Abstraction Relations Between Internal and Behavioural Agent Models for Collective Decision Making

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Abstract. For agent-based modelling of collective phenomena individual agent behaviours can be modelled either from an agent-internal perspective, in the form of relations involving internal states of the agent, or from an agent-external, behavioural perspective, in the form of input-output relations for the agent, abstracting from internal states. Illustrated by a case study on collective decision making, this paper addresses how the two types of agent models can be related to each other. First an internal agent model for collective decision making is presented, based on neurological principles. It is shown how by an automated systematic transformation a behavioural model can be obtained, abstracting from the internal states. In addition, an existing behavioural agent model for collective decision making incorporating principles on social diffusion is described. It is shown under which conditions and how by an interpretation mapping the obtained abstracted behavioural agent model can be related to this existing behavioural agent model for collective decision making.

1 Introduction

Agent models used for collective social phenomena traditionally are kept simple, and often are specified by simple reactive rules that determine a direct response (output) based on the agent's current perception (input). However, in recent years it is more and more acknowledged that in some cases agent models specified in the simple format as input-output associations are too limited. Extending specifications of agent models beyond the format of simple reactive input-output associations essentially can be done in two different manners: (a) by allowing more complex temporal relations between the agent's input and output states over time, or (b) by taking into account internal processes described by temporal (causal) relations between internal states. Considering such extended formats for specification of agent models used to model collective social phenomena, raises a number of (interrelated) questions:

(1) When agent models of type (a) are used in social simulation, do they provide the same results as when agent models of type (b) are used?
(2) How can an agent model of type (a) be related to one of type (b)?
(3) Can agent models of type (a) be transformed into agent models of type (b), by some systematic procedure, and conversely?

Within the context of modelling collective social phenomena, the internal states in agent models of type (b) do not have a direct impact on the social process; agent models that show the same input-output states over time will lead to exactly the same results at the collective level, no matter what internal states occur. This suggests that for modelling social phenomena the internal states could be hidden or abstracted away by transforming the model in one way or the other into a model of type (a). An interesting challenge here is how this can be done in a precise and systematic manner.

The questions mentioned above are addressed in this paper based on notions such as ontology mappings, temporal properties expressed in hybrid (logical/numerical) formats, and logical and numerical relations between such temporal properties. Here the idea to use ontology mappings and extensions of them is adopted from [15] and refined to relate (more abstract) agent models of type (a) to those of type (b), thus addressing question (2) above. Moreover, addressing question (1), based on such a formally defined relation, it can be established that at the social level the results for the two agent models will be the same. This holds both for simulation traces and for the implied temporal properties (patterns) they have in common. It will be discussed how models of type (b) can be abstracted to models of type (a) by a systematic transformation, implemented in Java, thus also providing an answer to question (3).

The approach is illustrated by a case addressing the emergence of group decisions. It incorporates from the neurological literature the ideas of somatic marking as a basis for individual decision making (cf. [4, 6, 7]), and mirroring of emotions and intentions as a basis for mutual influences between group members (cf. [10, 11, 12]).

The paper is organized as follows. Section 2 presents an internal agent model **IAM** for decision making in a group, based on neurological principles, and modelled in a hybrid logical/numerical format; cf. [3]. In Section 3 an existing behavioural agent model **BAM** for group decision making is briefly described, and specified in hybrid format. In Section 4 first the internal agent model **IAM** introduced in Section 2 is abstracted to a behavioural model **ABAM**, and next in Section 5 it is shown how this behavioural agent model **ABAM** can be related the the behavioural agent model **BAM**, exploiting ontology mappings between the ontologies used for **ABAM** and **BAM**. Section 6 concludes the paper.

2 The Internal Agent Model IAM for Group Decision Making

This case study concerns a neurologically inspired computational modeling approach for the emergence of group decisions, incorporating somatic marking as a basis for individual decision making, see [4], [6], [7] and mirroring of emotions and intentions as a basis for mutual influences between group members, see [10], [11], [12]. The model shows how for many cases, the combination of these two neural mechanisms is sufficient to obtain on the one hand the emergence of common group decisions, and, on the other hand, to achieve that the group members feel OK with these decisions.

Cognitive states of a person, such as sensory or other representations often induce emotions felt within this person, as described by neurologist Damasio [5], [6]. Damasio's *Somatic Marker Hypothesis* (cf. [4], [6], [7]), is a theory on decision making which provides a central role to emotions felt. Within a given context, each represented decision option induces (via an emotional response) a feeling which is used to mark the option. Thus the Somatic Marker Hypothesis provides endorsements or valuations for the different options, and shapes an individual's decision process.

In a social context, the idea of somatic marking can be combined with recent neurological findings on the *mirroring function* of certain neurons (e.g., [10], [11], [12]). Such neurons are active not only when a person prepares for performing a specific action or body change, but also when the person observes somebody else intending or performing this action or body change. This includes expressing emotions in body states, such as facial expressions. The idea is that these neurons and the neural circuits in which they are embedded play an important role in social functioning and in (empathic) understanding of others; (e.g., [10], [11], [12]). They provide a biological basis for many social phenomena; cf. [10]. Indeed, when states of other persons are mirrored by some of the person's own states that at the same time are connected via neural circuits to states that are crucial for the own feelings and actions, then this provides an effective basic (biological) mechanism for how in a social context persons fundamentally affect each other's actions and feelings, and, for example, are able to achieve collective decision making.

Table 1 State ontology used

notation	*description*
SS	sensor state
SRS	sensory representation state
PS	preparation state
ES	effector state
BS	body state
c	observed context information
O	option
c(O)	tendency to choose for option O
b(O)	own bodily response for option O
g(b(O))	other group members' aggregated bodily response for option O
g(c(O))	other group members' aggregated tendency to choose for option O

Given the general principles described above, the mirroring function can be related to decision making in two different ways. In the first place *mirroring of emotions* indicates how emotions felt in different individuals about a certain considered decision option mutually affect each other, and, assuming a context of somatic marking, in this way affect how by individuals decision options are valuated. A second way in which a mirroring function relates to decision making is by applying it to the *mirroring of intentions* or *action tendencies* of individuals for the respective decision options. This may work when by verbal and/or nonverbal behaviour individuals show in how far they tend to choose for a certain option. In the internal agent model **IAM** introduced below both of these (emotion and intention) mirroring effects are incorporated.

An overview of the internal model **IAM** is given in Fig. 1. Here the notations for the state ontology describing the nodes in this network are used as shown in Table 1, and for the parameters as in Table 2. Moreover, the solid arrows denote internal causal relations whereas the dotted arrows indicate interaction with other group members. The arrow from PS(A, b(O)) to SRS(A, b(O)) indicates an as-if body loop that can be used to modulate (e.g., amplify or suppress) a bodily response (cf. [5]).

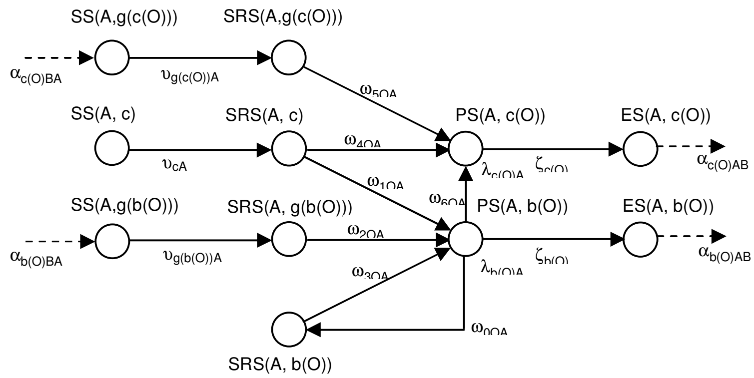

Fig. 1 Overview of the internal agent model **IAM**

Table 2 Parameters for the internal agent model **IAM**

description	*parameter*	*from*	*to*
strengths of connections within agent A	υ_{SA}	SS(A, S)	SRS(A, S)
	ω_{0OA}	PS(A, b(O))	SRS(A, b(O))
	ω_{1OA} ω_{2OA} ω_{3OA}	SRS(A, c) SRS(A, g(b(O))) SRS(A, b(O))	PS(A, b(O))
	ω_{4OA} ω_{5OA} ω_{6OA}	SRS(A, c) PS(A, g(b(O))) SRS(A, c(O))	PS(A, c(O))
	ζ_{SA}	PS(A, S)	ES(A, S)
strength for channel for Z from agent B to agent A	α_{ZBA}	sender B	receiver A
change rates for states within agent A	$\lambda_{b(O)A}$	change rate for PS(A, b(O))	
	$\lambda_{c(O)A}$	change rate for PS(A, c(O))	

This internal agent model **IAM** can be described in a detailed manner in hybrid logical/numerical format (cf. [3]) as follows.

IP1 From sensor states to sensory representations
SS(A, S, V) $\twoheadrightarrow$ SRS(A, S, υ_{SA}V)
where S has instances c, g(c(O)) and g(b(O)) for options O.

IP2 Preparing for an emotion expressed in a body state
SRS(A, c, V_1) & SRS(A, g(b(O)), V_2) & SRS(A, b(O), V_3) & PS(A, b(O), V)
$\twoheadrightarrow$ PS(A, b(O), V + $\lambda_{b(O)A}$ g($\omega_{1OA}V_1$, $\omega_{2OA}V_2$, $\omega_{3OA}V_3$, V) Δt)

IP3 Preparing for an option choice
SRS(A, c, V_1) & SRS(A, g(c(O)), V_2)) & PS(A, b(O), V_3) & PS(A, c(O), V)
$\twoheadrightarrow$ PS(A, c(O), V + $\lambda_{c(O)A}$ h($\omega_{4OA}V_1$, $\omega_{5OA}V_2$, $\omega_{6OA}V_3$, V) Δt)

IP4 From preparation to effector state
PS(A, S, V) $\twoheadrightarrow$ ES(A, S, ζ_{SA} V)
where S has instances b(O) and c(O) for options O.

IP5 From preparation to sensory representation of body state

$PS(S, V) \twoheadrightarrow SRS(S, \omega_{0OA}V)$

where S has instances $b(O)$ for options O.

Here the functions $g(X_1, X_2, X_3, X_4)$ and $h(X_1, X_2, X_3, X_4)$ are chosen, for example, of the form $th(\sigma, \tau, X_1 + X_2 + X_3) - X_4$, where $th(\sigma, \tau, X) = 1/(1 + e^{-\sigma(X - \tau)})$.

Next the following transfer properties describe the interaction between agents for emotional responses $b(O)$ and choice tendencies $c(O)$ for options O. Thereby the sensed input from multiple agents is aggregated by adding, for example, all influences $\alpha_{b(O)BA}V_B$ on A with V_B the levels of the effector state of agents $B \neq A$, to the sum $\Sigma_{B\neq A}$ $\alpha_{b(O)BA}V_B$ and normalising this by dividing it by the maximal value $\Sigma_{B\neq A}$ $\alpha_{b(O)BA}\zeta_{b(O)B}$ for it (when all preparation values would be *1*). This provides a kind of average of the impact of all other agents, weighted by the normalised channel strengths.

ITP Sensing aggregated group members' bodily responses and intentions

$\wedge_{B\neq A}$ $ES(B, S, V_B) \twoheadrightarrow SS(A, g(S), \Sigma_{B\neq A}\ \alpha_{SBA}V_B / \Sigma_{B\neq A}\ \alpha_{SBA}\zeta_{SB})$

where S has instances $b(O)$, $c(O)$ for options O.

Based on the internal agent model **IAM** a number of simulation studies have been performed, using MathLab. Some results for two simulation settings with 10 homogeneous agents with the parameters as defined in Table 3 are presented in Figure 3. The initial values for $SS(A, g(c(O)))$, $SS(A, c)$, $SS(A, g(b(O))$ are set to 0 in both settings. Note that a number of the connections strengths have been chosen rather low; for this reason also the activation levels shown in Fig. 3 are relatively low.

Table 3 The values of the parameters of model IAM used in two simulation settings

description	*parameter*	*setting 1*	*setting 2*
strengths of connections within agent A	$\upsilon_{g(c(O))A}$	0.5	0.9
	υ_{cA}	0.6	0.8
	$\upsilon_{g(b(O))A}$	0.9	0.7
	ω_{0OA}	0.8	0.6
	ω_{1OA}	0.3	0.5
	ω_{2OA}	0.3	0.2
	ω_{3OA}	0.4	0.3
	ω_{4OA}	0.6	0.4
	ω_{5OA}	0.2	0.3
	ω_{6OA}	0.2	0.3
	$\zeta_{c(O)A}$	0.6	0.8
	$\zeta_{b(O)A}$	0.9	0.4
strength for channel for Z from any agent to any other agent	α_{ZBA}	1	1
change rates for states within agent A	$\lambda_{b(O)A}$	0.7	0.9
	$\lambda_{c(O)A}$	0.4	0.9
parameters of the combination function based on threshold function $th(\sigma, \tau, X) = 1/(1 + e^{-\sigma(X - \tau)})$	σ	4	4
	τ	1.4	1.4

As one can see from Fig. 2, in both simulation settings the dynamics of the multi-agent system stabilizes after some time. Furthermore, in the stable state the agents from setting 1 demonstrate their emotional state more expressively than their intention to choose the option. In setting 2, the opposite situation can be observed in Fig. 2.

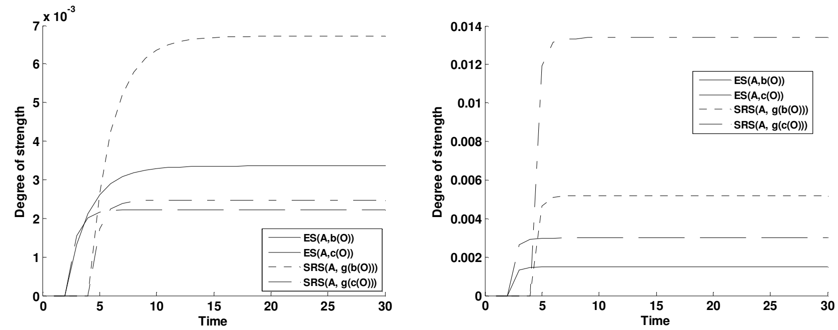

Fig. 2. The dynamics of ES(A, b(O)), ES(A, c(O)), SRS(A, g(b(O))) and SRS(A, g(c(O))) states of an agent A from a multi-agent system with 10 homogeneous agents over time for simulation setting 1 (left) and setting 2 (right) with the parameters from Table 3.

3 A Behavioural Agent Model for Group Decision Making: BAM

In [9], an agent-based model for group decision making is introduced. The model was designed in a manner abstracting from the agents' internal neurological, cognitive or affective processes. It was specified in numerical format by mathematical (difference) equations and implemented in MatLab. As a first step, the contagion strength for mental state S from person B to person A is defined by: $\gamma_{SBA} = \varepsilon_{SB} \cdot \alpha_{SBA} \cdot \delta_{SA}$ (1). Here ε_{SB} is the personal characteristic *expressiveness* of the sender (person B) for S, δ_{SA} the personal characteristic *openness* of the receiver (person A) for S, and α_{SBA} the interaction characteristic *channel strength* for S from sender B to receiver A. The expressiveness describes the strength of expression of given internal states by verbal and/or nonverbal behaviour (e.g., body states). The openness describes how strong stimuli from outside are propagated internally. The overall contagion strength γ_{SA} from the group towards agent A is $\gamma_{SA} = \sum_{B\neq A} \gamma_{SBA} = (\sum_{B\neq A} \varepsilon_{SB} \cdot \alpha_{SBA}) \cdot \delta_{SA}$ (2). This value is for the aggregated input $s_{g(S)A}(t)$ of the other agents upon state S of agent A:

$$s_{g(S)A}(t) = \sum_{B\neq A} \gamma_{SBA} \cdot q_{SB}(t) \;/\; \gamma_{SA} = \sum_{B\neq A} \varepsilon_{SB} \cdot \alpha_{SBA} \cdot q_{SB}(t) \;/\; (\sum_{B\neq A} \varepsilon_{SB} \cdot \alpha_{SBA}) \quad (3)$$

How much this external influence actually changes state S of the agent A may be determined by additional personal characteristics of the agent, for example, the tendency η_{SA} to absorb or to amplify the level of a state and the positive or negative bias β_{SA} for the state S. The dynamics of the value $q_{SA}(t)$ of S in A over time given as:

$$q_{SA}(t + \Delta t) = q_{SA}(t) + \gamma_{SA}\, \boldsymbol{c}(s_{g(S)A}(t), q_{SA}(t))\, \Delta t \quad (4)$$
$$\text{with } \boldsymbol{c}(X, Y) = \eta_{SA} \cdot [\beta_{SA} \cdot (1 - (1-X) \cdot (1-Y)) + (1 - \beta_{SA}) \cdot XY] + (1 - \eta_{SA}) \cdot X - Y$$

Note that for $\boldsymbol{c}(X, Y)$ any function can be taken that combines the values of X and Y and compares the result with Y. For the example function $\boldsymbol{c}(X, Y)$, the new value of the state is the old value, plus the change of the value based on the contagion. This change is defined as the multiplication of the contagion strength times a factor for the amplification of information plus a factor for the absorption of information. The absorption part (after $1 - \eta_{SA}$) considers the difference between the incoming contagion and the current level for S. The amplification part (after η_{SA}) depends on the bias of the agent towards more positive (part of equation multiplied by β_{SA}) or negative (part of equation multiplied by $1 - \beta_{SA}$) level for S. Table 4 summarizes the most important parameters and state variables within the model (note that the last two parameters will be explained in Section 3.2 below).

Table 4. Parameters and state variables

$q_{SA}(t)$	level for state S for agent A at time t
$e_{SA}(t)$	expressed level for state S for agent A at time t
$S_{g(S)A}(t)$	aggregated input for state S for agent A at time t
ε_{SA}	extent to which agent A expresses state S
δ_{SA}	extent to which agent A is open to state S
η_{SA}	tendency of agent A to absorb or amplify state S
β_{SA}	positive or negative bias of agent A on state S
α_{SBA}	channel strenght for state S from sender B to receiver A
γ_{SBA}	contagion strength for S from sender B to receiver A
$\omega_{c(O)A}$	weigth for group intention impact on A 's intention for O
$\omega_{b(O)A}$	weigth for own emotion impact on A 's intention for O

This generalisation of the existing agent-based contagion models is not exactly a behavioural model, as the states indicated by the values $q_{SA}(t)$ are internal states and not output states. After multiplication by the expression factor ε_{SA} the behavioural output states $e_{SA}(t)$ are obtained that are observed by the other agents. The model can be reformulated in terms of these behavioural output states $e_{SA}(t)$, assuming that time taken by interaction is neglectable compared to the internal processes:

$$s_{g(S)A}(t) = \sum_{B \neq A} \alpha_{SBA} \cdot e_{SB}(t) \;/\; (\sum_{B \neq A} \varepsilon_{SB} \cdot \alpha_{SBA}\;) \qquad (5)$$

$$e_{SA}(t + \Delta t) = e_{SA}(t) + \;\varepsilon_{SA}\; \gamma_{SA}\; \mathbf{c}(s_{g(S)A}(t),\, e_{SA}(t)/\varepsilon_{SA})\; \Delta t \qquad (6)$$

To obtain an agent-based social level model for group decision making, the abstract agent-based model for contagion described above for any decision option O has been applied to both the emotion states S for O and intention or choice tendency states S' for O. In addition, an interplay between the two types of states has been modelled. To incorporate such an interaction (loosely inspired by Damasio's principle of somatic marking; cf. [4], [7], the basic model was extended as follows: to update $q_{SA}(t)$ for an intention state S relating to an option O, both the intention states of others for O and the $q_{S'A}(t)$ values for the emotion state S' for O are taken into account. Note that in this model a fixed set of options was assumed, that all are considered. The emotion and choice tendency states S and S' for option O are denoted by $b(O)$ and $c(O)$, respectively. Then the expressed level of emotion for option O of person A is $e_{b(O)A}(t)$, and of choice tendency or intention for O is $e_{c(O)A}(t)$. The combination of the own (positive) emotion level and the rest of the group's aggregated choice tendency for option O is made by a weighted average of the two:

$$s_{g(c(O))A}{}^*(t) \;=\; (\omega_{c(O)A}/\omega_{OA})\, s_{g(c(O))A}(t) \;+ (\omega_{b(O)A}/\omega_{OA})\, e_{b(O)A}(t)\,/\varepsilon_{SA}$$

$$\gamma_{c(O)A}{}^* = \omega_{OA}\; \gamma_{c(O)A}$$

where $\omega_{c(O)A}$ and $\omega_{b(O)A}$ are the weights for the contributions of the group choice tendency impact and the own emotion impact on the choice tendency of A for O, respectively, and $\omega_{OA} = \omega_{c(O)A} + \omega_{b(O)A}$. Then the behavioural agent-based model for interacting emotion and intention (choice tendency) contagion expressed in numerical format becomes:

$$s_{g(b(O))A}(t) = \sum_{B\neq A} \alpha_{b(O)BA} \cdot e_{b(O)B}(t) \;/\; (\sum_{B\neq A} \varepsilon_{b(O)B} \cdot \alpha_{b(O)BA}) \quad (7)$$

$$e_{b(O)A}(t + \Delta t) = e_{b(O)A}(t) + \varepsilon_{b(O)A}\, \gamma_{b(O)A}\, \mathbf{c}(s_{g(b((O))A}(t),\, e_{b(O)A}(t)/\varepsilon_{b(O)A})\, \Delta t \quad (8)$$

with as an example

$$\mathbf{c}(X, Y) = \eta_{b(O)A}\cdot[\beta_{b(O)A}\cdot(1 - (1\text{-}X)\cdot(1\text{-}Y)) + (1 - \beta_{b(O)A})\cdot XY] \;+\; (1 - \eta_{b(O)A})\cdot X - Y$$

$$s_{g(c(O))A}(t) = \sum_{B\neq A} \alpha_{c(O)BA} \cdot e_{c(O)B}(t) \;/\; (\sum_{B\neq A} \varepsilon_{c(O)B} \cdot \alpha_{c(O)BA}) \quad (9)$$

$$e_{c(O)A}(t + \Delta t) = e_{c(O)A}(t) +$$
$$\varepsilon_{c(O)A}\, \omega_{OA}\, \gamma_{c(O)A}\, \mathbf{d}((\omega_{c(O)A}/\omega_{OA})\, s_{g(c(O))A}(t) + (\omega_{b(O)A}/\omega_{OA})\, e_{b(O)A}(t)/\varepsilon_{b(O)A},\, e_{c(O)A}(t)/\varepsilon_{c(O)A})\Delta t \quad (10)$$

with as an example

$$\mathbf{d}(X, Y) = \eta_{c(O)A}\cdot[\beta_{c(O)A}\cdot(1 - (1\text{-}X)\cdot(1\text{-}Y)) + (1 - \beta_{c(O)A})\cdot XY] + (1 - \eta_{c(O)A})\cdot X - Y$$

To be able to relate this model expressed by difference equations to the internal agent model **IAM**, the model is expressed in a hybrid logical/numerical format in a straightforward manner in the following manner, using atoms has_value(x, V) with x a variable name and V a value, thus obtaining the behavioural agent model **BAM**. Here s(g(b((O)), A), s(g(c((O)), A), e(b(O), A) and e(c(O), A) for options O are names of the specific variables involved.

BP1 Generating a body state
has_value(s(g(b(O)), A), V_1) & has_value(e(b(O), A), V)
→ has_value(e(b(O), A), V + $\varepsilon_{b(O)A}\, \gamma_{b(O)A}\, \mathbf{c}(V_1, V/\varepsilon_{b(O)A})\, \Delta t$)

BP2 Generating an option choice intention
has_value(s(g(c(O)), A), V_1) & has_value(e(b(O), A), V_2) & has_value(e(c(O), A), V)
→ has_value(e(c(O), A), V + $\varepsilon_{c(O)A}\, \omega_{OA}\, \gamma_{c(O)A}\, \mathbf{d}((\omega_{c(O)A}/\omega_{OA})\, V_1 + (\omega_{b(O)A}/\omega_{OA})\, V_2/\varepsilon_{b(O)A},\, V/\varepsilon_{c(O)A})\, \Delta t$)

BTP Sensing aggregated group members' bodily responses and intentions
$\wedge_{B\neq A}$ has_value(e(S, B), V_B) → has_value(s(g(S), A), $\Sigma_{B\neq A}\, \alpha_{SBA} V_B / \Sigma_{B\neq A}\, \alpha_{SBA}\varepsilon_{SB}$)

In Section 5 the behavioural agent model **BAM** is related to the internal agent model **IAM** described in Section 2. This relation goes via the abstracted (from **IAM**) behavioural agent model **ABAM** introduced in Section 4.

4 Abstracting Internal Model IAM to Behavioural Model ABAM

First, in this section, from the model **IAM** by a systematic transformation, an abstracted behavioural agent model **ABAM** is obtained. In Section 5 the two behavioural agent models **ABAM** and **BAM** will be related. In [14] an automated abstraction transformation is described from a non-cyclic, stratified internal agent model to a behavioural agent model. As in the current situation the internal agent model is not assumed to be noncyclic, this existing transformation cannot be applied. In particular, for the internal agent model considered as a case in Section 2 the properties IP2 and IP3 are cyclic by themselves (recursive). Moreover, the as-if body loop described by properties IP2 and IP5 is another cycle. Therefore, the transformation introduced here exploits a different approach. The two main steps in this transformation are: elimination of sensory representation atoms, and elimination of preparation atoms (see also Fig. 4).

1. Elimination of sensory representation atoms

It is assumed that sensory representation atoms may be affected by sensor atoms, or by preparation atoms. These two cases are addressed as follows

a) Replacing sensory representation atoms by sensor atoms

- Based on a property $\mathrm{SS}(A, S, V) \twoheadrightarrow \mathrm{SRS}(A, S, \upsilon V)$ (such as IP1), replace atoms $\mathrm{SRS}(A, S, V)$ in an antecedent (for example, in IP2 and IP3) by $\mathrm{SS}(A, S, V/\upsilon)$.

b) Replacing sensory representation atoms by preparation atoms

- Based on a property $\mathrm{PS}(A, S, V) \twoheadrightarrow \mathrm{SRS}(A, S, \omega V)$ (such as IP5), replace atoms $\mathrm{SRS}(A, S, V)$ in an antecedent (for example, in IP2) by $\mathrm{PS}(A, b(O), V/\omega)$.

Note that this transformation step is similar to the principle exploited in [14]. It may introduce new occurrences of preparation atoms; therefore it should preceed the step to eliminate preparation atoms. In the case study this transformation step provides the following transformed properties (replacing IP1, IP2, IP3, and IP5; see also Fig. 4):

IP2* Preparing for a body state

$$\mathrm{SS}(A, c, V_1/\upsilon_{cA})\ \&\ \mathrm{SS}(A, g(b(O)), V_2/\upsilon_{g(b(O))A})\ \&\ \mathrm{PS}(A, b(O), V_3/\omega_{0OA})\ \&\ \mathrm{PS}(A, b(O), V)$$
$$\twoheadrightarrow \mathrm{PS}(A, b(O), V + \lambda_{b(O)A}\, \mathbf{g}(\omega_{1OA}V_1, \omega_{2OA}V_2, \omega_{3OA}V_3, V)\, \Delta t)$$

IP3* Preparing for an option choice

$$\mathrm{SS}(A, c, V_1/\upsilon_{cA})\ \&\ \mathrm{SS}(A, g(c(O)), V_2/\upsilon_{g(c(O))A}))\ \&\ \mathrm{PS}(A, b(O), V_3)\ \&\ \mathrm{PS}(A, c(O), V)$$
$$\twoheadrightarrow \mathrm{PS}(A, c(O), V + \lambda_{c(O)A}\, \mathbf{h}(\omega_{4OA}V_1, \omega_{5OA}V_2, \omega_{6OA}V_3, V)\, \Delta t)$$

2. Elimination of preparation atoms

Preparation atoms in principle occur both in antecedents and consequents. This makes it impossible to apply the principle exploited in [14]. However, it is exploited that preparation states often have a direct relationship to effector states:

- Based on a property $\mathrm{PS}(A, S, V) \twoheadrightarrow \mathrm{ES}(A, S, \zeta V)$ (such as in IP4), replace each atom $\mathrm{PS}(A, S, V)$ in an antecedent or consequent by $\mathrm{ES}(A, S, \zeta V)$.

In the case study this transformation step provides the following transformed properties (replacing IP2*, IP3*, and IP4; see also Fig. 4):

IP2* Preparing for a body state

$$\mathrm{SS}(A, c, V_1/\upsilon_{cA})\ \&\ \mathrm{SS}(A, g(b(O)), V_2/\upsilon_{g(b(O))A})\ \&\ \mathrm{ES}(A, b(O), \zeta_{b(O)A} V_3/\omega_{0OA})\ \&\ \mathrm{ES}(A, b(O), \zeta_{b(O)A} V)$$
$$\twoheadrightarrow \mathrm{ES}(A, b(O), \zeta_{b(O)A} V + \zeta_{b(O)A}\, \lambda_{b(O)A}\, \mathbf{g}(\omega_{1OA}V_1, \omega_{2OA}V_2, \omega_{3OA}V_3, V)\, \Delta t)$$

IP3* Preparing for an option choice

$$\mathrm{SS}(A, c, V_1/\upsilon_{cA})\ \&\ \mathrm{SS}(A, g(c(O)), V_2/\upsilon_{g(c(O))A}))\ \&\ \mathrm{ES}(A, b(O), \zeta_{b(O)A} V_3)\ \&\ \mathrm{ES}(A, c(O), \zeta_{c(O)A} V)$$
$$\twoheadrightarrow \mathrm{ES}(A, c(O), \zeta_{c(O)A} V + \zeta_{c(O)A}\, \lambda_{c(O)A}\, \mathbf{h}(\omega_{4OA}V_1, \omega_{5OA}V_2, \omega_{6OA}V_3, V)\, \Delta t)$$

By renaming V_1/υ_{cA} to V_1, $V_2/\upsilon_{g(b(O)A}$ to V_2 , $\zeta_{b(O)A} V_3/\omega_{0OA}$ to V_3, $\zeta_{b(O)A} V$ to V (in IP2*), resp. $V_2/\upsilon_{g(c(O))A}$ to V_2, $\zeta_{b(O)A} V_3$ to V_3, and $\zeta_{c(O)A} V$ to V (in IP3*), the following is obtained:

IP2 Preparing for a body state**

$$\mathrm{SS}(A, c, V_1)\ \&\ \mathrm{SS}(A, g(b(O)), V_2)\ \&\ \mathrm{ES}(A, b(O), V_3)\ \&\ \mathrm{ES}(A, b(O), V)$$
$$\twoheadrightarrow \mathrm{ES}(A, b(O), V + \zeta_{b(O)A}\, \lambda_{b(O)A}\, \mathbf{g}(\omega_{1OA}\upsilon_{cA} V_1, \omega_{2OA}\upsilon_{g(b(O))A} V_2, \omega_{3OA}\omega_{0OA} V_3/ \zeta_{b(O)A}, V/\zeta_{b(O)A})\, \Delta t)$$

IP3 Preparing for an option choice**

$$\mathrm{SS}(A, c, V_1)\ \&\ \mathrm{SS}(A, g(c(O)), V_2)\ \&\ \mathrm{ES}(A, b(O), V_3)\ \&\ \mathrm{ES}(A, c(O), V)$$
$$\twoheadrightarrow \mathrm{ES}(A, c(O), V + \zeta_{c(O)A}\, \lambda_{c(O)A}\, \mathbf{h}(\omega_{4OA}\upsilon_{cA} V_1, \omega_{5OA}\upsilon_{g(c(O))A} V_2, \omega_{6OA}V_3/\zeta_{b(O)A}, V/\zeta_{c(O)A})\, \Delta t)$$

Based on these properties derived from the internal model **IAM** the specification of the abstracted behavioural model **ABAM** can be defined; see also Fig. 3, lower part.

Hybrid Specification of the Abstracted Behavioural Agent Model ABAM

Note that in IP2** V_2 and V have the same value, so a slight further simplification can be made by replacing V_3 by V. After renaming of the variables according to

ABP1			**ABP2**		
V_1	$\rightarrow$	W_0	V_1	$\rightarrow$	W_0
V_2	$\rightarrow$	W_1	V_2	$\rightarrow$	W_1
V_3	$\rightarrow$	W	V_3	$\rightarrow$	W_2
V	$\rightarrow$	W	V	$\rightarrow$	W

the following abstracted behavioural model **ABAM** for agent A is obtained:

ABP1 Generating a body state

$\mathrm{SS}(A, c, W_0)\ \&\ \mathrm{SS}(A, g(b(O)), W_1)\ \&\ \mathrm{ES}(A, b(O), W)$

$\twoheadrightarrow\ \mathrm{ES}(A, b(O), W + \zeta_{b(O)A}\, \lambda_{b(O)A}\, \mathbf{g}(\omega_{1OA}\upsilon_{cA}\, W_0,\ \omega_{2OA}\upsilon_{g(b(O))A}\, W_1,\ \omega_{3OA}\omega_{0OA}\, W / \zeta_{b(O)A},\ W/\zeta_{b(O)A})\ \Delta t)$

ABP2 Generating an option choice intention

$\mathrm{SS}(A, c, W_0)\ \&\ \mathrm{SS}(A, g(c(O)), W_1)\ \&\ \mathrm{ES}(A, b(O), W_2)\ \&\ \mathrm{ES}(A, c(O), W)$

$\twoheadrightarrow\ \mathrm{ES}(A, c(O), W + \zeta_{c(O)A}\, \lambda_{c(O)A}\, \mathbf{h}(\omega_{4OA}\upsilon_{cA}\, W_0,\ \omega_{5OA}\upsilon_{g(c(O))A}\, W_1,\ \omega_{6OA}\, W_2/\zeta_{b(O)A},\ W/\zeta_{c(O)A})\ \Delta t)$

ITP Sensing aggregated group members' bodily responses and intentions

$\wedge_{B\neq A}\ \mathrm{ES}(B, S, V_B)\ \twoheadrightarrow\ \mathrm{SS}(A, g(S), \Sigma_{B\neq A}\ \alpha_{SBA}V_B / \Sigma_{B\neq A}\ \alpha_{SBA}\zeta_{SB})$

where S has instances b(O), c(O) for options O.

Note that as all steps made are logical derivations, it holds **IAM** $\vdash$ **ABAM**. In particular the following logical implications are valid (shown hierarchically in Fig. 3):

IP1 & IP5 & IP2 $\Rightarrow$ IP2*	IP4 & IP2* $\Rightarrow$ ABP1
IP1 & IP3 $\Rightarrow$ IP3*	IP4 & IP3* $\Rightarrow$ ABP2

The transformation as described is based on the following of assumptions:

- Sensory representation states are affected (only) by sensor states and/or preparation states
- Preparation atoms have a direct relationship with effector atoms; there are no other ways to generate effector states than via preparation states
- The time delays for the interaction from the effector state of one agent to the sensor state of the same or another agent are small so that they can be neglected compared to the internal time delays
- The internal time delays from sensor state to sensory representation state and from preparation state to effector state within an agent are small so that they can be neglected compared to the internal time delays from sensory representation to preparation states

The transformation can be applied to any internal agent model satisfying these assumptions. The proposed abstraction procedure has been implemented in Java. The automated procedure requires as input a text file with a specification of an internal agent model and generates a text file with the corresponding abstracted behavioural model as output. The computational complexity of the procedure is $O(|M|*|N| + |L|*|S|)$, where M is the set of srs atoms in the **IAM** specification, N is the set of the srs state generation properties in the specification, L is the set of the preparation atoms and srs atoms in the loops in the specification, and S is the set of the effector state generation properties in the specification.

Using the automated procedure the hybrid specification of **ABAM** has been obtained. With this specification simulation has been performed with the values of parameters as described in Table 3. The obtained curves for ES(A, c(O)) and ES(A, b(O)) are the same as the curves depicted in Fig. 3 for the model **IAM**. This outcome confirms that both the models **ABAM** and **IAM** generate the same behavioural traces and that the abstraction transformation is correct.

5 Relating the Behavioural Agent Models BAM and ABAM

In this section the given behavioural agent model **BAM** described in Section 3 is related to the behavioural agent model **ABAM** obtained from the internal agent model **IAM** by the abstraction process described in Section 4. First the notion of interpretation mapping induced by an ontology mapping is briefly introduced (e.g., [8], pp. 201-263; [15]). By a basic ontology mapping π atomic state properties (e.g., a_2 and b_2) in one ontology can be related to state properties (e.g., a_1 and b_1) in another (e.g., $\pi(a_2) = a_1$ and $\pi(b_2) = b_1$). Using compositionality a basic ontology mapping used above can be extended to an interpretation mapping for temporal expressions. As an example, when $\pi(a_2) = a_1$, $\pi(b_2) = b_1$, then this induces a mapping π^* from dynamic property $a_2 \twoheadrightarrow b_2$ to $a_1 \twoheadrightarrow b_1$ as follows: $\pi^*(a_2 \twoheadrightarrow b_2) = \pi^*(a_2) \twoheadrightarrow \pi^*(b_2) = \pi(a_2) \twoheadrightarrow \pi(b_2) = a_1 \twoheadrightarrow b_1$. In a similar manner by compositionality a mapping for more complex temporal predicate logical relationships A and B can be defined, using

$$\pi^*(A \,\&\, B) = \pi^*(A) \,\&\, \pi^*(B) \qquad \pi^*(A \vee B) = \pi^*(A) \vee \pi^*(B)$$
$$\pi^*(A \Rightarrow B) = \pi^*(A) \Rightarrow \pi^*(B) \qquad \pi^*(\neg A) = \neg\, \pi^*(A)$$
$$\pi^*(\forall T\ A) = \forall T\ \pi^*(A) \qquad \pi^*(\exists T\ A) = \exists T\ \pi^*(A)$$

To obtain a mapping the given behavioural model **BAM** onto the abstracted **ABAM**, first, consider the basic ontology mapping π defined by :

$\pi(\text{has\_value}(e(S, A), V)) = ES(A, S, V)$ where instances for S are b(O), c(O) for options O

$\pi(\text{has\_value}(s(S, A), V)) = SS(A, S, V)$ where instances for S are g(b((O)), g(c((O)) for options O

Next by compositionality the interpretation mapping π^* is defined for the specification of the behavioural model **BAM** as follows:

Mapping BP1 Generating a body state

$$\begin{aligned}
\pi^*(\mathrm{BP1}) &= \pi^*(\text{has_value}(s(g(b(O)), A), V_1) \,\&\, \text{has_value}(e(b(O), A), V) \\
&\qquad \twoheadrightarrow \text{has_value}(e(b(O), A), V + \varepsilon_{b(O)A}\, \gamma_{b(O)A}\, \mathbf{c}(V_1, V/\varepsilon_{b(O)A})\ \Delta t)) \\
&= \pi(\text{has_value}(s(g(b(O)), A), V_1)) \,\&\, \pi(\text{has_value}(e(b(O), A), V)) \\
&\qquad \twoheadrightarrow \pi(\text{has_value}(e(b(O), A), V + \varepsilon_{b(O)A}\, \gamma_{b(O)A}\, \mathbf{c}(V_1, V/\varepsilon_{b(O)A})\ \Delta t)) \\
&= SS(A, g(b(O)), V_1) \,\&\, ES(A, b(O), V) \twoheadrightarrow ES(A, b(O), V + \varepsilon_{b(O)A}\, \gamma_{b(O)A}\, \mathbf{c}(V_1, V/\varepsilon_{b(O)A})\ \Delta t)
\end{aligned}$$

Mapping BP2 Generating an option choice intention

$$\begin{aligned}
\pi^*(\mathrm{BP2}) &= \pi^*(\text{has_value}(s(g(c(O)), A), V_1) \,\&\, \text{has_value}(e(b(O), A), V_2) \,\&\, \text{has_value}(e(c(O), A), V) \\
&\qquad \twoheadrightarrow \text{has_value}(e(c(O), A), V + \varepsilon_{c(O)A}\, \omega_{OA}\, \gamma_{c(O)A}\, \mathbf{d}((\omega_{c(O)A}/\omega_{OA})\, V_1 + (\omega_{b(O)A}/\omega_{OA})\, V_2/\varepsilon_{b(O)A}, V/\varepsilon_{c(O)A})\ \Delta t)) \\
&= \pi(\text{has_value}(s(g(c(O)), A), V_1)) \,\&\, \pi(\text{has_value}(e(b(O), A), V_2)) \,\&\, \pi(\text{has_value}(e(c(O), A), V)) \\
&\qquad \twoheadrightarrow \pi(\text{has_value}(e(c(O), A), V + \varepsilon_{c(O)A}\, \omega_{OA}\, \gamma_{c(O)A}\, \mathbf{d}((\omega_{c(O)A}/\omega_{OA})\, V_1 + (\omega_{b(O)A}/\omega_{OA})\, V_2/\varepsilon_{b(O)A}, V/\varepsilon_{c(O)A})\ \Delta t)) \\
&= SS(A, g(c(O)), V_1) \,\&\, ES(A, b(O), V_2) \,\&\, ES(A, c(O), V) \\
&\qquad \twoheadrightarrow ES(A, c(O), V + \varepsilon_{c(O)A}\, \omega_{OA}\, \gamma_{c(O)A}\, \mathbf{d}((\omega_{c(O)A}/\omega_{OA})\, V_1 + (\omega_{b(O)A}/\omega_{OA})\, V_2/\varepsilon_{b(O)A}, V/\varepsilon_{c(O)A})\ \Delta t)
\end{aligned}$$

Mapping BTP Sensing aggregated group members' bodily responses and intentions

$$\begin{aligned}
\pi^*(\mathrm{BTP}) &= \pi^*(\wedge_{B\neq A}\ \text{has_value}(e(S, B), V_B) \twoheadrightarrow \text{has_value}(s(g(S), A), \Sigma_{B\neq A}\ \alpha_{SBA} V_B / \Sigma_{B\neq A}\ \alpha_{SBA}\varepsilon_{SB})) \\
&= \wedge_{B\neq A}\ \pi(\text{has_value}(e(S, B), V_B)) \twoheadrightarrow \pi(\text{has_value}(s(g(S), A), \Sigma_{B\neq A}\ \alpha_{SBA} V_B / \Sigma_{B\neq A}\ \alpha_{SBA}\varepsilon_{SB})) \\
&= \wedge_{B\neq A}\ ES(B, S, V_B)) \twoheadrightarrow SS(A, g(S), \Sigma_{B\neq A}\ \alpha_{SBA} V_B / \Sigma_{B\neq A}\ \alpha_{SBA}\varepsilon_{SB})
\end{aligned}$$

So to explore under which conditions the mapped behavioural model **BAM** is the abstracted model **ABAM**, it can be found out when the following identities (after unifying the variables V_i, V and W_i, W for values) hold.

$$\pi^*(\mathrm{BP1}) = \mathrm{ABP1} \qquad \pi^*(\mathrm{BP2}) = \mathrm{ABP2} \qquad \pi^*(\mathrm{BTP}) = \mathrm{ITP}$$

However, the modelling scope of **ABAM** is wider than the one of **BAM**. In particular, in **ABAM** an as-if body loop is incorporated that has been left out of consideration for **BAM**. Moreover, in the behavioural model **BAM** the options O are taken from a fixed set, given at forhand and automatically considered, whereas in **ABAM** they are generated on the basis of the context c. Therefore, the modelling scope of **ABAM** is first tuned to the one of **BAM**, to get a comparable modelling scope for both models **IAM** and **ABAM**. The latter condition is achieved by taking the activation level W_0 of the sensor state for the context c and the strengths of the connections between the sensor state for context c and preparations relating to option O can be set at *1* (so $\upsilon_{cA} = \omega_{1OA} = \omega_{4OA} = 1$); thus the first argument of g and h becomes 1. The former condition is achieved by leaving out of **ABAM** the dependency on the sensed body state, i.e., by making the third argument of g zero (so $\omega_{0OA} = 0$).

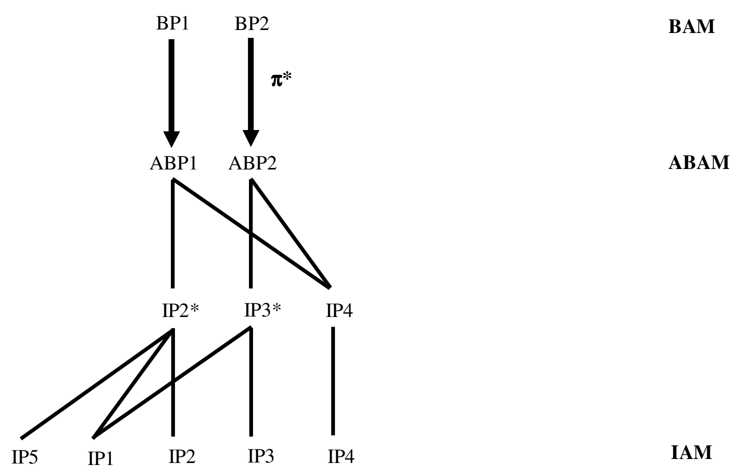

Fig. 3 Logical relations from network specification via internal agent model and abstracted behavioural model to behavioural agent model: **IAM** $\vdash$ **ABAM = $\boldsymbol{\pi}$(BAM)**

Given these extra assumptions and the mapped specifications found above, when the antecedents where unified according to $V_i \leftrightarrow W_i$, $V \leftrightarrow W$ the identities are equivalent to the following identities in V, V_i

$$\varepsilon_{b(O)A}\,\gamma_{b(O)A}\,\mathbf{c}(V_1,\ V/\varepsilon_{b(O)A}) \;=\; \zeta_{b(O)A}\,\lambda_{b(O)A}\,\mathbf{g}(1,\ \omega_{2OA}\upsilon_{g(b(O))A}\,V_1,\ 0,\ V/\zeta_{b(O)A})$$

$$\varepsilon_{c(O)A}\,\omega_{OA}\,\gamma_{c(O)A}\,\mathbf{d}((\omega_{c(O)A}/\omega_{OA})\,V_1 + (\omega_{b(O)A}/\omega_{OA})\,V_2/\varepsilon_{b(O)A},\ V/\varepsilon_{c(O)A}) =$$
$$\zeta_{c(O)A}\,\lambda_{c(O)A}\,\mathbf{h}(1,\ \omega_{5OA}\upsilon_{g(c(O))A}\,V_1,\ \omega_{6OA}\upsilon_{b(O)A}\,V_2/\zeta_{b(O)A},\ V/\zeta_{c(O)A})$$

$$\Sigma_{B\neq A}\ \alpha_{SBA}V_B\,/\,\Sigma_{B\neq A}\ \alpha_{SBA}\varepsilon_{SB} \;=\; \Sigma_{B\neq A}\ \alpha_{SBA}V_B\,/\,\Sigma_{B\neq A}\ \alpha_{SBA}\zeta_{SB}$$

The last identity is equivalent to $\varepsilon_{SB} = \zeta_{SB}$ for all S and B with $\alpha_{SBA} > 0$ for some A. Moreover, it can be assumed that $\varepsilon_{SB} = \zeta_{SB}$ for all S and B. There may be multiple ways in which this can be satisfied for all values of V_1, U_2, U. At least one possibility is the following. Assume for all agents A

$$\lambda_{b(O)A} = \gamma_{b(O)A} \qquad \upsilon_{b(O)A} = 1$$
$$\lambda_{c(O)A} = \omega_{OA}\,\gamma_{c(O)A} \qquad \upsilon_{g(S)A} = 1$$

for S is $b(O)$ or $c(O)$. Then the identities simplify to

$$\mathbf{c}(V_1, U) \;=\; \mathbf{g}(1, \omega_{2OA} V_1, 0, V)$$
$$\mathbf{d}((\omega_{c(O)A}/\omega_{OA}) V_1 + (\omega_{b(O)A}/\omega_{OA}) V_2, V) \;=\; \mathbf{h}(1, \omega_{5OA}V_1, \omega_{6OA}V_2, V)$$

Furthermore, taking $\omega_{2OA} = 1$, $\omega_{5OA} = \omega_{c(O)A}/\omega_{OA}$, $\omega_{6OA} = \omega_{b(O)A}/\omega_{OA}$, the following identities result (replacing $\omega_{5OA}V_1$ by V_1 and $\omega_{6OA}V_2$ by V_2)

$$\mathbf{c}(V_1, V) \;=\; \mathbf{g}(1, V_1, 0, V) \qquad\qquad \mathbf{d}(V_1 + V_2, V) \;=\; \mathbf{h}(1, V_1, V_2, V)$$

There are many possibilities to fulfill these identities. For any given functions $\mathbf{c}(X, Y)$, $\mathbf{d}(X, Y)$ in the model **BAM** the functions $\mathbf{g}$, $\mathbf{h}$ in the model **IAM** defined by

$$\mathbf{g}(W, X, Y, Z) \;=\mathbf{c}(W-1+X+Y, Z) \qquad\qquad \mathbf{h}(W, X, Y, Z) \;=\mathbf{d}(W-1\;+X+Y, Z)$$

fulfill the identities $\mathbf{g}(1, X, 0, Z) \;=\; \mathbf{c}(X, Z)$ and $\mathbf{h}(1, X, Y, Z) \;=\mathbf{d}(X+Y, Z)$. It turns out that for given functions $\mathbf{c}(X, Y)$, $\mathbf{d}(X, Y)$ in the model **BAM** functions $\mathbf{g}$, $\mathbf{h}$ in the model **IAM** exist so that the interpretation mapping $\boldsymbol{\pi}$ maps the behavioural model **BAM** onto the model **ABAM**, which is a behavioural abstraction of the internal agent model **IAM** (see also Fig. 3): $\boldsymbol{\pi}^*$(BP1) = ABP1, $\boldsymbol{\pi}^*$(BP2) = ABP2, $\boldsymbol{\pi}^*$(BTP) = ITP. As an example direction, when for $\mathbf{c}(X, Y)$ a threshold function th is used, for example, defined as $\mathbf{c}(X, Y) = th(\sigma, \tau, X+Y) - Y$ with $th(\sigma, \tau, V) = 1/(1 + e^{-\sigma(V-\tau)})$, then for $\tau' = \tau + 1$ the function $\mathbf{g}(W, X, Y, Z) = th(\sigma, \tau', W+X+Y+Z) - Z$ fulfils $\mathbf{g}(1, X, 0, Z) = \mathbf{c}(X, Z)$. Another example of a function $\mathbf{g}(V, W, X, Y)$ that fulfills the identity when $\mathbf{c}(X, Z) = 1 - (1-X)(1-Z) - Z$ is $\mathbf{g}(W, X, Y, Z) = W\,[1 - (1-W)(1-X)(1-Z)] - Z$. As the properties specifying **ABAM** were derived from the properties specifying **IAM** (e.g., see Figs. 2 and 3), it holds **IAM** $\vdash$ **ABAM**, and as a compositional interpretation mapping $\boldsymbol{\pi}$ preserves derivation relations, the following relationships holds for any temporal pattern expressed as a hybrid logical/numerical property A in the ontology of **BAM**:

$$\mathbf{BAM} \vdash A \;\Rightarrow\; \boldsymbol{\pi}(\mathbf{BAM}) \vdash \boldsymbol{\pi}(A) \;\Rightarrow\; \mathbf{ABAM} \vdash \boldsymbol{\pi}(A) \;\Rightarrow\; \mathbf{IAM} \vdash \boldsymbol{\pi}(A)$$

Such a property A may specify certain (common) patterns in behaviour; the above relationships show that the internal agent model **IAM** shares the common behavioural patterns of the behavioural model **BAM**. An example of such a property A expresses a pattern that under certain conditions after some point in time there is one option O for which both $b(O)$ and $c(O)$ have the highest value for each of the agents (joint decision).

6 Discussion

This paper addressed how internal agent models and behavioural agent models for collective desion making can be related to each other. The relationships presented were expressed for specifications of the agent models in a hybrid logical/numerical format. Two agent models for collective decision making were first presented. First an internal agent model **IAM** derived from neurological principles modelled in a network specification **NS** was introduced with **NS** $\vdash$ **IAM**, where $\vdash$ is a symbol for derivability. Next, an existing behavioural agent model **BAM**, incorporating principles on social contagion or diffusion, was described, adopted from (Hoogendoorn, Treur, Wal, and Wissen, 2010). Furthermore, it was shown how the internal agent model **IAM** can be systematically transformed into an abstracted behavioural model **ABAM**, where the internal states were abstracted away, and such that **IAM** $\vdash$ **ABAM**. This generic transformation has been implemented in Java. Moreover, it was shown that under certain conditions the obtained agent model **ABAM** can be related to the behavioural agent model **BAM** by an interpretation mapping $\boldsymbol{\pi}$, i.e., such that $\boldsymbol{\pi}$**(BAM) = ABAM**. In this way hybrid logical/numerical relations where obtained between the different agent models according to:

IAM $\vdash$ **ABAM** and **ABAM =** $\boldsymbol{\pi}$**(BAM)**

These relationships imply that, for example, collective behaviour patterns shown in multi-agent systems based on the behavioural agent model **BAM** are shared (in the form of patterns corresponding via $\boldsymbol{\pi}$) for multi-agent systems based on the models **ABAM** and **IAM**.

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