Input impedance of integrated elliptical lens antennas

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Abstract: An efficient method is presented for prediction of the input impedance of integrated elliptical lens antennas, excited by a focal radiator which consists of either a resonant slot or a slot-coupled patch. Simple equivalent networks are derived from this method which accounts for effects of multiple reflections inside the lens, which is useful for CAD purposes. Numerical results are presented to validate the equivalent networks. Furthermore, it is shown that these internal reflections significantly affect the input impedance of the primary source.

1 Introduction

Dielectric elliptical lens antennas have demonstrated significant potential for millimetre and submillimetre wave applications [1–5], owing to the possibility of integration with electronic components such as detecting diodes, oscillators and mixers. In addition, the elliptical shape of the lens gives a focusing property provided that the eccentricity is properly related to the dielectric constant. Under these conditions all rays leaving a focal source are refracted in the boresight direction, thus providing high directivity to the antenna.

In this paper an efficient method is presented for the design and analysis of an integrated dielectric elliptical lens antenna, excited by a standard resonant element located close to one focus. It is seen that the presence of the lens interface significantly affects the input impedance of the primary source especially for high dielectric constant (e.g. silicon technology). As pointed out in [6], the geometrical properties of the lens itself are such that all the doubly reflected rays launched by a focal source return in phase at the original focus. This means that a point caustic of doubly reflected rays exactly occurs where the feed source is located, thus presumably affecting its input impedance. Although the introduction of a proper matching layer [2] on the lens surface could reduce the internal reflections, its manufacture introduces practical problems of layer thickness control and the availability of specific materials applicable to millimetre waves. In view of this, and owing to the limited tuning possibilities intrinsic to the integrated technology, the availability of CAD tools for the accurate prediction of the antenna input impedance is particularly important.

In this paper, two kinds of feed are considered: a resonant slot fed by a microstrip, and a nonresonant slot-coupled patch [7–10]. Both these feed structures present several advantages over alternative arrangements, primarily due to the physical separation between the radiating region and the feed network. With respect to a coplanar waveguide (CPW) feed [4, 11], this separation provides radiation pattern regularity and polarisation purity, which can be driving features in submillimetre receiver front ends.

Due to the large dimensions of the lens in terms of wavelength, a complete full-wave analysis is very time-consuming and not well suited to design purposes. For this reason, the wave mechanisms involved in the lens volume are simplified using a mixed geometrical optics (GO)-physical optics (PO) method [6]. The primary source is studied using a spectral-domain full-wave method, but assuming a priori simplification of the equivalent current distributions that lead to simple and easily interpretable equivalent networks for the two feed sources. These networks allow for detection of the influence of the lens on the overall antenna impedance. However, the GO-PO method used here to include the lens effect can also be applied in a hybrid full-wave scheme when used to approximate the Green’s function of the lens region.

2 Lens reflection mechanisms

The geometries of the feeding structures are shown in Fig. 1. The elliptical dielectric lens is rotationally symmetric around the z-axis and it is fed by a primary source located as close as possible to its lower focal point. Two primary sources will be considered: (1) a resonant slot fed by a microstrip which is printed on an infinite dielectric substrate under the ground plane (2) a rectangular patch embedded in the dielectric lens, fed by a nonresonant slot. The latter is fed by a microstrip as in (1).

To understand and quantify the influence of the lens on the primary source and then on its input impedance, the reflection mechanisms inside the lens are considered first (Fig. 2). To this end, it is sufficient to assume that the phase centre of the primary field is exactly located at the focus. The rays emanating from the point source F1, after reflecting at the interface, cross the upper focus F2 of the ellipse. After that, the rays reflect again at the lens interface and then return in phase at the original focus, which is therefore a point caustic of doubly reflected rays. Note that rays launched inside the solid angle Ω (see Fig. 2) have a second reflection on the ground plane, and next undergo multiple incoherent reflections without focusing at F1. Consequently, their field contributions can be neglected. A more detailed presentation of this mechanism is given in [6] and its description is obtained by using a hybrid GO-PO...
technique. Both singly and doubly reflected ray contributions are described by GO, thus yielding two types of associate PO current distributions. The first type of currents \( J^{P1}, \dot{M}^{P1} \) are those associated through the equivalence principle to incident and singly reflected rays; these currents radiate the dominant field contribution in the far zone. The second type \( J^{P2}, \dot{M}^{P2} \) are associated through the equivalence principle to GO doubly reflected rays. The latter provide the dominant field contributions \( \{E^{P2}, H^{P2}\} \) around the lower focal point, so that is the most important in estimating the input impedance. According to the ray description depicted in Fig. 2, \( \{E^{P2}, H^{P2}\} \) are forced to zero in the region \( A_{Q} \). The field \( \{E^{P1}, \dot{M}^{P1}\} \) is obtained by means of a radiation integral of \( \{J^{P1}, \dot{M}^{P1}\} \) in a fully homogeneous dielectric.

3 Antenna equivalent network

The equivalent network of the overall antenna is depicted in Fig. 3. The feed microstrip is represented by a transmission line with characteristic impedance \( Z_{0} \) terminated by an open-ended stub, and coupled to the slot circuit by a series transformer with turn ratio \( n \). The secondary coil of the transformer is loaded by \( Y_{1} \) and \( Y_{2} \) in parallel representing the slot admittance in the upper and lower medium, respectively. When the primary source is the resonant slot (Fig. 3a), the admittance contribution \( Y_{1} \) towards the upper medium is constituted by the parallel of the admittance \( Y_{r} \) of the slot radiating in a homogeneous dielectric medium and of the admittance \( Y_{L} \) which represents the lens effect described in terms of double-reflection PO currents. When the primary source is the patch, the slot impedance is coupled to the patch impedance by a further transformer with turn ratio \( n_{p} \). The lens effect is accounted for by an impedance in series to the patch (Fig. 3b).

![Fig. 3 Equivalent networks](image)

According to these equivalent networks, the input impedance \( Z_{in} \) of the antenna is evaluated by

\[
Z_{in} = Z_{stub} + \frac{n^{2}}{Y_{1} + Y_{2}} \tag{1}
\]

where \( Z_{stub} \) represents the impedance of the open circuit stub. \( Z_{stub} \) has to be designed to compensate the reactance of the slot. In practical design including integrated elements (i.e. bolometers or diodes) the reactances of these elements should also be included in the design.

The various lumped elements of the equivalent network arise from full-wave type reaction integrals. By applying the equivalence principle, the slot is replaced by a metallic plug with unknown magnetic currents \( \dot{M} \) located on both sides. These currents have equal amplitude and opposite phase to ensure continuity of the electric field through the aperture. We consider slots that are either short in terms of a wavelength (slot coupled to the patch) or resonant (slot itself as primary source). Only one piecewise sinusoidal (PWS) \( x \)-directed mode is assumed in both cases:

\[
\dot{M}(x, y) = \dot{M}(x, y) \hat{x}
\]

where,

\[
\text{propagation constant of the quasi-TEM mode, whose definition is given by (eqn. 4)}
\]

\[
\text{in which } k_x \text{ and } k_y \text{ are the Fourier transforms of } M(x, y) \text{ and } G^H(k_x, k_y) \text{ is the pertinent spectral Green's function for the grounded dielectric slab (see Appendix, Section 8). Note that eqn. 4 applies to both types of feed (i.e. resonant slot or slot-coupled patch). The double spectral integral, as well as the others defined subsequently, has been calculated by using the standard polar change of variable } k_x = k_p \cos \alpha, k_y = k_p \sin \alpha. \text{ Furthermore, owing to its slow variation against frequency, an interpolation procedure has conveniently been applied in the frequency range of interest.}
\]

3.2 Transformer and microstrip line
The calculation of the transformer's turn ratio \( n \) that models the coupling between the slot and the microstrip can be simplified by assuming that the fringing field of the microstrip termination does not directly interact with the slot. Therefore, an infinite microstrip line can be used for its definition. The latter is substituted via equivalence principle by the homogenous space spectral Green's function for the magnetic field radiated in an infinite homogeneous dielectric half-space by \( M \) and by \( (\hat{M}^{P0}, \hat{M}^{PO}) \), respectively. These latter are the contributions of PO currents associated to the doubly reflected rays. In eqn. 8, admittance of \( M \) in the homogeneous dielectric half-space is calculated in the spectral domain by

\[
Y_s = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G^H(k_x, k_y) |M(k_x, k_y)|^2 dk_x dk_y
\]

3.3 Resonant slot
When the slot is the primary source (Fig. 3a), \( Y_1 \) is defined by

\[
Y_1 = Y_s + Y_{H^P} \int \hat{H}_s + \hat{M}^{P0} \text{ dS (8)}
\]

3.4 Slot-coupled patch
Let us now consider the case when the patch is placed over the slot at distance \( h_p \). The patch electric current distribution is described in terms of one resonant mode \( \tilde{J}_p(x, y) \)

\[
\tilde{J}_p(x, y) = J_p(x, y) \hat{\theta} \text{ for } \begin{cases} -\infty < y < \infty \\ |x| \leq W_x/2 \end{cases}
\]

\[
\text{in which } W_y \text{ is the width of the microstrip and } \hat{\theta} \text{ is the propagation constant of the quasi-TEM mode, whose approximate expression is given in the Appendix. With reference to Fig. 3, the transformer turn ratio } n = \frac{1}{W_y} \int_{W_y} |\hat{H}_s| \text{ dS - is defined by}
\]

\[
Z_p = Z_{p} + Z_{p} \text{ is then calculated by}
\]

\[
Z_p = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G^P(k_x, k_y) J_p(k_x, k_y) dk_x dk_y
\]
The integrals that define the parameters \( Y_{po} \) and \( Z_{po} \) in eqns. 8 and 11, respectively, involve the double-reflection PO current integration over \( A - A_{0} \) (see Fig. 2) and are actually performed numerically in the space domain. Owing to the fact that all the PO incremental contributions are almost in phase in the region occupied by the primary radiator, the integrand is nonoscillating and well behaved on almost all the domain \( A - A_{0} \). Exception is made for certain well identified lines of this domain that are characterised by critical angles of incidence for both singly- and doubly-reflected rays [6]. In particular, owing to the behaviour of the reflection coefficients at these critical angles, two closed curves \( C_{1} \), \( C_{2} \) occur in the integration domain where the integrand exhibits cusps (singularities in the first derivative). Consequently, the brute force use of adaptive integration routines slow down the calculation time for integration points around these curves. To speed up the procedure, it is convenient to split the domain \( A - A_{0} \) in parts having \( C_{1} \), \( C_{2} \) as boundaries.

The calculations of the PO integrals in a frequency range is accelerated by using interpolation procedures. In particular, we note that the total path of each doubly reflected ray is exactly \( 4T \), being \( T \) the major semi-axis of the ellipse, so that the quantities \( \exp(\pm jk_{0}d) \) exhibit slow variation in frequency, and may be easily interpolated. We emphasise again that a slow frequency variation is also presented by the spectral domain integrals which defines the other constituents of the network, so that other interpolation procedures against frequency have been used.

### 4 Numerical results and design considerations

Numerical results are presented hereinafter to validate the equivalent network models for both the primary feed.

#### 4.1 Resonant slot

For preliminary validation of the equivalent network in Fig. 3a, a comparison with experimental results found in literature [10] is presented in Fig. 4 for the case when the slot radiates in free space (the geometry is given in the caption). The agreement is clearly satisfactory. Fig. 5a presents the input impedance (real and imaginary parts) calculated according to eqn. 1 at the slot section when the slot itself radiates in presence of lenses with different larger semi-axes: \( T = T_{a} = 2126.8 \mu m \) (dashed-dotted line; A-case), \( T = T_{b} = 2091.3 \mu m \) (diameter = 4mm) (continuous line; B-case). For the sake of convenience, the case of slot radiating in a homogeneous dielectric is also included (dashed line; unperturbed case). The relative dielectric constant \( e_{r} = 4 \) (quartz) is chosen for both the lens and the microstrip slab. Note that the presence of the lens introduces oscillations on the input reactance and resistance as a function of frequency, which are directly related to the internal double reflections. The oscillations relevant to cases A and B are out of phase. Indeed the difference \( T_{a} - T_{b} \) is properly chosen equal to \( \pi/(4e_{r}\sqrt{k_{0}}) \), where \( k_{0} \) is the free space wavenumber at the resonant frequency (523GHz). This implies that the field scattered by the lens B is almost equal and opposite in phase with respect to that of the lens A. Note that the reactance curve B is flatter than the reactance curve A close to resonance, thus implying a larger bandwidth. This is more evident in Fig. 5b which presents the amplitude of the input reflection coefficients for the same configurations. Since the feeding microstrip is designed to have a characteristic impedance of 50 ohms, a quarter wavelength transformer is needed to match the impedance at the input microstrip port. In order to report the three different slot impedance levels at the same 50 ohm level, a specific width of the transformer is used for each of the three different cases. The -25dB bandwidths are 0.4%, 1.1% and 1.2% for case A, unperturbed case, and case B, respectively. This suggests that proper design of the lens may compensate the

\[ Y_{po} = \frac{1}{n_{p}} \int_{0}^{2\pi} \int_{-\infty}^{\infty} \exp(-k_{x}y) J_{p}(k_{x}x) J_{p}(k_{y}y) \times M(k_{x},k_{y}) dk_{x} dk_{y} \]

\[ B_{ppw} = \frac{1}{(2\pi)^{2}} \frac{1}{V_{0}} \int_{0}^{2\pi} \int_{-\infty}^{\infty} \exp\left(-|\rho|^{2} \right) \rho = \sqrt{x^{2} + y^{2}} \]

\[ \text{IM} \left( \int_{0}^{2\pi} \int_{-\infty}^{\infty} G_{ppw}(k_{x},k_{y}) [M_{s}(k_{x},k_{y})]^{2} dk_{x} dk_{y} \right) \]
influence of the internal reflections on the input impedance bandwidth.

4.2 Slot-coupled patch

In order to validate the equivalent network in Fig. 3b the input impedance obtained (continuous line) has been compared in Fig. 6 with that calculated by a full-wave analysis (dashed line) for the case when an infinite homogeneous dielectric with $\varepsilon_r = 11.7$ (silicon) is considered in place of the lens. The full-wave analysis has been obtained by using the software Ensemble 5. The microstrip line has a 50 ohm characteristic impedance. Next, the same patch is considered when radiating in the presence of a lens. Fig. 7 shows the amplitude of the reflection coefficient for the case of a silicon lens with $T = 2091.3\mu m$. The inset of the same Figure shows the Smith's chart representation. Note that the matching is sensitively affected by the presence of the lens due to the intrinsic narrow bandwidth of the patch.

impact of the lens internal reflections is revealed by the ciliated turns in the Smith's chart. Slightly changing the parameters of the feed line, allows one turn to lock the origin of the Smith chart thus improving the matching band (dashed line). In particular, increasing the stub length and decreasing the microstrip height translates the Smith's chart curve upward and rightward, respectively.

Finally, Fig. 8 presents the reflection coefficient relevant to a similar design realised in quartz ($\varepsilon_r = 4$), with lens (solid lines) and in the homogeneous space (dashed lines).
The lens presents the same dimension as before \((T_s = 2309.4\text{ mm})\), thus ensuring almost the same directivity. The geometry of the patch and its height have been properly scaled in order to have – in the infinite dielectric – approximately the same resonant frequency and bandwidth of the previous prototype. The magnitude of the ellipsoidal turns in the Smith chart (see the inset) is comparable to that of the silicon case, but their period is scaled by the ratio between the refraction indexes of silicon and quartz, as expected. Therefore, the reflection coefficient in presence of the lens appears to be less distorted compared with the case of silicon lens.

5 Conclusions

A simple model has been suggested for lens antennas fed by a resonant slot or by a slot-coupled patch. This model and the consequent equivalent networks are based on a simplified description of the electric and magnetic current distributions on the microstrip feeding line, on the coupling aperture and eventually on the patch. This description is shown to be adequately accurate by comparison with full-wave results. A mixed GO-PO approach is used for the description of the lens reflections toward the source. Numerical results have also shown the possibility of bandwidth enhancement by proper selection of the lens dimensions.

The method proposed here could be applied to other lenses (i.e. extended hemispherical lenses) when their profile remains close to that of the ellipse, as is usually desired for increasing directivity. Moreover, this approach is applicable to other feeding structures (i.e. coplanar waveguide excitations) with minor changes in the formulation. The equivalent network derived from this model provides a useful tool for CAD applications in designing integrated slot-patch coupled elliptical lens antennas, which can be easily implemented on PCs. To this regard, we emphasise that all the results presented in Figs. 5–8 required less than one minute on a Pentium 200MHz for the entire frequency scan.

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7 References