Abstract—This two-part sequence deals with the presentation of an efficient directive antenna that can be used to realize essentially non dispersive links over extremely large bandwidths. The antenna is a significantly enhanced version of previously proposed Leaky Lens antennas that use a frequency independent leaky slot radiation mechanism. A theoretical break-through now allows the use of this mechanism also in the presence of purely planar structures. This step allows the realization of the feed of a Leaky Lens antenna in a unique planar structure that is then glued to a standard circularly symmetric elliptical dielectric lens, as integrated technology requires in the mm and sub-mm wave domains. The first part of this sequence deals with a theoretical break-through, the consequent antenna concept and a description of the basic physical mechanisms inside and outside the lens antenna. It is shown that Leaky Lens antennas have the potential to be used to realize antenna links over bands exceeding a decade with minimal dispersion, high efficiency and high directivity. The second part of this sequence deals with the demonstration of these claims via the measurement of two prototypes.

I. INTRODUCTION

One of the key parts of high frequency (mm or sub-mm waves) sensing systems is the integrated antenna that couples the incoming radiation into the receiver. The combinations of elliptical dielectric lenses and slot or dipole feeds have been used for decades in harmonic or pulsed sensing systems. Resonant feeds were introduced by [1] and by now gained a significant level of maturity, [2], [3], [4], [5]. So far most of the studies presented in the literature involve antennas with 10 - 15 % operational bandwidth and good efficiencies. In recent years, however, the request for wider bands has been systematically driven by the desire for higher range resolution in radars or larger frequency characterization in spectrometers. In order to improve the usable bandwidth of dielectric lenses a particular leaky wave feed structure was proposed in [6], and [7] and the combination of feed and lens was called Leaky Lens antenna. While the antenna in [6] would operate over a band width of about an octave, the extension proposed in [7] leads to a decade of bandwidth (BW). However these previous Leaky Lenses were 3D structure with feed lines extending in different planes and thus not suited to be realized in integrated or printed circuit board (PCB) technology. Moreover, they presented asymmetric radiation patterns with high side lobe levels.

This paper presents an important breakthrough. The excellent performances of [7], derive from the very weak dispersivity intrinsic in the leaky wave slot radiation mechanism [8], [9]. The break-through is the realization that the velocity of propagation along a slot printed between two dielectric media can be accelerated acting locally on the reactive energy that surrounds the slot, without significantly reducing the characteristic low dispersivity of this type of radiation/propagation. Thanks to this theoretical insight it was possible to design an enhanced version of the UWB Leaky Lens antenna. The new antenna can be realized in planar integrated or printed technology. Next, it can be glued to a standard circularly symmetric lens, like those routinely used in mm and sub-mm wave technology i.e. as the ones in [1]–[5].

The novel antenna concept promises impedance matching over decades of bandwidth, very high efficiency, which is only limited by ohmic losses in the dielectric, directive and circularly symmetric patterns, and excellent pulse preservation properties which render it potentially suited for both pulsed and harmonic broad band instruments. For readers also interested in the practical realizability, a prototype demonstrator of the antenna has been designed to operate in the band (20 GHz-60 GHz) and some results of numerical simulations are presented at the end of this paper.

The prototype has then been manufactured with printed circuit board technology. The details of the prototype and the consequent characterization, which validates the discussion in this paper, is presented in Part 2 of this sequence [10].

II. PLANARLY FED DIELECTRIC LENSES

Figure 1 shows a cross section of an elliptical dielectric lens and the direct rays emerging from a focal slot. The eccentricity ε of the ellipse that focuses in broadside part of the rays emerging from the lower focus is defined by the dielectric constant of the lens, \( \epsilon = 1/\sqrt{\epsilon_r} \). Only the rays impinging on the dielectric air interface above the B-B’ section are focused in the far field, while rays that impinge on the interface below the B-B’ line are lost in spill over since they are scattered in useless directions.

A. Focused Feeds

In reception, a sub-wavelength antenna is able to capture power incoming in the focus area. The focus radius \( r \) depends on the focal distance to diameter ratio, F/D, of the system: essentially \( 2r \approx F/D\lambda_d \) with \( \lambda_d \) the wavelength in the dielectric. The F/D of these lenses cannot be approximated easily. However, a single pixel detection scheme would use the lens relatively efficiently if the dimension of the feed was in the order \( \lambda_d \) or smaller, when \( \epsilon_r \approx 10 \).

Resonant dipoles and slots are used to realize efficient integrated mixers in the spectroscopy domain. The twin slot configuration from [2], with the length of each of the slots being about one wavelength in the dielectric, is probably the...
most known design. In these cases all the available power from
the local oscillators and the incoming signal is effectively used.
Such efficient designs present narrow usable bandwidths. The
author of the present paper recently designed an extension of
the double slot of [2] in order to achieve an octave bandwidth
of operation. This could only be achieved at the price of the
efficiency decreasing to about 75% associated to an impedance
mismatch. Moreover, an additional cost is that the radiation
pattern from the slots becomes almost omnidirectional thus
inducing important spill over losses due to rays impinging
below the B-B’ line in Fig. 1. The details of these results are
not reported for the sake of brevity. Note that, for all resonant antennas located at an interface between two media, a
denser dielectric contrast means more power is radiated toward
the denser dielectric. Higher dielectric contrast corresponds to
Higher front to back ratio.

B. Planar Leaky Wave Slots: Standard and Enhanced

A different strategy for the excitation of these same lenses
is proposed here. The aim is to achieve an equivalent F/D of
these lenses larger than 1 over a broad band. The original idea
to adopt a leaky slot type of feed, as in [8] (in the following
described as standard leaky wave slot), in a circularly symmetric
elliptical lens antenna drove the present work. The leaky
slot would guarantee the broad bandwidth and, hopefully,
the circular symmetry of the lens should lead to reasonably
symmetric patterns, while maintaining a compatibility of the
feed manufacture with integrated technology. A simplified ray
picture of the near field radiated by a "standard" leaky slot
printed at the interface between air and a dense dielectric, is
shown in Fig. 2.

The region of co-existence of the leaky wave and the space
wave, as well as the region where only space waves exist are
highlighted in Fig. 2. The picture also shows the leaky wave
angle, $\gamma_{LW}$. As highlighted in section IV of [8] leaky
wave radiation is a much larger phenomenon than space wave
radiation when the slot is narrow in terms of the wavelength.
The leaky wave mode is associated to distributed radiation
which occurs ONLY in the denser medium, whatever is the
dielectric contrast, provided there is one. Thus the front to back
ratio in leaky slots is typically much higher than in resonant
type of feeds, and independent from the dielectric contrast.

When the observation point is moved to the far field, with
respect to the slot’s finite radiating length, the leaky wave
cannot be explicitly observed anymore, since it attenuates
exponentially as a function of $x$. However, a different but still
useful ray picture can be derived resorting to the space waves
only. The far field ray picture, associated to this same slot inside a dielectric lens is shown in Fig. 3a. This figure shows a
simplified two dimensional cut, representing the $H$ plane of a
dense, $\epsilon_r = 10$, elliptical lens, fed by the standard leaky wave
slot. The center of the slot would be located at the lower focus of
the elliptical lens so that the two separate beams, associated to
leaky waves propagating along $\pm 1\gamma$, could be separately
traced to the radiating aperture, $A$. The angle characterizing
the standard leaky wave, here indicated as $\gamma_{stan}$, defines the
width and the center location of the zones on the lens which are active. The angle $\gamma_{stan}$ can be evaluated analytically as shown in [8]. $\gamma_{stan} \approx 40^\circ$, is a typical value over a large frequency band for slots printed between air and a dielectric with $\epsilon_r = 10$. As described in Appendix A the -3 dB beam widths in the dielectric can also be calculated analytically. The beams of rays inside the dielectric can be imagined $12^\circ$ wide which can be representative for this type of slots. Following the rays to the aperture, one can identify a distribution which can be thought of as the superposition of two separate contributions. Let us assume that the separation between the two contributions is indicated with $d$ and that, for the sake of simplicity, each of the distributions is triangular with width $l$. Given these approximations, the analytical evaluation of the secondary radiation pattern, after the lens, can be performed by simple Fourier transformation of the aperture distribution in Fig. 3, leading to:

$$F_H(\theta, r) = \sin^2\left(\frac{k_0\sin\theta t}{2}\right)\cos\left(\frac{k_0\sin\theta d}{d}\right) e^{-jkr} r$$

(1)

Note that here $r$ indicates the distance of the observation
point from the center of the radiating aperture $A$, and $I$ is a
complex constant, unimportant at this point, which accounts
for the amplitude of the excitation, the propagation and reflections inside the lens and the field distribution in the $E$ plane.
If one substitutes $d = d_{stan} = 10\lambda_0$ and $l = l_{stan} = 2\lambda_0$ a
pattern as in Fig. 4(a) emerges, indicating that a large number of side lobes essentially renders this type of lens useless.
The interference between the two separate bundles of energy
emerging from the focal point gives rise to this.

In order to obtain a unique secondary main beam and much
lower side lobes, using a double leaky wave source, the angle
of the leaky waves with respect to the slot plane, \( \gamma_{LW} \), would have to be enhanced, i.e. be larger. Fig. 3(b) shows the ray picture in the case the angle of the leaky wave was \( \gamma_{enh} \approx 70^\circ \). A simple tracking of the geometrical rays shows that, in this case, the aperture distribution is well represented when the parameters \( d \) and \( t \) are taken as \( d_{enh} = d_{stan}/2 \) and \( t_{enh} = 2t_{stan} \). Resorting again to eq.(1) to estimate the pattern one finds the results in Fig. 4(b). In this enhanced case the side lobes are always lower than -10 dB thanks to the fact that the spatial separation of the ray bundles on A is much lower and these same bundles impinge on the dielectric-air interface almost normally.

![Fig. 3. Representation of the ray propagation inside an elliptical dielectric lens (\( \epsilon_r = 10 \)). Picture associated to the radiation from: a) a standard leaky wave slot, b) an enhanced leaky wave slot.](image)

One can note that any angle \( \gamma_{enh} < 70^\circ \) would give rise to side lobes higher than -10 dB. Since the radiation can be described as due to an array structure (composed of two elements), when the lens diameter becomes larger in terms of the wavelength grating lobes will eventually emerge. However the peak level of these grating lobes will be low as it is modulated by the element factor of this array. The element factor becomes more directive for larger lenses. Overall it appears that thus \( 70^\circ \) should be considered a minimum angle, as a rule of thumb.

III. ENHANCED LEAKY WAVE RADIATION: THE BREAKTHROUGH

The angle \( \gamma_{LW} \) of radiation of each leaky wave in Fig. 2 is associated to the real part of the propagation constant of the leaky wave propagating along the slot. In previous leaky lens structure [6] and [7] it was assumed that the propagation constant associated to this type of radiation is mainly the average of the propagation constant in the two dielectric media in contact through the slot. Apart from secondary dispersion effects, the propagation constant of the slot was assumed to be \( \beta = \sqrt{\epsilon_r^2 + \epsilon_s^2} \), where \( \epsilon_r \), for \( i = 1,2 \), were the propagation constants in the two media of dielectric constant \( \epsilon_r \) (with \( \epsilon_{r1} \) characterizing the less dense medium). Increasing the dielectric contrast would always lead to an upper boundary of \( 45^\circ \) for the leaky wave angle \( \gamma_{LW} = \cos^{-1} \frac{1}{\sqrt{\epsilon_r}} \). Thus a separation of at least \( 90^\circ \) between the two separate beams, as in Fig. 2 and 3(a), associated to a central feed of the slot is to be expected if the slot separates two different homogeneous dielectrics. However, if \( \beta \) is not the average between \( k_1 \) and \( k_2 \) but can be designed to assume a desired value things could look differently. If \( \beta \approx k_1 \) the separation between the two beams in the dense dielectric is much smaller. In fact \( \beta \approx k_1 \) implies waves propagating in the dielectric at angle, \( \gamma_{LW} = \cos^{-1} \frac{1}{\epsilon_r} \). This means that when waves are propagating with free space velocity of the less dense medium along the interface between the two media, waves propagate parallel to the critical angle in the denser dielectric. As an example, if on a slot separating free space and silicon (\( \epsilon_r = 12 \)), was propagating a wave with propagation constant equal to that of free space, the angle of leaky radiation in the dielectric would be \( 73^\circ \). In the case of quartz (\( \epsilon_r = 4 \)), the angle would be about \( 60^\circ \). It is not difficult to alter the slot’s geometry in order to render its propagation constant similar to the one of the free space in the less dense medium, \( \beta \approx k_1 \). It is sufficient to introduce a small air separation \( h \) between the slot and the dense dielectric as indicated in Fig. 5(a). If the slot width \( w_s \) is very small with respect to the wavelength and \( h \leq w_s \), the distributed capacitance associated to the slot is dominated by the very-near fields and thus by air. Consequently the

![Fig. 4. Simplified, normalized, H-plane secondary radiation patterns due to the lenses and feed represented in Fig. 3. a) a standard leaky wave slot feed \( (d_{stan} = 10\lambda_o, t_{stan} = 2\lambda_o) \), b) an enhanced leaky wave slot feed \( (d_{enh} = 5\lambda_o, t_{enh} = 4\lambda_o) \).](image)
real part of propagation constant is essentially that of free space, see Fig. 5 (b). The imaginary part of the propagation constant is associated to radiation in the dielectric. Power is increasingly leaving the slot as the wave propagates. In fact the pointing vector is not parallel to the slot but directed toward the dielectric in the presence of a dielectric contrast. Note that the introduction of a separation layer is just one way to alter the propagation constant and one of the most simple to realise at microwave frequencies. However, under-etching the slot in a dense crystal, or introducing a separation layer of low dielectric constant between the slot and the densest dielectric are equivalent implementation procedures.

In the rest of this paper we will refer to this type of radiation as to the enhanced leaky lens radiation, in contrast with the previous “standard” leaky lens radiation. For the interested reader, the link with Cherenkov radiation is explained in appendix B.

A. Green’s Function Investigations

The overall behavior of the leaky lens radiation has been described in the previous paragraph. A parametric investigation of the structure based on a rigorous Green’s Function (GF) analysis is of use when one wishes to actually design an antenna using the enhanced leaky slot. In this paragraph the dispersion equation and the radiation patterns are investigated using the formalism and calculational tools described in [8], and [9] and the specific extension to generic stratifications in [11]. It will be assumed that \( w_s = h \). This choice is just for the sake of fixing some variables. In practice the fine tuning of the ratio of choice for \( w_s \) and \( h \) depends on a third parameter, the thickness of the metal. This latter parameter is not easy to introduce in our evaluation of the GF, thus it is neglected in the present analysis.

1) Dispersion Study: The approximate but accurate solution of the dispersion equation that identifies the leaky wave pole can be obtained. The propagation constant in the slot can be expressed as

\[
\beta_{\text{LW}} \approx \beta_{\text{guess}} - \frac{D(\beta_{\text{guess}})}{D'(\beta_{\text{guess}})} \tag{2}
\]

where \( D \) is the function reported in eq. (21) of [11]. \( \beta_{\text{guess}} \) is an initial guess of the real part of the propagation constant. \( \beta_{\text{guess}} = \beta = k_1 \) would certainly be a good initial guess, however since the numerical evaluation of the derivative in (2) would imply the evaluation of the denominator function extremely close to a branch singularity \( k_x = k_1 \) in the complex domain. In the following numerical examples the initial point was taken to be 0.9\( k_1 \). The results are still excellent in comparison with much heavier full wave simulations performed with commercial tools.

The angle of radiation and the normalized attenuation constant as a function of the frequency are shown in Fig. 6 (a) and (b) respectively. The assumed dielectric contrast is \( \epsilon_{r1} = 1 \) and \( \epsilon_{r2} = 10 \) while \( w_s = h = 0.5 \text{mm} \). For comparison also the results associated to the case in which \( h = 0 \) are reported in order to appreciate the importance of the small separation layers. Considering the real part of the propagation constant, one can observe that the expected radiation angle \( \gamma_{\text{LW}} \) tends asymptotically to the critical angle in the dielectric (72°). From the observation of the attenuation constant one can realize that it decreases significantly as the slot is more separated from the dielectric in terms of the wavelength (high frequencies). This is because when the slot is farther away from the dielectric interface in terms of the wavelength, \( h \) is large, the portion of the field distributed inside the dielectric is lower and thus the propagation constant tends to be the one of free space.

2) Radiation Pattern in the Dense Dielectric: In this section the radiation from the slots whose dispersion has been investigated in section III-A.1 is studied. In particular the radiation patterns in absence of the dielectric lens are considered, assuming both the dielectric media, \( \epsilon_{r1}, \epsilon_{r2} \) and the slot infinitely extended. To evaluate the far fields, the same procedure as presented in [9] has been used, with the difference that this time the implemented Spectral Domain Magnetic Field GF includes the separation layer between the ground plane containing the slots and the denser dielectric. This spectral domain GF can be evaluated as discussed in [11]. Assuming the slot to be oriented along \( x \), (\( \phi = 0^\circ \)), Fig. 7(a),(b),(c) show the normalized total magnetic fields in the \( H \) (\( \phi = 0^\circ \)), the \( E \) (\( \phi = 90^\circ \)) and the diagonal (\( \phi = 45^\circ \)) planes respectively, as a function of the observation angle \( \theta \) in the case of \( \epsilon_{r1} = 1, \epsilon_{r2} = 10 \). The different curves in each figure refer to different values of \( w_s = h \). Note also that the angles from -90° to 90° refer to radiation in the denser dielectric, while the angle ranging -180°, -90° and 90°, 180° refer to radiation in the free space. The behavior of the filed in the \( H \) plane, Fig.7(a) is congruent with the dispersion analysis in sect. III-A.1: two beams are systematically found pointing at relatively small angles from broadside. As the values of \( w_s = h \) become larger the beams are more directive and pointing more clearly toward the critical angle. The behavior of the field...
in the $E$ plane, Fig. 7(b), is similar at least for certain values of the parameter $w_s = h$. This might come as a surprise to the reader, but in fact this was to be expected. The feed in the slot also excites a cylindrically spreading TM wave propagating away from the feed and that is initially bounded between the ground plane and the dense dielectric ($0 < z < h$). This propagation occurs with a phase velocity that is essentially the one of free space. In order to have continuity of the tangent fields at the air dielectric interface the waves in the denser dielectric, have to propagate toward the critical angle, thus realizing a directive beam in the dielectric not only in the direction of the slot ($H$-plane) but also in the other directions. In this plane the width of the slot plays a minor role and the $h$ parameter is more important. Note that the pattern in $E$-plane is approximately equivalent to the radiation pattern of an electric dipole printed at the interface between two different dielectric, see Fig 5 in [1]. For very small values of $w_s = h$ in terms of the wavelength, the radiation pattern in the $E$-plane tends to recover the shape it would have if the separation layer was equal to zero. This implies that a very small separation

Fig. 6. Leaky wave propagation constant as a function of the frequency: leaky wave angle, $\gamma_{lw}$ (a) and normalized attenuation constant (b). The dielectric constants are $\varepsilon_{r1}=1$ and $\varepsilon_{r2}=10$. The continuous lines are pertinent to slots parameters $w_s = h = 0.5 \text{mm}$, while the dashed lines are pertinent to $w_s = 0.5 \text{mm}$ and $h = 0$.

Fig. 7. The total magnetic field radiated in the far field by a set of different slots printed among $\varepsilon_{r1}=1$, $\varepsilon_{r2}=10$ and with $w_s = h$ are shown as a function of the elevation angles, $\theta$. a) represents the $H$ plane, b) the $E$ plane and c) the Diagonal plane.
Most of the peculiar properties of the planar UWB leaky lens, when compared to lenses fed by resonant feeds, stem from the fact that the effective F/D of the lens is relatively larger than for standard lenses. That is because the radiation pattern in the lens is more directive with respect to the cases mentioned in sect. II-A, and this implies that only the upper part of the lens is effectively used. As shown in the Appendix A, the equivalent F/D ratio for a circularly symmetric elliptical dielectric lens, with $\epsilon_r = 10$, fed by an enhanced leaky wave slot, is above 1 and increases as a function of the frequency. A larger F/D for the same lens clearly implies that the aperture efficiency is lower (a lower gain that the physical area would suggest). However the highest possible gain is rarely an incentive in higher frequencies, since the dimensions are very small.

1) **No Spill Over:** The main advantage of the rays illuminating the central part of the lens is that the amount of power, associated to rays emerging directly from the slot, that impinges at the dielectric air interface below the B-B’ line in Fig.1 is almost negligible. This implies that there is small amount of power lost in spill over. In standard lens antennas fed by sub-wavelength antennas, the only way to achieve low spill over is to either use array types of feeds which further diminish the bandwidth, or to use non elliptical lenses, which also lead to a lower directivity.

2) **Reflection at the Dielectric Air Interface:** As explained [13] and [14] the double reflections at the dielectric-air interface normally play a major role in characterizing the input impedance of the antenna as well as its radiation pattern. However, in the present case only the upper part of the lens is active. As explained in Fig.2(b) of [13] the top region of the lens does not contribute to the refocusing of doubly reflected rays. As a consequence, the new configuration of circularly symmetric lenses is not expected to suffer from resonances associated to doubly reflected rays even if a dense dielectric is used. However, as also explained in [4] the first reflections can be an important cause of performance degradation.

3) **Dielectric Layers:** When possible, it is better to include matching layers [16] to diminish all reflections. In the present case the antenna was expected to operate over a very broad band, thus a three-layers structure is proposed. The matching layers were chosen to be of heights $h_1 = 0.8$ mm, $h_2 = 1.1$ mm, $h_3 = 0.75$ mm and dielectric constants $\epsilon_{r1} = 6.5$, $\epsilon_{r2} = 3.5$, $\epsilon_{r3} = 2.5$. Fig. 9 shows the reflection coefficient associated to a plane wave normally incident to the dielectric stratification. It is apparent that a reduction of the reflection coefficient from -6 dB to -10 dB can be obtained over most of the band from 12 to 80 GHz. Also the transmission efficiency associated to the same plane wave is reported in Fig.9. A transmission higher than 90% can be obtained over the same band. The matching layers included in the design are also visible in Fig.8.

4) **Front-to-back Ratio:** Normally sub-wavelength or resonant slots printed at the interface between two dielectrics radiate with front-to-back ratios proportional to the dielectric
contrast as $\epsilon_r^{3/2}$ ($\epsilon_r = 10$ corresponding to -15 dB). From the curves in Fig. 7 one can observe that the front-to-back ratios for the magnetic fields ($H(\theta = 0^\circ)/H(\theta = 180^\circ)$) are moderate if one considers the radiation in the lens. However the rays propagating with angles close to 17°, the critical angle, are also directed toward broadside by the focusing lens. Accordingly, one can expect that the eventual front-to-back ratio for the magnetic fields of the overall system will be higher than 15 dB everywhere, corresponding to ($H(\theta = 17)/H(\theta = 180)$) from Fig. 7. If instead to referring to the magnetic fields one refers to the radiated power, or Poynting vectors $\mathbf{S}$, the front to back ratios are higher that 25 dB.

### B. Details of the Feed

Some of the geometrical details of the structure, excluding the metallic holder are shown in Fig. 10. Below the metallic thick plate a printed circuit board is glued, which is composed of a thin, $h_{\mu}$, dielectric layer of dielectric constant $\epsilon_r$, metallized ($h_{gp}$) on the side where the radiating slot is etched. The slot’s width and length are $w_s$ and $l_{slot}$, respectively. The metallic holding block keeps the slot’s plane at $h$ m of distance from the bottom of the lens. A micro-strip line, shorted to the slot’s plane via a vertical metallic pin, feeds the slot. The qualitative behavior of the phase of the field inside the aperture field as radiated by the source. Thus in the approximate equation (1) for the far field radiated pattern, including the lens, was discussed. If matching layers are used, the transition between dense dielectric and air is practically reflection-less. Since all the rays arrive at the radiating aperture, see Fig. 12, with the same electrical path length, one can use the central ray to estimate their phase. Note that, unlike the examples in section III, $w_s \neq h$.

### V. Dispersivity of a Link with Two Leaky Lenses

A detailed discussion on the performance of this antenna will be presented in [10] based on the measurements of hardware demonstrators. Here, we will discuss the potentials for this antenna to be used as basic element for a non distorting radiated link. The link between a transmitter and a receiver does not distort a short time domain pulse if the S12 parameter of the link in the frequency domain presents

- Linear phase variation (no dispersivity).
- Constant amplitude (no amplitude distortion).

#### A. Phase Center and Phase Linearity

The qualitative behavior of the phase of the field inside the lens but far from the feed is the same as that shown in [9]. The propagation constant in the dielectric $k_d$ and the distance of the observation point from the feed point $r_f$ relates to the phase as $e^{-jk_d r_f}$. This represents a spherical spreading in the lens. Moreover, in section II-B the $E$-plane radiation pattern, including the lens, was discussed. If matching layers are used, the transition between dense dielectric and air is practically reflection-less. Since all the rays arrive at the radiating aperture, see Fig. 12, with the same electrical path length, one can use the central ray to estimate their phase. Thus in the approximate equation (1) for the far field radiated by the lens, one can substitute the expression of the phase of the aperture field as radiated by the source.

$$F_I(\theta, r) = F(\theta)e^{-jk_d D \frac{\sqrt{r^2 + h^2}}{r}} e^{-jk_0 r} I_n$$  \hspace{1cm} (3)

where the subscript $n$ in $I_n$ differs from $I$ in equation (1). $F(\theta)$ in (3) is a real function for a symmetric distribution. The only variation of the phase of the far field $\phi$ as a function of the frequency and of the observation point can be expressed as

$$\phi(r) = -k_0 r - k_d \frac{D}{2} \left( \frac{\sqrt{r^2 + h^2}}{r} + 1 \right)$$ \hspace{1cm} (4)

We note that this expression is valid in the far field with respect to the radiating antenna and that $r$ is taken at the center of the top of the lens. For quasi optical systems, the far field region is defined as $r > 2D^2/\lambda_0$, where $D$ is the diameter of the entire aperture. Thus the far field region is at greater
distance for higher frequencies. However, in the present leaky lens case, the equivalent diameter of the radiating aperture decreases for higher frequencies due to the attenuation and thus the far field zone remains essentially at constant distance for all frequencies.

The dependence from the frequency in equation (4) can be highlighted.

\[ \phi(r) = -\omega \sqrt{\mu_0 \epsilon_0} r - \omega \sqrt{\mu_0 \epsilon_0} \frac{D}{2 \sqrt{\epsilon_r + 1}} \]  

(5)

This last equation shows that:
- the phase center is constant for all frequencies and observation points in the main beam. It is located on top of the lens as shown Fig. 11;
- the antenna can be described as non dispersive, since the variation of the phase as a function of the frequency is linear.

Overall, if the far field radiated by an antenna can be expressed as a radiation integral from a planar aperture distribution with constant phase, the phase center of the antenna does not move as a function of the frequency. If the phase of the distribution also depends linearly from the frequency then the link is dispersion less.

Overall, if the far field radiated by an antenna can be expressed as a radiation integral from a planar aperture distribution with constant phase, the phase center of the antenna does not move as a function of the frequency. If the phase of the distribution also depends linearly from the frequency then the link is dispersion less.

\[ \phi(r) = -\omega \sqrt{\mu_0 \epsilon_0} r - \omega \sqrt{\mu_0 \epsilon_0} \frac{D}{2 \sqrt{\epsilon_r + 1}} \]  

Fig. 11. Phase center location for the planar leaky lens.

**Differences with TEM Horns:** These properties do not apply for the most widely used antennas for UWB applications that require low dispersivity, TEM horns (or derived types like the Vivaldi). These horns, like all horns, are characterized by phase centers that move as a function of the frequency. The reason why they are considered weakly dispersive antennas is that their performance is typically observed in the broadside direction, see Fig. 12 (a). For simplicity we can assume that the frequency spectrum can be divided in two bands only. Also we can assume that the high frequency components emerge as rays from the high frequency phase center \( x_{\text{high}} \), while low frequency components emerge as rays from a point further along the tapering, the low frequency phase center \( x_{\text{low}} \). Before emerging as rays, the waves are guided with a phase velocity of a coplanar line \( k_g \). In the Fig. 12 (a) and (b) the paths from the antenna feeding point to the observation points followed by the high frequency rays are indicated by the solid lines. The paths followed by the low frequency rays are indicated by a dotted line. For observation points at broadside, \( r_{\text{obs}} = x_{\text{obs}} \), the high frequency and the low frequency rays arrive to the observer after having followed the same path length: as anticipated the low frequency waves are partly guided and partly radiated while the high frequency waves can be considered entirely radiated. But for TEM horns

\[ k_g = k_0 \]  

So high frequency rays and low frequency waves arrive to the observation point with the same phase delay. This shows that in the broadside case the dependence of the phase from the frequency is linear even if the phase center moves.

This only happens at broadside, as is evident when observing Fig. 12(b), which depicts the situation for observation points not at broadside. The path lengths for high frequency, solid line, and low frequency, dotted line, differ. In other terms, it would not be possible to obtain a correct representation of the far field radiated by a horn based on the radiation integral from a planar aperture distribution with constant phase, as was done in the case of the leaky lens.

**B. Calculated Gains and Directivities**

The antenna presented in the previous section has been analyzed by means of the commercial code CST, [17] based on the FDTD method. Two configurations, with and without matching layers, have been investigated. The only parameters which are important to finalize the theoretical discussion are the calculated gain and directivity. Fig. 13 shows the calculated gain and directivity for the antenna described in section IV, with and without the matching layers.

The important aspect is that, with matching layers, the achieved gain increases almost linearly with frequency in the band from 15 GHz to 70 GHz. This is unusual for dielectric lens antennas. As anticipated, this is due to the fact that as the frequency increases the slot feed becomes more directive and only the central part of the lens is illuminated.
EQUATION 4.7
\[ j(x, t) \approx q v_o e^{-\alpha t} \delta(x - v_o t) i_x \]

where the small argument approximation of the exponential has been used, because the deceleration is assumed to be small. Taking the Fourier transform of the decreasing current leads to a representation in the frequency domain

\[ j(x, \omega) = q v_o e^{-j \beta o x} e^{-\frac{j \omega}{v_o} x} i_x \]

Thus the spectral representation of the current associated to a decelerating charge is the same as that of a current exponentially decreasing in amplitude. Clearly there is a connection between the phenomenon of field amplitude decay in leaky waves and charge deceleration. This link is not obvious.

REFERENCES