistortion, the weighted beamforming method was demonstrated through hardware to give effective results for both LFM and P4 polyphase-coded waveforms, showing that the presented wideband beamforming method has potential application in any wideband active array system when a precise beampattern is desired.

REFERENCES

Waveform Weighting for a Frequency-Invariant Transmit Beampattern


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