Scanning Performances of Wide Band Connected Arrays in the Presence of a Backing Reflector

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Abstract—The scanning performances of connected arrays that include backing reflectors are investigated. The comparison between connected dipoles and slots is based on the properties of the corresponding Green’s Functions. The investigation reveals that connected arrays of dipoles are better suited to scan to wide angles (45°), retaining the minimal number of Transmit/Receive modules. The paper quantifies and motivates this preference. Eventually, the theoretical design of a fully efficient, 40% bandwidth, planar array with the lowest possible cross polarization is presented.

I. INTRODUCTION

A NUMBER of applications have been recently arising that require antennas with high directivity, wide bandwidth (BW), low profile, low cross polarization and electronic beam scanning. These applications range from multifunction defense and security radars in X-band and lower, to communication applications in Ku-bands, from earth based deep space investigation (e.g. Square Kilometer Array, [1]), to satellite based sub-mm wave instruments (e.g. SPICA, [2]). Connected arrays are now considered as one of the most promising antenna solutions for such applications. While their origin stems from the concept of self complementarity, [3], [4], recently it was R. C. Hansen, [5], that brought the concept of connected arrays of dipoles to the attention of the antenna community. A detailed study of finite array of dipoles connected end-to-end by reactive loads was presented in [6]. Such investigation showed that Non-Foster coupling between contiguous elements allows optimizing the performance over a decade bandwidth. The design strategy for arrays of disconnected dipoles presented in [7], while appearing radically different because it is based on capacitively loaded dipoles, presents some similarities with the one based on connected arrays. Indeed, the purpose and effect of the capacitive loading in [7] is to obtain almost continuous currents among the different dipole elements, thus realizing the continuous sheet current from Wheeler, [8]. This is the same scope of the connected dipole arrays. In [9] the connected dipole concept was extended to the dual structure, based on slots. In [10] and [11] the Green’s functions (GF) of such long slot arrays was derived and presented in analytical form, for both finite and infinite arrays, starting from a spectral representation of the field in each slot [12]. Recently, other authors, [13], have been investigating the GF of connected array structures with full-wave techniques, obtaining models which also include the details of the feeding structures. The availability of the Green’s function greatly facilitates the design steps and eventually the first functioning demonstration of a planar connected array was presented in [14]. This consisted of a 4 × 8 connected array of slots, with a backing reflector, radiating at broadside with reasonable efficiency, on a bandwidth that spanned from 150 MHz to 650 MHz.

In this paper, the properties of connected slots and dipoles are investigated and compared, focusing, for the first time, on the array scanning properties. The dependence of the performances of this type of arrays on a number of factors...
like the array sampling, the distance of the radiating aperture from the ground plane and the energy stored in the feeding points is analyzed and described in detail. In particular, it is shown how the capacitive energy stored in the feeding gaps of a connected dipole structure can be used to match the array for wide scanning angles. This finding sets a preference for connected dipoles over connected slots. A design with a BW in the order of 40% and wide sampling periods (\(d_z = d_y \approx 0.5\lambda_0\) at the highest useful frequency) is presented and discussed, showing its full functionality also when scanning up to \(\pm 45^\circ\).

The radiation patterns of such an \(8 \times 8\) finite array are investigated using the appropriate analytical expressions for the far field co- and cross-polarized directivities. These expressions are derived directly from the GF and thus remain valid even for a small connected array. On the contrary, the very widely used approximations based on infinite array theory [15] and consequent windowing [16], although very efficient from the computational point of view, would fail in equivalently small array cases. This is because of the intrinsic nature of connected arrays: they are characterized by a strong mutual coupling which does not allow neglecting global array effects. Eventually, the low cross-polarization level when scanning in the diagonal plane is highlighted as the main characterizing feature of these broadband arrays.

The findings of this paper constitute the basis for a prototype that is presently being manufactured and will be the object of a future publication.

II. IMPEDANCE OF CONNECTED ARRAYS

The geometries of the slot and dipole arrays that we consider are shown in Fig. 1(a) and (b), respectively, together with the pertinent reference system and characterizing parameters. The Green’s Function (GF) of an array of long slots periodically fed, in the presence of a backing reflector, was derived in [11]. From the GF, the input impedance was also derived, which can be explicitly expressed as follows:

\[
\frac{Z_{\text{slot}}^{br}}{\text{d}_{\text{slot}}} = \frac{k_0\zeta d_y}{d_x} \sum_{m_y = -\infty}^{\infty} \sin^2(k_{x,m}d_y/2) \cdot \frac{1}{(k_0^2 - k_{x,m}^2) \sum_{m_y = -\infty}^{\infty} J_0(k_{y,m} \cdot \text{d}_y) / k_{z,m}}
\]

(1)

where \(J_0\) is the Bessel function of zero-th order, \(k_0\) is the free space propagation constant, \(k_{x,m} = k_0 \sin \theta \cos \phi - \frac{2\pi m_y}{\text{d}_y}\), \(k_{y,m} = k_0 \sin \theta \sin \phi - \frac{2\pi m_y}{\text{d}_y}\), and \(k_{z,m} = \sqrt{k_0^2 - k_{x,m}^2 - k_{y,m}^2}\). For the dipole structure, with backing reflector as in Fig.1(b), the GF can be evaluated once the electric current on a single infinitely extended dipole fed by a \(\delta\)-gap voltage source is known. The derivation of this current is shown in the appendix (eq. (19)). The extension to an infinite periodic array of dipoles periodically fed is obtained extending the procedure shown in [11]. Eventually, the following expression for the active input admittance at each feed can be written as:

\[
\frac{Z_{\text{dipole}}^{br}}{\text{d}_{\text{dipole}}} = \frac{k_0 \zeta d_y}{d_x} \sum_{m_y = -\infty}^{\infty} \sin^2(k_{x,m}d_y/2) \cdot \frac{1}{(k_0^2 - k_{x,m}^2) \sum_{m_y = -\infty}^{\infty} J_0(k_{y,m} \cdot \text{d}_y) / k_{z,m} (1 - j \cot(k_{0}h_x \cos \theta))}
\]

(2)

Note that here it is assumed that the electric dipoles, Fig. 1(b), are oriented along \(x\), implying that the connected array of dipoles operates in a polarization orthogonal to the one associated with the connected array of slots in Fig. 1(a).

A. Dominant Floquet Wave

Both expressions (1) and (2) present a clear resonance condition. The condition can be gathered by considering only the first mode, \(m_x = m_y = 0\) in the double summations. Note that the dominant mode representation is a realistic hypothesis only when the array is extremely well sampled. The slot array impedance becomes

\[
Z_{\text{slot}}^{br} = \frac{\zeta d_y}{d_x} \left(1 - \sin^2 \theta \cos^2 \phi \right) \left(1 - j \cot(k_0 h_x \cos \theta) \right)
\]

(3)

The resonance of the impedance in (3) is clearly given by the condition \(k_0 h_x \cos \theta = \pi/2\), which implies \(h_x = \lambda_0/4\) for broadside radiation. In this case \(Z_{\text{slot}}^{br} = \frac{\zeta d_y}{d_x} \left(1 - \sin^2 \theta \cos^2 \phi \right)\), which is twice as large as the asymptotic value for low frequency of a connected array of slots without backing reflector, \(Z_{\text{slot}}^{br} \rightarrow 2Z_{\text{slot}}\). The factor 2 derives from the fact that, with a backing reflector, all the power provided to the array radiating aperture is radiated in the upper half space. Also for the dipole case, retaining only the dominant mode, one obtains an expression extremely similar to the one in eq. (3):

\[
Z_{\text{dipole}}^{br} = \frac{\zeta d_y}{d_x} \left(1 - \sin^2 \theta \cos^2 \phi \right) \left(1 - j \cot(k_0 h_x \cos \theta) \right)
\]

(4)

The only difference with respect to (3) resides in the changed \(\phi\)-dependence, congruent with the fact that the \(E-\) and \(H\)-planes are inverted in the dipole configuration. This implies that no major differences can be observed, as far as the bandwidth of connected dipoles and slots are concerned.

For any given \(\theta\) or \(\phi\), eq. (3) and (4) state that the array can be matched with a real transmission line to present a reflection coefficient lower than \(-10 \text{dB}\) over about a 60% relative BW. The transmission lines characteristic impedance that guarantees the widest frequency BW is the one associated with the matching condition at broadside (377\(\Omega\)).

The most important impact on the matching of the arrays as a function of the scanning derives from the variation of the resonance frequency. As the scanning angle grows, as an example to 45°, the electrical length \((k_0 h_x \cos \theta)\) becomes smaller. This implies that the resonance condition is achieved for frequency 30% higher with respect to broadside.

Thus, simplifying, the maximum possible overlap between the matching BW achieved at broadside and at 45° scanning, on both planes, is about 30%. An example of this behavior is shown in Fig. 2 where the reflection coefficient, with respect to a 400 0 transmission line, of a well sampled array is shown.
of the different feeds implies that the propagation of waves along the longitudinal direction of the slots or dipoles, \( i_s \), in both cases (Fig. 1), strongly influences also the infinite array behavior. Here we will discuss the nature of these propagations highlighting the main differences between connected slots and dipoles.

A. Leaky wave poles in Connected Slots

A parallel plate waveguide is compatible with a TEM guided mode. This mode, when perturbed by the presence of periodic slots, originates leaky waves which radiate power in unwanted directions. Accordingly, connected arrays of slots backed by a ground plane, Fig. 1 (a), can support leaky waves. To investigate the properties of such leaky waves, one must study their propagation along the slots. At this aim, it is necessary to do one step back and consider each of the slots in the array, as if it was excited by only one feed. In that case, the representation of the voltage along each slot would be obtained as an inverse integral in \( k_x \), by using Eq. (1) and Eq. (18) from [11]. Note that in the present case Eq. (18) of [11] is the kernel of the \( m_x \) summation in Eq. (1).

We will concentrate the attention to the case in which the array is phased to radiate toward \( \phi = 0 \) (so that \( k_y \delta = k_0 \sin \theta \sin \phi = 0 \)) and we will investigate the spectrum in \( k_x \). Expected branch point singularities appear in \( \pm k_0 \). However, also polar singularities emerge as solutions of the dispersion equation that is obtained equating to zero the denominator in equation (1):

\[
(k_0^2 - k_x^2) \sum_{m=-\infty}^{\infty} J_0(k_{ym} w_s) \left(1 - j \cot(k_{zm} h_s)\right) k_{zm} = 0 \quad (5)
\]

where \( k_{zm} = \sqrt{k_0^2 - k_x^2 - k_y^2} \).

The dispersion equation can be solved approximately using a first order Newton method, with a procedure similar to the one used in [12]. Fig. 3 shows the results of the dispersion analysis in the portion of the complex plane \(-k_0 < Re[k_x] < k_0\) and \(-k_0 < Im[k_x] < k_0\). The continuous curves, as a function of the slots widths, \( w_s \), and parameterized for different heights, \( h_s \), of the antenna with respect to the backing reflector, indicate the location of pole singularities, \( k_{xp}^s \), of the GF. The array is characterized by \( d_x = d_y = 0.5\lambda_0 \), with \( \lambda_0 = 2\pi/k_0 \).

The poles represented in Fig. 3 are of the leaky wave type, in the sense that they are characterized by a propagation, \( Re[k_{xp}^s] \), and by an attenuation constant, \( Im[k_{xp}^s] \), this latter associated with radiation losses.

When the backing reflector is farther away from the radiating slots, the propagating mode of the slot, \( k_{xp}^s \), is quasi-TEM, travelling almost parallel to the slot direction. As \( h_s \) becomes...
smaller, the mode field distributions are much more affected by the presence of the backing reflector. As a consequence the poles present an increasingly larger imaginary part. Most importantly, they present a smaller real part, meaning that they can be relevant also for arrays scanning in directions closer to broadside, since \( \sin \theta = \text{Re}(k_{xm}/k_0) \). The propagation becomes more frequency dependent. An increasing width of the slots is also associated with an increased dispersivity. In fact, as the slot becomes wider, the field is less tightly bound to the slots, and the presence of the ground plane has more impact.

Curves similar to those in Fig. 3 have been obtained also for \( \phi \neq 0 \), i.e. scanning in planes different from the \( H \)-plane. However, since equivalent qualitative conclusions could have been obtained, they are not reported here. Lower values of \( h_s \) imply poles in the visible region of the \( k_x \) spectrum.

To highlight the negative effect due to the compatibility of connected slot arrays with leaky wave poles, Fig. 4 shows the real parts of the input impedance of a connected slot array, over a broad frequency range. Named \( \lambda_c \) the wavelength at the desired maximum operating frequency, the curves are given for \( h_s = 0.1\lambda_c \) and \( h_s = 0.4\lambda_c \). The remaining array parameters are fixed at \( w_s = 0.2\lambda_c \), \( d_s = d_y = 0.5\lambda_c \) and \( \delta_s = 0.05\lambda_c \). The curves are also given for different scanning (broadside and \( 45^\circ \) in the \( E \)- and \( H \)-planes). It is apparent that for smaller heights the resonances are indeed shifted toward higher frequencies. However, smaller heights imply the drastic narrowing of the useful band when scanning in the \( E \)-plane.

Given these findings, it is clear that reducing \( h_s \) to achieve matching at higher frequencies, and accordingly to reduce the number of T/R modules, it is not a viable solution since the scanning performances are seriously degraded.

### B. TEM Poles in Connected Dipoles

If a similar dispersion analysis is performed for the connected dipole array, in the presence of a backing reflector, no leaky wave pole are found for \( h_d \) relatively small. This can be shown fairly easily. In the case of dipoles, one needs to find the solutions of the following dispersion equation, which is obtained equating to zero the denominator of the expression in eq. (2):

\[
(k_0^2 - k_{xm}^2) \sum_{m_y=-\infty}^{\infty} J_0 \left( k_{ym} \frac{w_s}{2} \right) k_{zm}(1 - j\cot(k_{zm} h_d)) = 0 \tag{6}
\]

Despite being similar to eq. (5) the analytic solutions of eq. (6) are simpler to characterize. Solutions are found explicitly, for arbitrary values of frequency, scanning angle \( \phi \), and distance from the ground plane \( h_d \), in correspondence of \( k_x = \pm k_0 \). They are associated with guided micro-strip like (TEM) modes. Other poles emerge at higher frequencies. In fact they arise from the reduced dispersion equation

\[
\sum_{m_y=-\infty}^{\infty} J_0 \left( k_{ym} \frac{w_s}{2} \right) k_{zm}(1 - j\cot(k_{zm} h_d)) = 0 \tag{7}
\]

The possible solutions of this reduced dispersion equation are found observing that
- \( 1 - j\cot(k_{zm} h_d) = 2/(1 - e^{-jk_{zm} 2\lambda_h}) \);
- for a well sampled radiating array, \( k_{zm} \) is always purely imaginary except in correspondence of \( m_y = 0 \), in which case \( k_{z0} = k_0 \cot \theta \);
- both real and imaginary parts of eq. (7) must be zero in order to have a solution of the dispersion equation.

From the last two conditions it follows that the contribution to the summation associated with the mode of index \( m_y = 0 \) must present a zero by itself. This is only possible when \( (1 - e^{-j2\lambda_h \cot \theta}) = 0 \), which leads to \( h_d = \frac{n \lambda_h}{2 \cos \theta} \), where \( n \) is integer. Thus, the distance between the ground plane and the dipoles should be at least larger than half wavelength to have another pole, different from the micro-strip TEM mode \( (k_x = \pm k_0) \), to be compatible with the structure.

These micro-strip like TEM modes are important since they guide power from one feed to the next, but they do not imply important degradation of the main focused radiation, since they contribute to radiation toward \( \theta \approx 90^\circ \). Accordingly the reduction of the height, \( h_d \), does not present drawbacks specifically associated with scanning. However, a dipole array, whether connected or not, has a significantly reduced radiation BW when the \( h_d \) is small with respect to the wavelength. This can be observed in Fig. 5 where the real part of the

![Fig. 4. Active impedance of a connected slot array as a function of the frequency for two heights.](image1)

![Fig. 5. Active impedance of a connected dipole array as a function of the frequency for two heights.](image2)
impedances associated with two arrays of different heights but same sampling are compared. Differently from the slot case, for the dipoles we observe not only a shift of the resonance when varying the distance $h_d$ from the ground plane, but also much higher impedance value for lower $h_d$. Here the growth of impedance is the manifestation of a transverse, non radiating, resonance due to the mutual coupling between different dipoles.

Overall, one can conclude that the nature of the dominant waves in connected arrays of dipoles is much simpler (pure TEM) than the one (leaky) of connected arrays of slots. However, for neither structures, a lower height brings significant advantages.

IV. BW DEPENDENCE ON THE GAP WIDTH

So far it has been established that the impedances of grounded connected dipoles and slots have similar dependences on the periodicity and the distance from the ground plane. Where they truly differ is in their dependence on the reactive energy stored in the feeding gaps. In the connected slot case this reactive energy has only a marginal role, while in the case of connected dipoles the reactive energy can be used to effectively achieve wider matching bandwidths.

The reactive energy is mathematically represented in the admittance of the connected dipoles, eq. (2), or in the impedance of the connected slots, eq. (1), by the higher order Floquet modes. This energy is localized in the proximity of each gap. Thus, for instance in a connected slot array, one can assume that the concentrated inductance in each of the $\delta$-gaps in every slot is the same as that localized in a single $\delta$-gap of a unique infinitely extended slot etched on an equivalent ground plane in absence of the backing reflector. Analogously, one can assume that the concentrated capacitance associated with the $\delta$-gaps in a connected dipole array is the same as that of a single feed of an infinite dipole. For the interested reader, these lumped equivalent impedances were provided in [18].

A. Slot case: lumped inductance

In the connected slot array case, the lumped inductance associated with each feed gap is in series with respect to an equivalent circuit representing the excitation for the gap. That is because in a well sampled array, the real part of the impedance is only associated with the dominant ($m_z = 0$) term of the Floquet mode summation in eq. (1). The imaginary part depends also on all the other additive modes in $m_z$. In Fig. 6(a) the equivalent circuit is shown, introducing the dynamic portion of the impedance as the difference between the input impedance and the concentrated inductive reactance associated with the gap. In order to highlight the effects of this inductance, Fig. 7 shows a parametric investigation of the input impedance as a function of the frequency for increasing dimension of the gap $\delta_s$. It can be noted that the real part of the impedance is indeed not dependent from the dimension of the gap. At the lower frequencies, for all values of $\delta_s$, the imaginary part of the impedance is inductive (positive). That is because the slots are operating in the close vicinity of the backing reflector, which acts as a short circuit (just considering the dominant Floquet wave representation in (3) is sufficient to verify this aspect). Then, as the frequency grows, the waves reflected from the ground plane tend to add less destructively to the ones directly radiated toward free space, until a resonant condition is achieved. Eventually, the actual resonant condition, when the imaginary part of the input impedance is equal to zero, depends on the specific width of the $\delta$-gap. It is apparent that for larger (less inductive) gaps the resonance occurs at slightly higher frequencies.

Note that the active input impedance cannot be seen as purely inductive, but only the reactive energy localized at the feeding points is modelled as an inductance.

B. Dipole case: lumped capacitance

In the case of connected dipoles, the gaps can instead be truly used as a design parameter to enlarge the BW. Here a lumped capacitance representing the stored energy is in parallel with respect to the equivalent circuit of the excitation, see Fig. 6(b). That is because the capacitance is associated with the summation of the higher order contributions to the input admittance. Note that, in this case, in the equivalent circuit the dynamic impedance $Z_{\text{dipole}}^{\text{dyn}}$ is equal to $\frac{1}{j\omega C_{\text{gap}}}$. The dynamic admittance $Y_{\text{dipole}}^{\text{dyn}}$ is the difference between the input admittance and the static admittance $Y_{\text{dipole}}^{\text{stat}}$. 

![Equivalent circuit and active impedance graphs](https://example.com/fig6.png)

**Fig. 6.** Equivalent circuit representing the active impedance of each feed of a connected array of slots with inductive gap feed (a) and of dipoles with capacitive gap feed (b).

![Impedance graph](https://example.com/fig7.png)

**Fig. 7.** Active impedance of a connected slot array as a function of the frequency for different gap widths, given $h_s = 0.3\lambda_c$, $w_s = 0.2\lambda_c$, $d_y = 0.5\lambda_c$. 

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**References:**

[18] Some published works discussing equivalent circuits and impedances for slots and dipoles.
would be achieved for implements this controlled capacity condition.

One should aim at a configuration which apparent that, if one is interested in broad bandwidth and the scanning angle to which the array is pointing. It is thus allows to enlarge the impedance bells independently from matching bandwidth of the connected dipole array.

is a key design parameter that can be tuned to obtain wider frequencies. The same effect is not present in the slot case, case (Fig. 7). However, when the dimensions of the gaps are define a relatively narrow bell as in the connected slot array case (Fig. 7). However, when the dimensions of the gaps are non negligible with respect to the wavelength, and accordingly the gaps store less capacitive energy, the overall impedance bell appears lower, much wider, and shifted toward higher frequencies. The same effect is not present in the slot case, where the series inductance would not affect the real part of the impedance. Thus, we can already see that the gaps capacitance is a key design parameter that can be tuned to obtain wider matching bandwidth of the connected dipole array.

The nicest aspect about this design parameter is that it allows to enlarge the impedance bells independently from the scanning angle to which the array is pointing. It is thus apparent that, if one is interested in broad bandwidth and wide angle scanning, one should aim at a configuration which implements this controlled capacity condition.

V. DOUBLE FEED FOR THE DIPOLES

From the curves in Fig. 9, it appears that the best matching would be achieved for \( \delta_d = 0.15 \lambda_0 \). However, the practical realization of such large gaps is not simple, since it would require feeding lines (for instance coplanar strip lines) with inter-conductor distances so large that they would start radiating strongly cross-polarized fields. The solution that we envisage is to excite a connected array of dipoles with a double feed in each periodic cell. The schematic view of the feeding arrangement is shown in Fig. 10. In this case, the effective capacitance at each cell is essentially divided by two, since the two feeds are connected in series. It is clear that, due to the logarithmic dependence of the capacitance from the gap width, [18], much narrower gaps can be used in the double feed case to obtain similar impedance curves, with respect to the single feed configuration. With such small gaps, the dipoles can be fed by transmission lines with negligible losses for cross-polar radiation. Fig. 11 shows the comparison between the input impedances in the two cases of single feed with \( \delta_d = 0.125 \lambda_0 \), and double feed with \( \delta_d = 0.01 \lambda_0 \). An optimized array configuration based on a double feed per each unit cell was found to have 40% relative BW, scanning up to 45°, as show in Fig. 12. The feeds are at a distance \( \Delta = 0.29 \lambda_0 \), the gap size \( \delta_d \) is chosen equal to 0.01\( \lambda_0 \), the dipole width is \( w_d = 0.05 \lambda_0 \), and the height from the ground plane is \( h_d = 0.31 \lambda_0 \). In order to simplify the analysis with only minimal loss of generality, the double-feed curves in Fig. 11 and in Fig. 12 have been obtained representing the double-feed configuration by two delta gap generators in each cell. Then the cell input impedance is the sum of two equal feed impedances, each of them defined as the inverse of
the average active admittance observed at the two ports. The description, the meaning and the mathematical representation of this implementation are discussed in the next section.

A. Active admittance for multiple feeds per unit cell

The excitation of two feeding points can be modelled as two impressed electric fields $e_{1,2}(x)$, such to create a voltage drop of 0.5 Volt at each feed. Assuming that the separation between the two feed points inside the same cell is $\Delta$, the $x$-dependence of the electric field can be expressed as

$$e_{1,2}(x) = \frac{1}{\delta_d} \Pi_{\delta_d} \left( x \pm \frac{\Delta}{2} \right),$$

where $\Pi_{\delta_d}(x) = \begin{cases} 1 & \text{if } x \in [-\frac{\delta_d}{2}, \frac{\delta_d}{2}] \\ 0 & \text{otherwise} \end{cases}$ (8)

Accordingly, the Fourier transform (FT) of the entire impressed electric field on the cell is

$$E(k_x) = \int e_1(x')e^{jk_x x'}dx' + \int e_2(x')e^{jk_x x'}dx' =$$

$$e^{jk_x \frac{\Delta}{2}} \sinh \left( k_x \frac{\delta_d}{2} \right)$$

In a two-dimensional periodic array, following the same procedure in [11], the current distribution along the dipole associated with the excitation in eq.(8) can be written as

$$i(x) = \frac{k_0d_y}{\zeta_0dx} \sum_{m_x = -\infty}^{\infty} 2 \cos \left( \frac{k_{xm}\Delta}{2} \right) \sinh \left( \frac{k_{xm}\delta_d}{2} \right)$$

(10)

Proceedings in duality with what was shown in [19], the admittance in a gap feed of a dipole excited by unitary impressed voltage drop can be calculated as the average electric current flowing in the gap. Accordingly, the active admittance in each of the two feeds can be expressed as

$$y_{1,2} = \frac{1}{\delta_d} \int i(x')dx'$$

(11)

Substituting eq.(10) in eq.(11), and performing the integration in the $x'$-variable, we obtain

$$y_{1,2} = \frac{k_0d_y}{\zeta_0dx} \sum_{m_x = -\infty}^{\infty} 2 \cos \left( \frac{k_{xm}\Delta}{2} \right) \sinh^2 \left( \frac{k_{xm}\delta_d}{2} \right) \int_0^{+\infty} e^{-jk_{xm}x}$$

(12)

By comparing eq.(12) with eq.(2), it is apparent that, for a well sampled array in which only the $m_x = m_y = 0$ mode is dominant, the admittance at each feed point, in this double feed configuration, is twice as large as the admittance of the same array with a single feed per cell. As a result, within this approximation, the input impedance at each pf the two feeds roughly becomes half the single-feed value. Only when the two ports are connected in series, the single-feed value is approximately recovered.

In practice, when the cell is fed at two points which are physically separated, the maintenance of a voltage excitation

![Fig. 11. Real and Imaginary part of the active impedance of a connected dipole with single feed ($\delta_d = 0.125\lambda_0$), and with double feed ($\Delta = 0.29\lambda_0$, $\delta_d = 0.01\lambda_0$).](image)

![Fig. 12. Active reflection coefficients, with respect to a 350 $\Omega$, of a dipole array characterized by $d_x = d_y = 0.45\lambda_0$, $w_d = 0.05\lambda_0$, $\delta_d = 0.01\lambda_0$, $\Delta = 0.29\lambda_0$ and $h_d = 0.31\lambda_0$. Curves are shown for scanning at broadside and toward $\theta = 45^\circ$ in the two main planes.](image)

![Fig. 13. Schematic representation of the double feed excitation in a unit cell and related electric field vector lines.](image)
at the same amplitude and phase for the two ports can be achieved by means transmission lines that implement a series feeding scheme, as shown in Fig. 13. In that case, the active input admittance at the section A'A' cannot be evaluated without introducing an explicit phase shift.

However, it might be convenient and useful to refer to the concept of the average input admittance \( y_{in}^{av} \) for each feed, which is defined as

\[
y_{in}^{av} = \frac{y_1 + y_2}{2} = \frac{k_0 d_y}{\zeta_0 d_x} \sum_{m_x = -\infty}^{\infty} 2 \cos^2 \left( \frac{k_{xm} \Delta}{2} \right) \sin^2 \left( \frac{k_{xm} \delta d_x}{2} \right) \cdot \left( k_0^2 - k_{xm}^2 \right) \frac{1}{\sum_{m_y = -\infty}^{\infty} J_0(k_{ym} \frac{\delta d_y}{2}) J_0(k_{ym} \delta d_y)}, \tag{13}
\]

and consequently the average input impedance of each feed in a double cell is \( z_{in}^{av} = \frac{1}{\pi y_{in}^{av}} \).

Finally, the cell input impedance is \( z_{cell} = 2 \pi y_{in}^{av} \). This is the impedance plotted in dashed lines in Fig. 11 and to which Fig. 12 refers.

Since for broadside radiation, the double feed structure is actually a practical implementation of a feed sampling twice as dense, one might think that it is simply the effect of the denser sampling that helps the matching. However, this is only partly true. In fact, when the array is phased to scan, the two feed points are associated with the same phase shifting. The practical realization of such a double feed, similar to the one in [14] requires the connection of the two feeds via an appropriate transmission line, as shown in Fig. 13.

**B. Practical considerations**

This final configuration that includes the double feed represents a schematic design, without explicit feeding network, that has been taken as baseline for the development of a prototype demonstrator that is presently being manufactured, [20], in the framework of a contract with the European Space Agency.

For the practical realization of the feeding network in a connected array, it is important to note that differential lines also support common mode propagation. This latter, while unwanted, is always present. A detailed representation of the effects of common mode resonances in connected arrays has been recently discussed in [21]. The authors of the present paper have also encountered such resonances in their designs and different strategies are being investigated to overcome the problems associated with them. The strategies depending on the specific applications may range from proper balun designs to the use of differential amplifiers very close to the radiating elements. This specific problem will be the object of follow on contributions, focusing on the implementations rather than the general theory of connected arrays.

**VI. ACTIVE ARRAY FAR FIELD PATTERNS**

The single most important reason for developing phased arrays based on connected dipoles or slots is the possibility to obtain radiation patterns of the highest polarization purity for wide scanning angles and over large frequency bands. Using the present Green’s function formalism, the radiation patterns can be evaluated analytically, which not only saves computational design time, but eventually rigorously demonstrates the polarization purity properties of these arrays.

Before proceeding to the derivation of the radiated fields, it is worth noting that in this section we will focus on active far fields radiated by active currents. This means that, as typically done in large phased arrays, the radiating currents are those present when all feeds are excited with ideal, uniform amplitude, impressed voltages (electric field). In reality, one can only use real generators. Thus, the realization of a feeding network that implements such uniform voltage generators independently from the scan angle and the frequency must be properly designed and it could turn out to be very challenging. Particularly more so in the present case of connected arrays, for which the mutual coupling between the elements is very high. The practical realization of a connected dipole array prototype will be the subject of a separate investigation. For the present time we will make use of the active current ideal model.

The electric far field can be simply obtained from the expression of the electric potential and in particular:

\[
E_{\theta} = -j \omega \cos \theta \cos \phi A_x \tag{14}
\]

\[
E_{\phi} = j \omega \sin \phi A_y \tag{15}
\]

After a few algebraic manipulations, it can be shown that the potential in the far field can be expressed analytically as:

\[
A_x(r) = \frac{\mu}{4\pi} \frac{e^{-jk_0 r}}{r} \left( 1 - e^{-jk_0 2h_x \cos \theta} \right) \cdot J_x(k_0 \sin \theta \cos \phi, k_0 \sin \theta \sin \phi) \tag{16}
\]

where \( J_x(k_x, k_y) \) is the FT of the electric current distribution on the array. The key step in the evaluation of the far field is the approximation of the equivalent currents on the finite array. \( J_x(k_x, k_y) \) can be expressed as the product of two separate variables: i.e. \( J_x(k_x, k_y) = J_x^0(k_y) J_y(k_x) \).

Windowing approach, [16], can be safely used in approximating the current distribution transverse to the array, \( J_y^0(k_y) \). For \( N_y \) elements along the transverse direction, remembering that in each dipole the distribution verify the edge singular condition, it results:

\[
J_y^0(k_y) = AF(k_y - k_y0, N_y, d_y) J_0 \left( \frac{1}{2} w_d k_y \right) \tag{17}
\]

where \( AF(k_y - k_y0, N_y, d_y) = \sum_{n_y = 0}^{N_y/2} I_{n_y} e^{jk_y(k_y - k_y0)n_y d_y} \).

The longitudinal direction is instead more critical. In fact in section III, we have demonstrated analytically that in any connected array, slots or dipoles, a dominant propagating mode is supported by the structure with backing reflector, even at low frequencies. This mode is a leaky wave in the slot case or a bounded micro-strip mode in a connected dipole array. In particular, in the dipole case, the waves travelling from the edges in the longitudinal direction, \( \pm k_z \), are not attenuated. This suggests that an infinite array approximation could be much less accurate as it completely neglects longitudinal standing waves and associated heavy current tapering, that could exist even in the presence of uniform excitations.
The complication in the analysis is only apparent since we have actually provided the analytical closed form expression for the Green’s function of an infinitely extended dipole (or slot) in the spectral domain (see appendix), from which one can extract the spectrum of the longitudinal current in the case of a finite number of δ-gap sources.

\[ I(k_x) = V_0|AF(k_x - k_{20}, N_x, d_x)| \cdot \frac{\sin c(k_x \delta_d/2)}{D_\infty(k_x)} \] (18)

This expression based on the exact GF is more accurate than those achieved using a windowing approximation and it also implies reduced computational cost.

The far field copolar directivity patterns radiated by an array of 8 dipoles each one of them fed by 8 δ-gap feeds is then investigated. The chosen frequency is \( f_0 = c_0/\lambda_0 \), where \( c_0 \) is the free space speed of light. The array dimensions in terms of the wavelength are \( d_x = d_y = 0.5\lambda_0, t = w_d = 0.05\lambda_0, h_d = 0.31\lambda_0 \).

In Fig. 14 we compared the E-plane radiation patterns obtained with the windowing approximation and the exact Green’s function in eq. (18). When observing in the E-plane, broadsid or at scanning, the use of the complete GF shows higher side lobe levels and also broader beams at 45°. The use of the GF predicts also higher radiation at grazing angles. The actual values of grazing radiation are not very significant, since they will be eventually affected by the finite length of the dipoles, however their visibility suggests the necessity to control end point longitudinal radiation when dealing with connected arrays of small dimension.

Finally, Fig.15 presents a comparison between the copolar and the cross polar directivity obtained when the array is scanned to \( \theta = 45° \) in the diagonal plane, \( \phi = 45° \). The third Ludwig definition has been used, [22]. The cross polarized field is at least 15 dB lower than the co-polarized field. This level of low cross polarization is the same of a single elementary dipole operating in free space and thus the value theoretically achievable by planar array of linearly polarized dipoles.

VII. CONCLUSIONS

Based on rigorous Green’s Function formulations, a comparison between the performances in scanning of connected arrays of dipoles and slots in the presence of a backing reflector is shown. The investigation reveals that the reactive energy contained in the feeds of the connected dipoles can be tuned in order to achieve broad band matching, independently from the scan angle. Eventually, the case of a planar connected array of dipoles that presents the lowest theoretical cross polarization levels for a planar array, on a 40% bandwidth even when scanning in elevation to 45° is presented. The number of required T/R modules remains limited with \( 5\lambda_0 \) samplings at the highest operational frequency. The use of the rigorous GF in the prediction of the far fields in the E-plane provides more accurate results with respect to a standard windowing technique and is even less computationally intensive. The significance of the active patterns will be discussed and validated in a following paper that will include also the design of a prototype demonstrator.

APPENDIX: GREEN’S FUNCTION OF A CONNECTED DIPOLE ARRAY WITH A BACKING REFLECTOR

The derivation of the Green’s Function of an infinitely long dipole fed by a single δ type gap source is summarized here. The procedure is a simple extension of the one shown for the slot case in [12], to which the reader is referred to a more in depth derivation; the extension to arrays of dipoles is also straightforward, similarly to what was presented in [11]. The canonical geometry and the pertinent reference system considered here is depicted in Fig. 16. An infinitely long dipole of width \( w_d \), directed along \( x \) is located in the x-y plane at distance \( h_d \) from an infinite backing reflector. The dipole is fed by a \( x \)-directed electric field at its origin (uniformly distributed along \( w_d \)).

A separable functional dependence for the transverse (across the dipole) and longitudinal electric current distribution is
Substituting in (20) the explicit Green’s function, we obtain:

\[
D(k_z) = \frac{-\delta_0}{4\pi k_0} \int_{-\infty}^{\infty} \frac{k_0^2 - k_z^2}{k_0} (1 - e^{-2ik_zh_d}) J_0(k_y \sqrt{\frac{w_d}{2}}) dk_y
\]  

(21)

where \( k_z = \sqrt{k_0^2 - k_y^2 - k_h^2} \) with \( Im[k_z] < 0 \).

Substituting eq. (21) in (19), the explicit double integral representation of the current in the infinitely long dipole can be obtained. For the rest of this paper, there is no interest in evaluating these integrals analytically, since only the double spectral summations arising from the periodic extensions are discussed in the text.

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