Abstract—Connected arrays constitute one of the most promising options for wideband phased arrays. Like most phased arrays, they are designed using infinite array theory. However, when finiteness is included, edge effects perturb their behavior. These effects are more severe when the arrays are designed to operate over very broad frequency ranges, since the mutual coupling between the elements facilitates the propagation of edge-born waves that can become dominant over large portions of the arrays. Finite array simulations, which would predict these behaviors, are computationally unwieldy. In this paper we present a Green’s function based procedure to assess edge effects in finite connected arrays. First the electric current distribution on the array is rigorously derived. Later on, the introduction of a few simplifying assumptions allows the derivation of an analytical approximation for the current distribution. This latter provides meaningful insights in the induced dominant edge-wave mechanism. The efficiency of connected arrays as a function of their dimension in terms of the wavelength and of the loading feed impedances is investigated.

1. INTRODUCTION

In the last years, connected arrays have attracted a growing attention for wideband applications ranging from wide-angle scanning arrays [1], [2], to focal plane arrays for multi-beam imaging [3]. Their wideband performance is due to the fact that the connections between neighboring elements allow currents to remain nearly constant with frequency [4] and each element is effectively larger than its physical dimension. The connections also support the propagation of guided waves from one element to the other. However, as discussed in [5]–[7], these guided waves can be very strongly excited at the edges of the array. As a consequence, the overall behavior of a finite wideband array can be dramatically different with respect to the design based on infinite array analysis. Even if not in the context of connected arrays, [8] and [9] also investigated in detail the effects of strong guided waves associated to the finiteness of wideband dipole arrays.

In this paper, we present an accurate and analytical procedure to accurately assess edge effects, already in the preliminary design phase of connected arrays. This appears to be particularly needed because finite array full-wave simulations with general purpose tools, even when possible, are too demanding in terms of computational resources to be a useful design tool. Specifically, this paper will focus only on finiteness effects along the longitudinal direction (along which the dipoles are connected), $x$-axis in Fig. 1, to present properties that specifically characterize connected arrays. In fact, the effects of finiteness associated with the transverse direction, $y$ in Fig. 1, are dominated by space wave coupling. These effects have been extensively discussed in the dated literature [10] (and there cited references), more recently resorting to windowing type of approximations [11], and lately with analytically enhanced full-wave solutions in [12]–[14]. The latter works heavily relied on the ray field representations introduced by [15], and then refined in a number of more detailed works [16]–[19].

Here we investigate an infinite number of dipoles along the transverse direction, with each dipole fed at a finite number of points in the longitudinal direction, as in Fig. 1. The starting base for the analysis is the availability of transmission lines Green’s Functions (GF) of infinitely extended connected arrays. The derivation of these GF was initiated for the slotted case in [5], then extended to dipoles in [20], and generalized for both transmitting and receiving arrays including loads in [21]. Here, the effects of the array finiteness are explicitly addressed for the first time.

In the first part of the paper, the current distribution is rigorously derived resorting to the transmission line GF formalism. The global current distribution is obtained via a MoM-like numerical inversion procedure, which requires only one unknown per elementary cell, independently from the cell geometrical...
parameters. This is possible thanks to the use of an integral equation with Kernel characterized by the appropriate connected array GF. Results obtained using this methodology are compared with full-wave simulations using commercial software, showing excellent agreement at much lower computational costs. For practical designs, there is no limit to the longitudinal number of elementary cells that can be studied with this method. Both the cases of connected dipole arrays with and without backing reflectors are considered. Using this procedure, important design considerations regarding the role of the loads in the propagation of edge waves are provided. The method allows one to estimate the efficiency of connected arrays that are large or small in terms of the wavelength at very limited computational cost.

In the second part of the paper, in order to gain a deeper physical insight into the wave mechanisms occurring in connected arrays, a different approach is proposed. This latter method is based on the representation of the electric current along each long dipole as the superposition of an infinite array contribution plus edge-born waves. While infinite array current components are rigorously derived resorting to the full GF formalism, edge-born waves are approximated as a staircase distribution. It is important to note that this approximation would be totally inadequate if referred to the entire current distribution. However, it leads to small errors in absolute terms when applied only to the edge born contributions. Thanks to this simplification, a single spectrally analytical approximation of the edge currents is obtained. The singularities of this spectrum can be investigated and the pertaining inverse Fourier integrals can be asymptotically evaluated to provide the analytical expressions for the spatial currents. These latter analytical steps are performed only in the cases of arrays in free space and scanning in the $E$-plane, in order to maintain the analytical formulation as simple as possible, while still highlighting the main mechanism. Important potentials remain for future developments of the theoretical formulations.

II. SET UP OF THE SPECTRAL EQUATION: THE FINITE × INFINITE ARRAY CASE

The geometry of the problem under analysis is depicted in Fig. 1, for arrays of connected dipoles operating in transmission (Tx). The reception (Rx) case will be discussed in Appendix B, since it does not present particular difficulties, but requires a somewhat different notation. The dipoles, of width $w$ and separated by distance $d_y$ along $y$, are electrically connected along the longitudinal direction ($x$). When the array is transmitting, each dipole is fed at $N_x$ points ($0. N_x - 1$), spaced by period $d_y$. The excitations on the zeroth dipole ($y = 0$) are realized by lumped voltage generators with internal source impedance $Z_0$ and voltages $v_{n'z}$ for all other dipoles along the $y$ direction, a progressing phase is imposed: $e^{-jk_{0}z_{x}dz}$, where $k_{0} = k_{0}\sin\theta\sin\varphi$, $k_{0}$ is the free space wave number, and $(\theta, \varphi)$ indicate the pointing direction of the main beam. The equivalent planar problem is shown in Fig. 2 for an array in free space. Note that, even if the dipoles are fed at a finite number of points ($N_x$), it is assumed that the loads are periodically distributed over the entire length of the infinitely extended dipoles. The impact of this hypothesis in the actual solution is minimal, just like the infinite length of the dipoles themselves, and it is only retained for the sake of a clearer and simpler formulation. The problem could also be set up with the dipole assumed to be backed by an infinite ground plane at distance $h$. In the present modelling, we assume that both the ground plane and the dipoles are infinitely extended and thus the finiteness of the metallizations is not accounted for.

The derivation of the GF for doubly infinite, periodically excited, connected arrays with the inclusion of the loads was presented in [21]. In the case of a finite number of feeds ($N_x$), a similar integral equation for the unknown current $i(y')$ along the zeroth dipole ($y = 0$) can be used. One should only take care that in the right hand side (RHS) of (9) in [21], the incident field is now defined over a finite number of feeds, 0 to $N_x - 1$. The incident field can be assumed to be concentrated in the dipole gaps and uniformly distributed ($\delta$-gap excitations). Thus, the relevant integral equation is given by:

\[
\int_{-\infty}^{\infty} i(x')d_{x}(x, x')dx'
\]

\[
= - \sum_{n'z=0}^{N_x-1} \frac{v_{n'z}}{8} \Pi_{x,n'z}(x) + \frac{Z_{0}}{8} \sum_{n'z=-\infty}^{\infty} i_{n'z} \Pi_{x,n'z}(x), \quad (1)
\]

- $d_{x}$ is the space domain Green’s function once the dependence from the transverse dimension ($y$) is accounted for (6) in [21];
- the incident electric field at each feeding gap $v_{n'z}/\delta$ is expressed via $v_{n'z}/\delta = v_{n'z}e^{-jk_{0}z_{dx}dz}$, with $k_{0} = k_{0}\sin\theta\sin\varphi$ if the array is scanning towards $(\theta, \varphi)$.
- $i_{n'z}$ is the average current flowing in the $n_{x}z$-th gap (see (3));
- $\Pi_{x,n'z}(x) = 1$ for $x \in (n_{x}d_{x} - \delta/2, n_{x}d_{x} + \delta/2)$ and 0 elsewhere.

Resorting to the same techniques shown in [21], (1) can be solved in the Fourier domain leading to an expression for the spectral current distribution along the dipoles. The spectrum of the current can be written as follows:

\[
I(k_{x}) = \frac{\sin(\frac{d_{y}}{2})}{D_{f}(k_{x})}
\]
where the explicit expression of $D_j(k_x)$ is given in (18) and (17) of Appendix A, for the cases with and without backing reflector, respectively.

The expression of the current spectrum in (2) is given only implicitly, since it depends on the unknown terms $i_{k_x}^{n_x}$. In the remainder of this paper, we propose two different methods to derive an explicit expression for the spatial current distribution over the dipoles. In Section III, a rigorous numerical solution that involves a matrix inversion is presented. Finally, in Section V, we discuss the analytical approximations that allow to explicitly highlight the finiteness effects.

III. NUMERICAL SOLUTION

A simple numerical procedure to solve (2) is provided here. The terms $i_{k_x}^{n_x}$ can be expressed as functions of the spectrum at the left hand side (LHS). To this goal, let us recall the definition of the average currents on the gaps

$$i_{k_x}^{n_x} = \frac{1}{\delta} \int_{n_x \delta - \frac{\delta}{2}}^{n_x \delta + \frac{\delta}{2}} i(x) dx.$$  \hspace{1cm} (3)

Substituting in (3) the spatial current distributions $i(x)$, expressed as inverse Fourier Transform of (2), after a few simple algebraic manipulations, leads to

$$i_{k_x}^{n_x} = \sum_{n_x = 2}^{N_x+1} \frac{Y_{n_x} n_x}{\delta} \left( v_{n_x} - Z_l i_{k_x}^{n_x} \right)$$  \hspace{1cm} (4)

where the infinite summation of loads has been restricted to $N_x + 4$ elements, including two dummy elements at each edge of the array. These are typically sufficient, for non negligible values of $Z_l$, to replace the infinite summation in the RHS of (2). The mutual admittance in (4) is defined as

$$Y_{n_x, n_x'} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin^2 \left( \frac{k_x \delta}{2} \right)}{Z_l(k_x)} e^{-j k_x (n_x - n_{x'}) \delta} d\delta.$$  \hspace{1cm} (5)

Equation (4) can be written in matrix form, leading to a system of linear equations that can be solved by matrix inversion as

$$i_{k_x} = \left[ I + Z_l Y \right]^{-1} i_{k_x} = 0 \Rightarrow Y_{n_x} = v_{n_x} / i_{k_x}^{n_x}$$  \hspace{1cm} (6)

where $i_{k_x} = 0 = Y_v$, $I$ is the identity matrix, $i_{k_x} = [i_{k_x}^1, i_{k_x}^2, \ldots, i_{k_x}^{N_x+1}]$ and $v = \{v_0, v_1, \ldots, v_{N_x-1}, 0, 0\}$ is the vector of the impressed voltages. The inversion leads to the exact solution for the average currents on the gaps including the effects of the loads.

The elements of the admittance matrix in (5) can be evaluated numerically by performing the spectral integral with convenient deformations of the original integration path on the real $k_x$ axis. Fig. 3 shows the complex topology and branch cuts associated with the first (for $ny = 0$) square root of the GF that appears in (17) and (18) of Appendix A. The branch points are in $\pm k_3$ in the case an array scanned in the longitudinal direction ($k_y = 0$, or $\varphi = 0$).

For highly-coupled elements (small factors $n_x - n_{x'}$), an integration path deformation as in Fig. 3(a) has been used to avoid the branch cuts. For large distances ($n_x - n_{x'}$), the integrands present faster oscillations on the real axis. Thus, a path deformation as in Fig. 3(b) usually guarantees faster convergence in the free space cases and whenever poles of the stratification’s GF are not captured in the deformation. In the case of the array operating in the presence of a backing reflector, the height of the array from the reflector is typically such that no poles are expected to be found in the real $k_x$ axis. Further poles could arise in the case the array is printed on a grounded dielectric slab. The presence of these poles would correspond to possible excitation of surface and leaky waves. However, these configurations are not considered useful from a design point of view, regardless of the theoretical interest their analysis can arise.

A. Results of the Numerical Solution

The active currents at the gaps calculated via (6) can be used to evaluate the active impedances of the finite array, given by $Z_{n_x} = v_{n_x} / i_{k_x}^{n_x} \Rightarrow Z_4$ [21].

Fig. 4 shows a comparison between the numerical solution presented here and simulated result obtained via Ansoft HFSS [22]. Fig. 4(a) refers to an array in free space that is excited at 15 feeding points along y and is infinite along y. The array periods are $d_x = d_y = 0.45 \lambda_9$, where $\lambda_9$ is the wavelength at 10 GHz. The other geometric parameters of the array are $w = 0.2 \lambda_9$ and $d = 0.2 \lambda_9$. Fig. 4(b) refers to the same array where a backing reflector is included at a distance $h = 0.25 \lambda_9$ from the dipoles. Curves in Figs. 4(a) and (b) are shown for broadside radiation and scanning to $45^\circ$ on the $E$-plane, while Fig. 4(c) refers to the same array in free space scanning to $45^\circ$ on the diagonal plane ($\varphi = 45^\circ$). A very good agreement can be observed when comparing full-wave HFSS simulations and the numerical solution presented here. Note that, once the average currents on the gaps have been obtained, the total current on the array can be expressed using (2), and consequently all important parameters of the array, including the radiation patterns, can be obtained.
IV. EFFICIENCY OF FINITE CONNECTED ARRAYS

In the previous section we have derived a reliable and fast solution for the current distributions at the feeds of a finite connected array. The main advantage of this formalism is that the efficiency of a scanning connected array can be evaluated much more accurately than would be possible with only infinite array solutions. In the present context, by the term array efficiency we refer to the impedance mismatch at each of the \( N_x \) feeding points of the array. The array is assumed to be fed by transmission lines with characteristic impedance \( Z_0 \) that ensures the widest usable BW at broadside. For each feed we can define an active reflection coefficient \( \Gamma_{\text{act}} = (Z_{\text{act}} - Z_l)/(Z_{\text{act}} + Z_l) \), in which \( Z_{\text{act}} \) is the active impedance at the \( n_x \) element. We can also associate with the same element a mismatch efficiency \( \eta_{\text{mismatch}} = 1 - |\Gamma_{\text{act}}|^2 \). Clearly, the matching of each element will depend on the frequency and the scanning angle. As a consequence, the average efficiency of the array as follows:

\[
\eta(f, \theta) = \frac{1}{N_x} \sum_{n_x=1}^{N_x} \left( 1 - \left| \frac{Z_{\text{act}}(f, \theta) - Z_l}{Z_{\text{act}}(f, \theta) + Z_l} \right|^2 \right),
\]

Especially for array scanning to wide angles and for arrays composed by only a few elements, the current distributions over the finite arrays are significantly different from the infinite array ones. As a consequence, the active impedances are different from those that would be expected only on the base of infinite array designs.

Fig. 5 presents the overall array efficiency, defined as in (7), as a function of the scanning angle on the \( E \)-plane, for different frequencies. The array under analysis is composed of 8 elements and is operating in the presence of a backing reflector. The specific dimensions are taken from an array design discussed in [20] \((w = 0.05\lambda_0, d = d_y = 0.45\lambda_0, h = 0.31\lambda_0, \delta = 0.125\lambda_0)\), with \( \lambda_0 \) being the wavelength at the frequency \( f_0 \) and refer to a load resistance of \( Z_L = 400 \Omega \). It is apparent that, for larger scanning angles, the finite array simulations show important differences with respect to the infinite array ones. In practice, the array differences between the exact and approximate modelling are significant when the arrays are not perfectly matched. The availability of an accurate and rapid finite array modelling tool is key in real designs especially if the threshold of acceptable functionality is defined for scanning toward \( -60^\circ \).

Fig. 5 presents the resulting overall array efficiency, as a function of the number of elements of the array, for different scanning angles. The figure presents results for two different arrays, both with backing reflector at \( h = 0.31\lambda_0 \) and \( h = 0.41\lambda_0 \), designed in such a way that the active impedances are well fed by 400 \( \Omega \) and 100 \( \Omega \) characteristic impedance lines, respectively. A first predictable consideration is that, when the number of elements of the array tends to be large, the simulations assuming infinite or finite arrays imply similar efficiencies. A second non obvious design aspect emerges from these simulations. For arrays designed to operate well when fed by low impedance feeding lines, the edge effects are more important than for arrays designed to be fed by high impedance lines. Thus
a designer should avoid antenna design that apparently (within finite arrays simulations) require low input impedances. Because in reality in these cases the edge effect dominate a much larger portion of the array and the asymptotic behavior of a large array is only achieved for unrealistically large arrays. This can only be explained by digging deeper into the physics of finite connected arrays.

V. SPECTRAL INTEGRAL APPROXIMATION

Although the numerical solution presented in Section III is efficient (one unknown per array element) and accurate, it does not provide physical insight on the nature of the edge-waves. In order to obtain an alternative, more insightful, representation it is useful to recall how the infinite array auxiliary problem is set up. By simple extension of (1), the current $i_{\infty}(x')$ can be represented as the solution of

$$\int_{-\infty}^{\infty} i_{\infty}(x') dt(x, x') dx' = \sum_{n_{\delta} = \infty}^{\infty} \frac{Z_{l}n_{\delta}^2 - v_{n_{\delta}}}{\delta} \Pi_{s,n_{\delta}}(x).$$

(8)

Once the solution for the current $i_{\infty}(x')$ is assumed to be known, as shown in [20], the electric currents in a finite connected array can be expressed in a form that highlights edge effects as follows:

$$i(x') = i_{\infty}(x') + i_{\text{edge}}(x').$$

(9)

The edge term represents the perturbation induced by finiteness effects. Proceeding as in Appendix C and assuming the validity of the following conditions

$$\left\{ \begin{array}{l} d_{y} \ll \lambda_{0} \\ d_{z} = \delta \end{array} \right.$$  

(10)

it is possible to achieve a rigorous representation of the spatial current distribution. In these cases, the edge currents can be expressed as a single spectral integral, avoiding the necessity to perform the matrix inversion in (6)

$$i_{c,\text{edge}}^{s_{x}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin^2 \left( \frac{k_{y}d_{y}}{D_{y}(k_{z})} \right)}{D_{y}(k_{z})} \frac{Z}{d_{z}}$$

$$\times \sum_{n_{\delta}} \frac{e^{j(k_{x} - k_{\delta})n_{\delta}d_{x} - jk_{s}n_{\delta}d_{z}d_{y}}}{n_{\delta}} dk_{z}.$$  

(11)

This last spectral integral can be performed numerically along the path defined in Fig. 3(a). A validation of the procedure is shown in Figs. 7 and 8, which show the magnitude of the average currents in the gaps, normalized to the infinite array solution. The results are for connected arrays in free space and in the presence of a backing reflector, when scanning toward broadside and toward $\theta = 60^\circ$, respectively. The arrays are fed at 31 points, spaced by $d_{x} = \delta = 0.2\lambda_{0}$. For the array in free space and the one backed by a reflector (at $h = 0.1\lambda_{0}$) source impedances of $Z_{l} = 188 \Omega$ and $Z_{l} = 377 \Omega$ are assumed, respectively. The remaining geometrical parameters of the arrays considered are $d_{y} = 0.2\lambda_{0}$, $w = 0.08\lambda_{0}$.

An excellent agreement is obtained between the results predicted by the integration procedure and the fully numerical inversion when (10) are verified. It can be noted that in the considered cases the finiteness of the array can cause variations of the current distribution, with respect to the infinite array solution, corresponding to unity in the graphs, of up to 70% for large scanning angles. These variations are still well represented with the integral solution since this latter does not include any important approximations in the simple cases. This accuracy is maintained also for much smaller or larger arrays. Similar curves, describing edge effects in finite arrays have been first observed in [23], [24] and interpreted with a heuristic Gibbsian model.
Fig. 9. Comparison of the numerical solution and the spectral integral solution for a connected array in free space at four different frequencies: (a) 0.2f_0, (b) 0.4f_0, (c) 0.8f_0, (d) f_0; the load impedance is equal to 200 Ω.

A. Extrapolation of the Simple Case Solution

Equation (11) was obtained thanks to the simplifications in (10). Specifically, the second hypothesis is instrumental for the algebraic operations on the spectra. If one assumes that it makes sense to have the load impedance \( Z_l \) distributed over the entire cell \( (d_e) \), while the feeding is only applied to a region \( (δ) \), with these two parameters being different, (11) can be extrapolated as follows:

\[
P_{\text{edge}}^{n_x} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin^2 \left( \frac{k_z d_e}{2} \right)}{D_{\text{load}}(k_z)} \prod_{n_x} \left( e^{j(k_z - k_{z0}) n_x} - e^{-j(k_z - k_{z0}) n_x} \right) e^{j k_z n_x d_e} d_k_z
\]

where \( D_{\text{load}}(k_z) = D_l(k_z) - Z_l/d_e \) and \( δ \), possibly different from \( d_e \), is now in the argument of the sinc function.

Fig. 9 shows a comparison of the numerical solution and the spectral integral solution for the currents in the gaps normalized to the infinite array solution. The connected array is in free space with 15 feeds along \( x \) and loaded by 200 Ω impedances and \( δ \), scanning toward 45° on the \( E \)-plane. The four graphs refer to different frequencies at which the array periodicity and the gap dimensions are \( d_e = 0.1, 0.2, 0.4, 0.5 \lambda_0 \) and \( δ = 0.04, 0.08, 0.16, 0.2 \lambda_0 \), respectively.

The agreement in Fig. 9 is excellent at low frequencies and shows only minor deviations at higher frequencies where the critical small period approximation that justifies the stair case distribution begins to fail. Overall, it appears that while the low frequency approximation is instrumental to the spectral expression to work, the non verification of the second hypothesis in (10) \( (δ = d_e) \) is not important from the point of view of the results accuracy. A possible explanation is that, to the first order, a good approximation of the reactive energy associated with the cell is already included in the infinite array approximation and the gap dimension, and accordingly \( δ \) play only a minor role in defining the edge-born currents.

VI. FREE SPACE CASE: UNIFORM ASYMPTOTIC EVALUATION OF THE INTEGRAL APPROXIMATION

Equation (12) represents a single integral expression for the edge-born currents in a connected array excited at a finite number feeding points. The asymptotic evaluation of this integral can provide the physical insight that is now missing. In order to maintain the formulation as simple as possible, only the case of connected arrays operating in free space will be considered. The first step to the evaluation is the recognition of two separate terms, each associated with one edge of the array.

A. Contributions From the Two Edges

The summation over the auxiliary contributions from sources external to the array can be expressed in closed form leading to two contributions associated with the left and right edges of the array as follows:

\[
\sum_{n_x}^{\text{ext}} e^{j(k_z - k_{z0}) n_x d_e} d_z
\]

where \( D_{\text{load}}(k_z) = D_l(k_z) - Z_l/d_e \) and \( δ \), possibly different from \( d_e \), is now in the argument of the sinc function.

Noting the first summation is convergent for \( \delta = d_e \), while the second one for \( \delta = d_e \). The introduction of (13) in (12) leads to

\[
k_{z,\text{edge}} = k_{z,\text{edge}1} + k_{z,\text{edge}2}
\]

This representation highlights the presence of a number of poles, which emerge from the zeroes of \( (1 - e^{j(k_z - k_{z0}) d_e}) \) associated with the Floquet Waves (FWs) in \( k_{z,xm} = k_{z0} + (2\pi m)/(d_e) \). Besides FW poles, the integrand also possesses other types of singularities, specifically the branch points as in Fig. 3 and complex poles associated to the dispersion equation \( D_{\text{load}}(k_z) = 0 \). Their location in the complex plane depends on the actual loads \( Z_l \) that characterize the feeding lines. The approximate solution of this dispersion equation is reported in Appendix D. The real and imaginary part of the dominant pole are plotted in Fig. 10, for a case characterized by \( d_e = d_0 = 0.45 \lambda_0 \) and \( m = δ = 0.2 \lambda_0 \). The figure compares the analytical solution provided in (38) with a numerical solution based on a simple descent along the gradient.
Fig. 10. Real and imaginary part of the pole \( k_{z_p} \) versus load resistance for an array in free space.

following an accurate starting point. For small loads \( k_{z_p} \approx k_0 \) and unattenuated waves are supported by connected arrays. However for large loads, the imaginary part of this propagating mode is highly negative. It should be noted that these poles correspond to purely attenuating waves due to losses (feed absorption). These are not leaky waves.

After the discussion on the singularities, also the approximate analytical evaluation of the two integrals in (14) is derived in Appendix D. The analytical expressions for the current contribution born from the left edge of the array is given by

\[
\left. \begin{array}{l}
\tilde{j}_{\text{left}} \approx \frac{e^{-jk_0X}}{X \sqrt{X}} \\
\quad \times \left( C_1 \frac{F_1(X(k_0 - k_{zp}))}{(k_0 - k_{zp})} + C_2 \frac{F_2(X(k_0 - k_{z0}))}{(k_0 - k_{z0})} \right) \end{array} \right\} (15)
\]

with \( X = (m_z + 1)d_z \). In (15) the slope Fresnel function is introduced: \( F_n(x) = 2jxF_n(1 - F_n(x)) \), where \( F_n(x) \) is the Koyoumian Fresnel function [26]. This latter is defined in (49) in Appendix D.

Also in (15)

\[
C_1 = C' \left( R(k_0) - \frac{j}{d_x(k_{zp} - k_{z0})} \right),
\]

\[
C_2 = \frac{jC'}{d_x(k_{zp} - k_{z0})}
\]  

(16)

with \( C' = C\sqrt{2k_0}(\text{e}^{-jk_0/4\pi}) \) and \( R(k_z) \) defined in (45). The current contribution born at the right edge of the array can be similarly expressed and is reported in the Appendix D.

B. Comments on the Analytical Solution

A comparison between the results obtained resorting to the analytical expressions in (15), or (50) and (52), and the numerical method is shown in Fig. 11. The array is radiating toward broadside and \( \theta = 60^\circ \) on the \( E \)-plane. The array is assumed to be composed by \( N_z = 101 \) feeding points, and the geometrical parameters are \( d_x = d_y = 0.4\lambda_0 \), where \( \lambda_0 \) is the calculation frequency, and \( w = \delta = 0.2\lambda_0 \). It is apparent that the proposed analytical solution is extremely accurate also for arrays that are scanning at very wide angles.

The availability of an analytical expression allows one to give qualitative considerations on the nature of the electric current distribution. For high values of the loads \( Z_i \), and observation points close to broadside, \( k_{z0} \approx 0 \) and the arguments of the \( F_n \) functions are large. This means that the current distribution from each edge is dominated by the spreading factor \( x^{-3/2} \), which is associated with a rapid decay as a function of the distance from the end points. For lower values of \( Z_i \), the load induced pole \( k_{zp} \) can be close to the branch point \( k_0 \). Also, for observation toward wide scanning angles, \( k_{z0} \) is close to \( k_0 \). When either of the two situations occurs, the transition functions argument tends to zero, and the Fresnel function can be approximated by \( F(X) \approx \sqrt{\pi X}e^{j(\pi/4 + X)} \). This implies that the dominant term to the current distribution is of the type \( e^{-jk_{zp}x} \) or \( e^{-jk_{zp}x} \), which present no geometrical spreading and only a small exponential attenuation \( \langle k_{zp} \rangle \) or no attenuation at all \( (k_{z0} \in \mathbb{R}) \).

Specifically the \( e^{-jk_{zp}x} \) dependence of the current distribution is shown here to emerge from an analytical Green’s function for the first time. It express the idea that the load/source impedances attenuate the edge waves consuming their energy. This mechanism is probably occurring in all arrays, not only in connected arrays, but to our knowledge was never given explicit evidence or demonstrated for any array.
VII. Conclusion

This paper has presented a novel analytical methodology to assess edge effects in connected arrays, which provides important guidelines to broadband phased array design.

Analysis: Starting from the knowledge of the connected arrays GF, this paper has first presented the derivation of a general purpose numerical procedure for the assessment of the finiteness effects in connected arrays. This procedure is of general applicability in terms of type of array and scanning conditions. In fact, arrays with and without backing reflectors or dielectric stratifications and scanning in the E-, H- or diagonal planes can be analyzed. The numerical cost of the analysis is only the inversion of a matrix of dimension \( N_x \times N_x \), where \( N_x \) is the number of feeding points in the array along the longitudinal direction. The availability of such numerical procedure can provide unique design opportunities. It is particularly convenient when the performance of wideband wide-angle scanning arrays needs to be assessed in advance of measurements or full-wave, all inclusive, numerical simulations.

In a second step, the representation of the total current in terms of the infinite array plus edge-born waves has been introduced. Thanks to this representation, simplifications that would otherwise be unreasonable can be adopted. These simplifications lead to a single analytical expression for the spectrum of the edge-born waves in cases of general applicability.

Finally for the specific case of a connected array of dipoles array operating in free space, and scanning only in the E-plane, a rigorous analytical approximation of the pertinent inverse Fourier transforms allows an analytically rigorous expression of the edge-born waves. The expression is given in terms of standard Fresnel functions which highlight similarities between the edge-induced currents in connected arrays and the edge-born currents in the canonical problems of diffraction from half planes.

Design: From the design point of view the main findings of this investigation are as follows.

1) Edge effects are fundamental to assess the behavior of connected array in wide angle scanning situations. Especially when the arrays are composed of a small number of elements \([8, \ldots, 16]\), infinite array simulations are just not good enough to predict the performance. The free space case, treated here analytically, gives good physical understanding and guidelines.

2) The intensity of the edge-born waves is only mitigated by the source load impedances. The information of the load impedance is crucial to assess finite array effects. This was previously anticipated by Hansen in [2] and Munk [8]. The origin of this phenomenon is believed to be explained here for the first time.

3) The intensity of the edge waves is more important for low loading and source impedances. High impedance (400 Ohms) reduces edge-born waves in connected arrays with a backing reflector.

APPENDIX A
CONNECTED ARRAY GF’S

This appendix reports the explicit expression of the GF for connected arrays, for both cases in which the array is operating in free space and in the presence of a backing reflector.

For arrays in free space, the explicit expression of the GF \( D_\ell(k_z) \) is given by

\[
D_\ell(k_z) = -\frac{\zeta_0}{2k_0d_y} \sum_{m_y=-\infty}^{\infty} J_\ell\left(\frac{k_{nu}m_y}{2}\right) \frac{J_\ell\left(\frac{k_{nu}m_y}{2}\right)}{\sqrt{k_0^2-k_{nu}^2-k_{nu}^2}} \tag{17}
\]

while in the case of a backing reflector at a distance \( h \) from the array, the expression becomes as follows:

\[
D_\ell(k_z) = -\frac{\zeta_0}{2k_0d_y} \sum_{m_y=-\infty}^{\infty} J_\ell\left(\frac{k_{nu}m_y}{2}\right) \left(1 - e^{-j2h\sqrt{k_0^2-k_{nu}^2-k_{nu}^2}}\right) \times \sqrt{k_0^2-k_{nu}^2-k_{nu}^2} \tag{18}
\]

APPENDIX B
RECEIVING MODE

In the receiving mode, the source is assumed to be an incident plane wave from the direction \((\theta, \phi)\), since the response of the structure to any other source can be represented as the superposition of responses to a spectrum of plane waves.

For plane wave incidence and arrays in free space the incident electric field can expressed spectrally as

\[
E_x^i(k_z, y = 0, z = 0) = e^{+j\delta(k_z - k_{0z})} \tag{19}
\]

where \( E_x^i \) is the amplitude of the incoming plane wave. A more general expression of the incident field will be dependent on the specific stratification considered. The case of connected arrays in the presence of backing reflector is of particular interest in this paper and would imply a multiplying factor \((1 - e^{-j2h\delta})\) in (19). The average currents in the gaps can be expressed as

\[
\int_{b_{\ell,\nu}}^{b_{\ell,\nu}} d^2x = \int_{\text{ann..Z_0=0}}^{\text{ann..Z_0=0}} -\int_{\text{ann..Z_0=0}}^{\text{ann..Z_0=0}} \sum_{\nu_0=0}^{N_x-1} Y_{\nu_0\nu_0} \int_{b_{\ell,\nu}}^{b_{\ell,\nu}} d\nu_0 \tag{20}
\]

where

\[
\tilde{i}_{\nu}^{\nu,\ell} = e^{+\frac{\nu_0}{\tilde{D}_\ell(k_0z_0)}} e^{-j_{\nu_0\nu_0\nu_\ell}z_0}. \tag{21}
\]

The solution for the spatial current distribution in the receiving mode is obtained substituting \( \text{ann..Z_0=0} = \text{ann..Z_0=0} \) in (6), where the elements of the vector \( \text{ann..Z_0=0} \) are defined as in (21).

APPENDIX C
SPECTRAL REPRESENTATION OF THE EDGE CURRENT

Using (9), the integral (1) for the finite array can be re-expressed as

\[
\int_{-\infty}^{\infty} \left( i_{\text{ann..Z_0=0}}(x') + i_{\text{edge..Z_0=0}}(x') \right) d_\ell(x', x') dx' \tag{22}
\]

\[
= \sum_{\nu_0} \frac{e^{+i\nu_0\pi}}{\rho} \Pi_{\delta_\nu_0}(x') + \int_{b_{\ell,\nu_0}}^{\text{edge..Z_0=0}} \left( i_{b_{\ell,\nu_0}}^{\nu_0} + i_{b_{\ell,\nu_0}}^{\nu_0} \right) \Pi_{\delta_\nu_0}(x) \tag{22}
\]
where the following notation was used:

$$\sum_{n'_z=-\infty}^{\infty} = \sum_{n'_z=-\infty}^{N_z+1} + \sum_{n'_z<0,n'_z>N_z+1} + \sum_{n'_z=0}^{\infty} + \sum_{n'_z=0}^{\infty}. \quad (23)$$

Subtracting (8) from (22) leads to:

$$\int_{-\infty}^{\infty} i_{\text{edge}}(x') d_j(x, x') dx' = \sum_{n'_z=0}^{\infty} i_{\text{edge}}(x') + \frac{Z_1}{\delta} \sum_{n'_z=0}^{\infty} n'_z \ll l_{\text{edge}} \Pi_{N_z, n'_z}(x). \quad (24)$$

In this equation the only unknown is $i_{\text{edge}}(x')$ and in general it cannot be solved in a simple spectral way. However, a particular geometrical case can be solved exactly in a spectrally analytical form.

**Simple Case: Low Frequency and Generator Distributed Over the Entire Cell**

The simpler case that can be solved in a closed form is obtained with the configuration in which

$$\begin{cases} d_x \ll \lambda_0 \\ d_x = \delta \end{cases}. \quad (25)$$

The first hypothesis leads to the possible use of a star case approximation for the edge born currents

$$i_{\text{edge}}(x') \approx \sum_{n'_z} i_{\text{edge}} n'_z \Pi_{N_z, n'_z}(x'). \quad (26)$$

The second hypothesis, which implies that the load is distributed over the entire elementary cell, allows one to identify the same $i_{\text{edge}}(x)$ on the LHS and the RHS of (24). This leads to the following simplified integral equation

$$\int_{-\infty}^{\infty} i_{\text{edge}}(x') d_j(x, x') dx' = \sum_{n'_z} i_{\text{edge}} n'_z \Pi_{N_z, n'_z}(x') + \frac{Z_1}{\delta} i_{\text{edge}}(x'). \quad (27)$$

This equation can be simply solved analytically once it is expressed in the spectral domain

$$I_{\text{edge}}(k_z) = \frac{\nu_0 \sin\left(\frac{k_d x}{2}\right)}{D_j(k_z) - \frac{Z_1}{\delta}} \sum_{n'_z} \delta(k_z - k_{z, n'_z}) n'_z d_z. \quad (28)$$

The spatial current distribution is then given by an inverse Fourier integral. By projecting the spatial current distribution on the gaps, assuming $\nu_0 = 1$ Volt, we can evaluate the expression of the currents at each feeder of the finite array as follows

$$i_{\text{edge}}(x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{k_d x}{2}\right)}{D_j(k_z) - \frac{Z_1}{\delta}} \sum_{n'_z} \delta(k_z - k_{z, n'_z}) n'_z d_z e^{-jk_z n'_z d_z} dk_z, \quad (29)$$

Fig. 12. Original the integration path for the evaluation of (29).

**APPENDIX D**

**Uniform Evaluation of the Integral**

The integral defining $i_{\text{edge}}^{(x z)}$ and $i_{\text{edge}}^{(y z)}$ in (14) are converging over different integration paths. The convergence of the integral $i_{\text{edge}}^{(x z)}$ requires a counter-clock circling of the poles (see Fig. 12). On the contrary, the integration path of $i_{\text{edge}}^{(y z)}$ should be deformed in the half plane $3m \{k_z\} > 0$. These different complex plane topologies suggest two specular uniform asymptotic evaluations for the two integrals. In the following the focus will be on $i_{\text{edge}}^{(x z)}$, since the evaluation of $i_{\text{edge}}^{(y z)}$ can be performed in exactly the same way, once the change of variable $k_z' = -k_z$ is introduced.

**Dominant Singularities**

Before proceeding with the asymptotic evaluation, it is useful to isolate the zeroth order $(m_y = 0)$ from the higher order FWs $(m_y \neq 0)$ in the transverse GF terms pertaining to the free space case (17). This is obtained by representing $D_j(k_z)$ as follows:

$$D_j(k_z) = -\frac{\nu_0}{2k_0 k_d} \left( k_z^2 + k_{z, 1}^2 \right) \times \sum_{m_z \neq 0} \frac{J_0(k_{z, m_z} / 2)}{\sqrt{k_z^2 - k_{z, m_z}^2}}. \quad (30)$$

Using this representation it is simple to express the loaded transverse GF $D_{\text{load}}(k_z)$ in terms of $k_z = \sqrt{k_z^2 - k_{z, 1}^2}$ as follows:

$$D_{\text{load}}(k_z) = -\frac{\nu_0}{2k_0 k_d} \left( k_z + k_{z, 1}^2 j\Psi(k_z) + 2k_0 d_j Z_1 \right) \quad (31)$$

where

$$j \Psi(k_z) \approx \sum_{m_z \neq 0} \frac{J_0(k_{z, m_z} / 2)}{\sqrt{k_z^2 - k_{z, m_z}^2}}. \quad (32)$$

Since for well sampled arrays the function $\Psi(k_z)$ is slowly varying with $k_z$, it is legitimate to approximate $\Psi(k_z) \approx \Psi(0) \equiv \Psi$. This approximation helps to recognize (31) as a second degree polynomial function of $k_z$, which can be expressed highlighting its roots

$$D_{\text{load}}(k_z) = -\frac{j\nu_0}{2k_0 k_d} (k_z - k_{z, 1})(k_z - k_{z, 2}) \quad (33)$$
where

$$k_{zp1} = \frac{1}{2\Psi} \left( 1 \pm \sqrt{1 - j\frac{\Psi k_0 d_y Z_1}{\zeta d_x}} \right) - jk_0 \psi_0$$  \hspace{1cm} (34)

Using (33), the contribution due to the first edge in (14) can be then expressed as follows:

$$\begin{align*}
\hat{\eta}^{\text{nz}}_{\ell, \text{edge}1} &= \frac{j k_0 d_y e^{jk_0 d_y Z_1}}{\Psi_0 \Pi} \\
&\times \int_{-\infty}^{\infty} \frac{\sin^2 \left( k_z^2 \right) e^{-jk_z (n_z+1) d_x}}{(k_z - k_{zp1})(k_z - k_{zp2})} \left( 1 - e^{-j(k_z - k_{z0}) d_x} \right) dk_z.
\end{align*}$$  \hspace{1cm} (35)

In (35) the denominator $D_{\text{load}}$ is expressed with explicit roots in $k_z$. These roots define the branch cuts of the complex plane, in particular the top/bottom Riemann sheet, i.e., $\text{Im}(k_z) \leq 0$. These roots are also associated with poles in $k_z$ plane, as in will be discussed in the following.

**Load Dependent Pole**

It is simple to verify that $k_{zp1}$ is associated with values of $k_z$ far from the the branch points for any values of $Z_1$. It is then useful to locate in the $k_z$ plane the poles associated with $k_{zp2}$, as a function of the load impedance $Z_l$. For $Z_l = 0$, from (34), it results $k_{zp2} = 0$ and consequently $k_{zp} = k_0$. For small values of $Z_l$ a second order approximation of the square root function in (34) for small argument leads to

$$\begin{align*}
\sqrt{1 - j\frac{\Psi k_0 d_y Z_1}{\zeta d_x}} &\approx 1 - \frac{1}{8} \left( j\frac{\Psi k_0 d_y Z_1}{\zeta d_x} \right)^2 \approx -k_0 (n_r + jn_i) \\
\end{align*}$$  \hspace{1cm} (36)

which implies

$$k_{zp2} = \frac{2k_0 d_y Z_1}{\zeta d_x} - j\Psi \left( k_0 d_y Z_1 \right)^2 = -k_0 (n_r + jn_i)$$  \hspace{1cm} (37)

where $n_r$ and $n_i$ are very small real positive functions of the geometrical parameters at play. Consequently, the approximate expression of $k_{zp}$ is now as follows:

$$k_{zp} = \sqrt{k_0^2 - k_{zp2}^2} = \sqrt{k_0^2 \left( 1 - (n_r + jn_i)^2 \right)}$$

$$\approx k_0 \left( 1 - \frac{(n_r + jn_i)^2}{2} \right)$$

$$= k_0 \left( 1 - \frac{n_r^2 - n_i^2}{2} \right) - jk_0 n_r n_i$$  \hspace{1cm} (38)

which explicitly shows that, for small values of the load impedance $Z_l$, the dominant poles are located close to the branch point $k_0$ and show small imaginary parts that tend to become more negative as $Z_l$ increases. The pole $k_{zp}$ has a negative imaginary part. The corresponding root in $k_z$ was chosen with negative imaginary part, implying that the pole $k_{zp}$ represented in (38) is not associated with a leaky-wave, but with a damped wave. Since the damped wave is located in the top Riemann sheet of the complex $k_z$-plane, $k_{zp}$ is not captured when deforming the integration path along the

**Steepest Descent Path (SDP)**

In order to perform an uniform asymptotic evaluation of the integral as in Fig. 13.

Multiplying and dividing the integrand of (35) for the factor $(k_z + k_{zp2})$ we obtain, after a few algebraic steps, the following expression:

$$\begin{align*}
\hat{\eta}^{\text{nz}}_{\ell, \text{edge}1} &= \frac{-j k_0 d_y e^{jk_0 d_y Z_1}}{\Psi_0 \Pi} \\
&\times \int_{-\infty}^{\infty} \frac{\sin^2 \left( k_z^2 \right) e^{-jk_z (n_z+1) d_x}}{(k_z + k_{zp2})(k_z - k_{zp1})} \left( 1 - e^{-j(k_z - k_{z0}) d_x} \right) dk_z.
\end{align*}$$  \hspace{1cm} (39)

This integral can be deformed into the SDP around the saddle point $k_{zp}$ as in Fig. 13. Note that in the deformation none of the poles associated with the FWs are captured since the original integration path Fig. 12) surrounds all poles counter-clockwise. The poles defined by $k_{zp1}$ and $k_{zp2}$ in (34) are also not captured in the deformation. From (39) we can then define two contributions as follows:

$$\begin{align*}
\hat{\eta}^{\text{nz}}_{\ell, \text{edge}1} &\approx \hat{\eta}^{\text{nz}, a}_{\ell, \text{edge}1} + \hat{\eta}^{\text{nz}, b}_{\ell, \text{edge}1} \\
\end{align*}$$  \hspace{1cm} (40)

where

$$\hat{\eta}^{\text{nz}, a}_{\ell, \text{edge}1} = C \int_{\text{SDP}} \frac{\sqrt{k_0 k_z}}{k_{zp2} - k_z} e^{-jk_z (n_z+1) d_x} \left( 1 - e^{-j(k_z - k_{z0}) d_x} \right) dk_z,$$  \hspace{1cm} (41)

$$\hat{\eta}^{\text{nz}, b}_{\ell, \text{edge}1} = C k_{zp2} \int_{\text{SDP}} \frac{\sqrt{k_0 k_z}}{k_{zp2} - k_z} \left( 1 - e^{-j(k_z - k_{z0}) d_x} \right) dk_z.$$  \hspace{1cm} (42)

The term

$$C = \frac{-j k_0 d_y e^{jk_0 d_y Z_1} \sin^2 \left( k_0 d_y / 2 \right)}{\Psi_0 \Pi (k_{zp1})(k_0 + k_{zp2})}$$  \hspace{1cm} (43)

includes both the constants and the slower varying portions of the integrand from (39), approximated in $k_z \approx k_0$. The integrand in (42) presents no square root type of branches. Accordingly, in the upward and downward path that define the SDP the integrand is the same, so that the two half paths contributions can be.

The integral in (41), instead, requires an uniform asymptotic evaluation since the poles in $k_{zp}$ and in $k_{z0}$ can be close to the branch point $k_0$ for particular geometrical, loading or scanning configurations. Before performing the evaluation, it is convenient to express the integrand in a form where the mentioned poles in $k_{zp}$ and in $k_{z0}$ are shown explicitly [25]. This can be
achieved by adding and subtracting the quantity \( \frac{j}{dx} / (k_0 - k_{x0}) \) as follows:

\[
\frac{1}{1 - e^{-j(k_x - k_{x0}) dx}} = R(k_x) + \frac{j}{dx} \left( \frac{1}{k_x - k_{x0}} \right)
\]

(44)

where we defined the function

\[
R(k_x) = \frac{1}{1 - e^{-j(k_x - k_{x0}) dx}} + \frac{j}{dx} \left( \frac{1}{k_x - k_{x0}} \right).
\]

The function \( R(k_x) \) is a smooth regular function in the vicinity of the SDP and consequently can be approximated with its value at the saddle point \( R(k_{x0}) \approx R(k_0) \). The integral in (41) can be split into two contributions as follows:

\[
i_{x}^{z} \approx C \sqrt{2k_0} R(k_{x0}) \int_{-\infty}^{\infty} \frac{\sqrt{k_0 - k_x}}{k_x - k_{x0}} e^{-j k_0 x dx} d{k_x} - \frac{jC}{\sqrt{X}} \int_{-\infty}^{\infty} \frac{\sqrt{k_0 - k_x}}{k_x - k_{x0}} e^{-j k_0 x dx} d{k_x},
\]

(46)

While the first of the two integrals in (46) is already in a canonical form, the second one can be brought to the same form by recognizing that

\[
\frac{1}{(k_x - k_{x0})(k_x - k_{xp})} = \frac{1}{k_{x0} - k_{xp}} \left( \frac{1}{k_x - k_{x0}} - \frac{1}{k_x - k_{xp}} \right).
\]

(47)

After these manipulations, the three terms composing (41) can be all expressed analytically resorting to the following mathematical identity [16]:

\[
\int_{-\infty}^{\infty} \frac{\sqrt{k_0 - k_x}}{k_x - k_{x0}} e^{-j k_{x0} x dx} d{k_x} = -e^{-j\pi/4} \sqrt{\frac{k_0 - k_{xp}}{k_{x0} - k_{xp}}} F_0(x'(k_0 - k_{xp})),
\]

(48)

where the slope Fresnel function is introduced: \( F_0(x) = 2jx(1 - F'(x)) \). Here \( F'(x) \) is the Kouyoumjian Fresnel function [26], which is defined as

\[
F'(x) = 2j \sqrt{x} e^{jx^2} \int_{0}^{\infty} e^{-jt^2} dt, \quad (-3\pi/2 < \arg(x) \leq \pi/2).
\]

(49)

Using (48), after a few simple algebraic manipulations, the final expression of the current contribution born from the left edge of the array is given by

\[
i_{x}^{z} \approx C \sqrt{2k_0} e^{-j\pi/2} \sqrt{X} \left( C_1 F_0 \left( \frac{X(k_0 - k_{xp})}{(k_0 - k_{x0})} \right) + C_2 F_0 \left( \frac{X(k_0 + k_{x0})}{(k_0 - k_{x0})} \right) \right)
\]

(50)

with \( X = (n_x + 1) dx, \) In (50)

\[
C_1 = C' \left( \frac{R'(k_0) - j}{dx(k_{x0} - k_{x0})} \right),
\]

\[
C_2 = \frac{jC'}{dx(k_{x0} - k_{x0})}.
\]

(51)

and \( C' = C \sqrt{2k_0} e^{-j\pi/4} \). Proceeding in the same way the expression for the current born at the right edge of the array can be expressed as

\[
i_{x}^{z} \approx e^{-j k x X} X \sqrt{X} \left( C_1 F_0 \left( \frac{X(k_0 - k_{xp})}{(k_0 - k_{x0})} \right) + C_2 F_0 \left( \frac{X(k_0 + k_{x0})}{(k_0 - k_{x0})} \right) \right)
\]

(52)

where \( X = (n_x + 1) dx, \) \( C_1 \) and \( C_2 \) are defined with the substitution \( k_{x0} \rightarrow -k_{x0} \) and in the definition of \( C \) in (43) the exponential \( e^{j k_{x0} d x} \) is replaced by \( e^{-j k_{x0} d x} \).

REFERENCES


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