Managing the Complexity of Large-Scale Agent-based Social Diffusion Models with Different Network Topologies

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Abstract. Social diffusion models have been extensively applied to study biological and social processes on a large scale. Previously two issues with these models were identified: understanding emerging dynamic properties of complex systems, and a high computational complexity of large-scale social simulations. Both these issues were tackled by abstraction techniques developed previously for social diffusion models with underlying random networks. In the paper it is shown that these techniques perform poorly on scale-free networks. To address this limitation, new model abstraction methods are proposed and evaluated for three network types: scale-free, regular and random. These methods are inspired by node centrality measures from the social networks area. The proposed methods increase the computational efficiency of the original model significantly (up to 40 times for regular networks).

Keywords: group dynamics, model abstraction, social diffusion, large-scale agent-based simulation

1 Introduction

Social diffusion models have been extensively applied to study diverse biological and social processes on a large scale, such as spread of innovation [10], dynamics of epidemics, formation and spread of opinions [4-7, 9, 12, 13]. These models describe a gradual spread of states (e.g., information) of agents in a network. Previously two issues with large-scale social diffusion models were identified [12, 13]: understanding emerging dynamic properties of systems, and a high computational complexity of large-scale social simulations. To address these issues, model abstraction techniques were proposed for social diffusion models with underlying random networks [12, 13]. The main idea behind these techniques was to identify dynamic clusters of agents with similar states, and to replace them by super-agents that were used in simulation as aggregate objects. The dynamic properties of these super-agents were inferred from properties of the agents of which they comprised. When some of the agents forming a super-agent changed their states, dynamic re-grouping of the super-agent took place.

The existing abstraction techniques for social diffusion models [12, 13] and model reduction methods [1, 2] are based on direct calculation of the equilibrium state of a

network. However, as shown in the paper, when a network does not converge to equilibrium quickly and/or the equilibrium state is disturbed frequently, these abstraction methods perform poorly. To address this limitation, three methods for approximating the dynamics of a large-scale social diffusion model are proposed, which do not rely on the system reaching equilibrium. For the illustration of the methods a social decision making model based on social diffusion is used in the paper. The methods proposed are based on determining a relative contribution of each agent from a group to the joint group's opinion. The agent's contribution depends on the agent's degree of influence in the network. The abstraction methods proposed differ only in the way how the agent's degree of influence is defined. In contrast to the existing model abstraction techniques, the abstraction methods proposed are inspired by node centrality measures used in social network analysis [3, 8].

The second contribution of the paper is that the proposed abstraction methods, as well as the most promising existing abstraction technique from [12], are evaluated on three network types, which often occur in many fields [3]: scale-free, regular and random. A few works exist that investigate effects of network topologies on the dynamics of social diffusion models [6]. To our best knowledge, analysis of abstraction techniques for social diffusion models was only attempted for random networks [12, 13]. Our analysis showed that the network type is essential for the applicability of the model abstraction techniques: whereas equilibrium-based techniques are well-suited for random and regular networks, these techniques perform poorly on scale-free networks. For the latter networks, the abstraction methods proposed in this paper show the best approximation results. Furthermore, the developed abstraction methods increase the computational efficiency of the original model significantly. The acceleration factor is the highest for regular networks (~40).

In many applications the size of dynamic groups, which could be numerous, is (much) smaller than the total number of agents. Previously, abstraction techniques were applied in a large-scale crowd evacuation study (~10000 agents) [11]. Although the number of agents was significant, the maximal size of emergent dynamic groups was 174. The maximal size of groups considered in this paper is 1000 agents.

The paper is organized as follows. An agent-based social decision making model based on social diffusion is described in Section 2. The proposed model abstraction methods are explained in Section 3 and evaluated in Section 4. Section 5 concludes the paper.

2 A Social Decision Making Model Based on Social Diffusion

In this section first a model description is provided in Section 2.1. Then, in Section 2.2 model analysis is given.

2.1 Model Description

The model describes decision making by agents in a group as a process of social diffusion. Two decision options – *s1* and *s2* - are considered by the agents. The

opinion $q_{s,i}$ of an agent i for a decision option $s \in \{s1,s2\}$ is expressed by a real number in the range $[0, 1]$, reflecting the degree of the agent's support for the option. For each option each agent communicates its opinion to other agents, and thus influences their opinions. Agents communicate only with those agents to which they are connected in a social network representing a group. In this study we consider three network topologies (see Fig.1):

- *Scale-free network*: a connected graph with the property that the number of links originating from a given node representing an agent has a power law distribution. In such networks the majority of the agents have one or two links, but a few agents have a large number of links.
- *Regular network*: a connected graph with the property that most nodes have approximately the same number of links (i.e., a homogeneous network). In this study each node has the number of links equal to the half of the number of agents.
- *Random network*: a graph, in which links between nodes occur at random, with equal probability $p=0.5$. Only connected graphs are considered in this study.

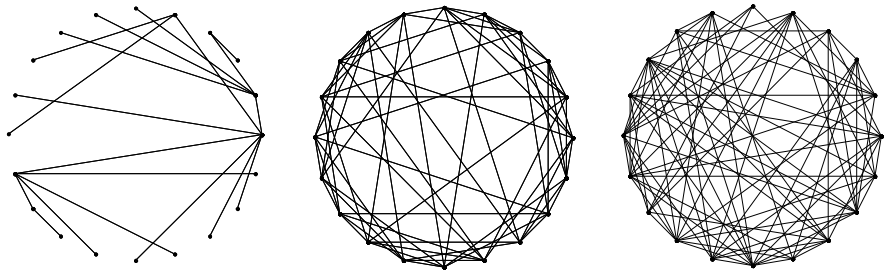

Fig. 1. Examples of different network topologies with 20 agents

It is assumed that the agents are able to both communicate and receive opinions to/from the agents, to which they are connected (i.e., the links between agents are bidirectional). Furthermore, a weight $\gamma_{i,j} \in [0,1]$, indicating the degree of influence of agent i on agent j, is associated with each link for each direction of interaction. This weight determines to which extent the opinion of agent i influences the opinion of agent j for each option.

In the literature on social diffusion two modes of interaction of agents have been repeatedly considered:

- *synchronous* or *parallel mode*, in which all agents interact with each other at the same time (synchronously), as in [9];
- *asynchronous* or *sequential mode*, in which at every time point, only one pair of connected agents chosen randomly interacts, as in [4].

Both these modes are considered in this paper. In the parallel mode, the opinion states of the agents are updated at the same time as follows:

$$q_{s,i}(t+\Delta t) = q_{s,i}(t) + \eta_i \, \delta_{s,i}(t)\Delta t \quad (1)$$

Here η_i is an agent-dependent parameter within the range $[0,1]$, which determines how fast the agent adjusts to the opinion of other agents, $\delta_{s,i}(t)$ is the amount of change of the agent i's opinion due to the influence of other agents defined as:

$\delta_{s,i}(t) = \sum_{j \neq i} \gamma_{j,i}(q_{s,j}(t) - q_{s,i}(t)) / \sum_{j \neq i} \gamma_{j,i}$ when $\sum_{k \neq i} \gamma_{k,i} \neq 0$

and

$\delta_{s,i}(t) = 0$ when $\sum_{k \neq i} \gamma_{k,i} = 0$

In the sequential mode two agents interact at each time point. The opinion states of interacting agents i and j are updated similarly to (1):

$$q_{s,i}(t+\Delta t) = q_{s,i}(t) + \eta_i \gamma_{j,i} / \sum_{j \neq i} \gamma_{j,i} (q_{s,j}(t) - q_{s,i}(t)) \Delta t \quad (2)$$

$$q_{s,j}(t+\Delta t) = q_{s,j}(t) + \eta_i \gamma_{i,j} / \sum_{i \neq j} \gamma_{i,j} (q_{s,i}(t) - q_{s,j}(t)) \Delta t \quad (3)$$

The opinion states of the agents not interacting remain the same:

$q_{s,k}(t+\Delta t) = q_{s,k}(t)$ when $k \neq i$, $k \neq j$

In the real world, opinions of human agents are influenced not only by their peers with whom they have mutual connections, but also by external information sources, such as media. To represent such sources in the model, a special agent type is introduced called *information source agent*. Such agents can influence normal agents, but cannot be influenced by any of the agents, i.e., $\gamma_{i,a}=0$ for an information source a and all agents i. It is assumed that interaction between normal agents and information source agents lasts shortly (1 time point in the simulation). For the time point, when agent i interacts with information source a, the following formula is used to update the agent i's opinion state instead of the formulae (1)-(3) in both parallel and sequential modes:

$$q_{s,i}(t+\Delta t) = q_{s,i}(t) + \eta_i \gamma_{a,i} (q_{s,a}(t) - q_{s,i}(t)) \Delta t \quad (4)$$

Different interaction frequencies of agents with information sources are examined by simulation in Sections 2.2 and 4.

2.2 Model Analysis

The model abstraction methods developed previously (e.g., [12, 13, 2]) are based on approximation of the behaviour of a model by the model's equilibrium state, calculated explicitly. However, if a model does not reach the equilibrium state quickly and/or the equilibrium state is disturbed frequently, the precision of the equilibrium-based abstraction methods may decline drastically (more precise results are given in Section 4). In this section we examine by simulation how the speed of convergence of the agent-based social diffusion model from Section 2.1 depends on the topology of the underlying network and on the number of agents in the network.

It was shown in [12] that every static network, in which each agent is influenced by at least one another agent, reaches an equilibrium state. Thus, since only connected networks are considered in this study, every model in this study reaches an equilibrium state. Also, previously it was proved for social diffusion models that the agents are in an equilibrium state if and only if their opinions are the same [12]. Based on this result, the degree of convergence cd of the model is evaluated in this study by the sum of the variances of the opinions of the agents for each option:

$$cd(t)=\sum_{s=\{s1,s2\}} var(q_{s,1..n}(t)),$$

where n is the number of agents.

To evaluate the degree of convergence for the model, 18 simulation settings were introduced. Each setting is characterized by the mode (parallel, sequential), network type (scale-free, regular, random) and the number of agents (100, 500, 1000). Then, each setting was simulated 1000 times and *cd* was calculated and averaged over all simulations for every 100th time point. The results are given in Figures 2-5.

As can be seen from Fig.2, in the parallel mode, during the first 100 time points the variance of the agents' opinions decreases very quickly to a very low value, and then remains low and decreases slowly. The speed of convergence of the model to equilibrium is higher for the random and regular networks in both sequential and parallel modes. The results for the regular networks, not shown in the figures, are very close to the ones for the random networks. This explains why equilibrium-based abstraction methods developed previously (as in [12, 13]) perform well on random networks. The rate of convergence of the model decreases with an increase in the number of agents in all simulation settings. In the sequential mode (see Fig.3), the model stabilizes much slower than in the parallel mode, especially when the number of agents is high.

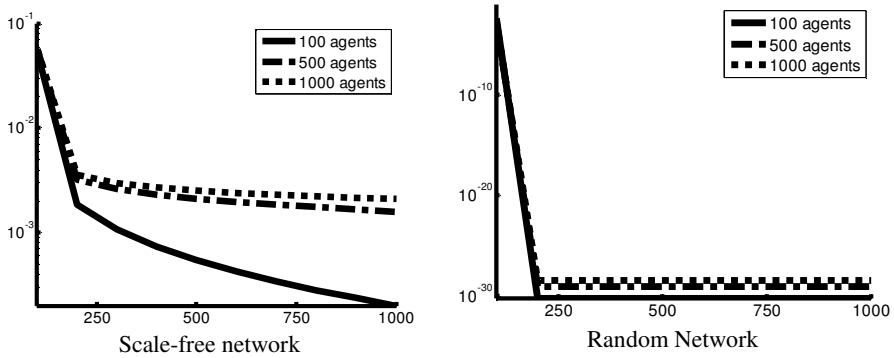

Fig. 2. Change of the variance of the opinion states of the agents over time in the parallel mode; the horizontal axis is time, the vertical axis is variance (log scale)

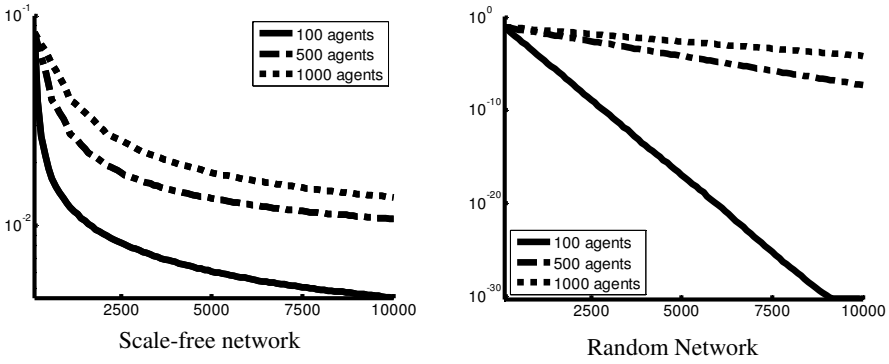

Fig. 3. Change of the variance of the opinion states of the agents over time in the sequential mode; the horizontal axis is time, the vertical axis is variance (log scale)

When information source agents send messages to other agents, they may disturb the convergence of the network. In Fig. 4 and 5 it is shown how the degree of convergence of the model is influenced by information source agents. All simulations were performed for the model with 500 normal agents and 50 information source agents. The average time between messages (ATBM) was varied between 10 and 5000. As can be seen in Fig. 4, in the parallel mode, the model stabilizes quickly after each interaction with information source agents. Thus, the number of messages does not influence the degree of convergence of the model in the parallel mode greatly. In the sequential mode (see Fig. 5), the number of messages has a greater effect on the degree of convergence than in the parallel mode.

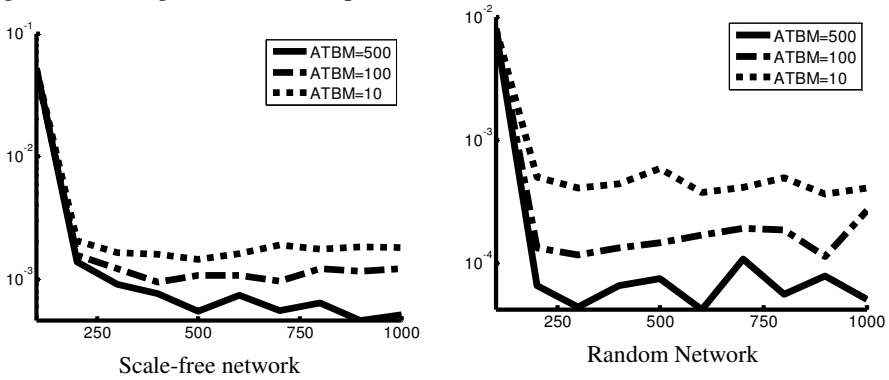

Fig. 4. Change of the variance of the opinion states of 500 agents over time in the parallel mode with the average time between messages (ATBM) > 0; the horizontal axis is time, the vertical axis is the variance of the opinion states (log scale)

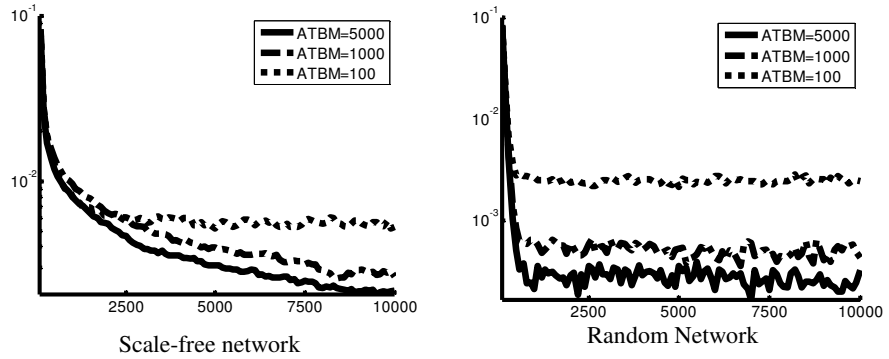

Fig. 5. Change of the variance of the opinion states of 500 agents over time in the sequential mode with the average time between messages (ATBM) > 0; the horizontal axis is time, the vertical axis is the variance of the opinion states (log scale)

In conclusion, whereas equilibrium-based abstraction methods may be effective for approximating the model with random and regular networks in the parallel mode, other abstraction methods need to be developed for models with scale-free networks. Three such methods are proposed in the next section.

3 Model Abstraction Methods

In this section three new model abstraction methods are proposed. The methods approximate the averaged opinion states of a model with network G with n agents: $q_{s,G}(t)=\sum_{i\in G} q_{s,i}(t)/n$. The approximated states $q^*_{s,G}$ are determined at the beginning of the simulation by weighted aggregation of the agents' initial opinions:

$$q^*_{s,G}(1)=\sum_{i\in G} sv_i\, q_{s,i}(1)/\sum_{i\in G} sv_i \qquad (5)$$

The weight sv_i of an agent i (significance value) is an indication of the agent's amount of influence in the network. The abstraction methods proposed in this section differ only in the ways how the significance values are calculated.

The approximated states are updated after every interaction of an agent j with an information source agent:

$$q^*_{s,G}(t)= (q^*_{s,G}(t-\Delta t)\sum_{i\in G\setminus j} sv_i + sv_j\, q_{s,j}(t))/\sum_{i\in G} sv_i \qquad (6)$$

where $q_{s,j}(t)$ is calculated by (4), and the agent j's opinion states before the interaction are equal to the approximated states $q^*_{s,G}(t-\Delta t)$.

For the rest of the time the approximated states do not change: $q^*_{s,G}(t)= q^*_{s,G}(t-\Delta t)$.

In the first abstraction method *M1*, significance value sv_i of agent i is determined by the sum of the degrees of the agent's influence on the other agents in the network:

$$sv^{M1}{}_i = \sum_{j\in G} \gamma_{i,j} \qquad (7)$$

This measure is similar to the degree centrality measure often used in social networks [3]. In [8] a centrality measure for weighted networks is proposed, in which both weights of links and the number of links are taken into account. Preliminary experiments with this measure produced results worse than the ones of method *M1*. Instead of this measure, the sum of relative degrees of influence $\gamma_{i,j}/\sum_{k\in G} \gamma_{k,j}$ was used as the basis for method *M2*:

$$sv^{M2}{}_i = \sum_{j\in G} \gamma_{i,j}^*, \qquad (8)$$

where $\gamma_{i,j}^* = \gamma_{i,j}/\sum_{k\in G} \gamma_{k,j}$, if $\sum_{k\in G} \gamma_{k,j} > \gamma_{i,j}$, and $\gamma_{i,j}^* = \gamma_{i,j}$ otherwise.

This measure stems from the model itself, and thus reflects better its dynamics than the measures from both *M1* and [8].

Both measures $sv^{M1}{}_i$ and $sv^{M2}{}_i$ are calculated for each agent locally, by taking into account direct influences only. In the third abstraction method *M3*, also indirect influences of agents in the network are considered. In *M3* the significance values of the agents are propagated through the network. When the value is propagated, it is multiplied by the strength of the link. Thus, agents, which have a high influence on influential agents, have also a high significance value. The significance value $sv^{M3}{}_i$ is calculated by an iterative algorithm provided below.

Initially, the significance values of the agents are set to $sv^{M2}{}_i$. Then, at every iteration the agent j's significance value from the previous iteration multiplied by the relative degree of influence of agent i on agent j (i.e., $sv^{M3}{}_j(iter\text{-}1)\gamma_{i,j}/\sum_{k\in G} \gamma_{k,j}$) is propagated to agent i. The new significance value of each agent is the sum of the received (propagated) significance values. Thus, with every iteration, indirect

influences of more and more distant agents in the network are taken into account in the calculation of the significance values.

Algorithm 1: Calculating significance values $sv^{M3}{}_i$

1: for all agents i: $sv^{M3}{}_i(1) \Leftarrow sv^{M2}{}_i$
2: **for** *iter*=2 to <*number-iterations*> **do**
3: For all agents i: $sv^{M3}{}_i(iter) = \sum_{j \in G} sv^{M3}{}_j(iter\text{-}1)\ \gamma_{i,j}^*$,
$\gamma_{i,j}^* = \gamma_{i,j} / \sum_{k \in G} \gamma_{k,j}$, if $\sum_{k \in G} \gamma_{k,j} > \gamma_{i,j}$, and $\gamma_{i,j}^* = \gamma_{i,j}$ otherwise.
4: **end for**

The precision of the abstraction methods is evaluated by calculating root-mean-square error (RMSE) defined as

$$(\sum_{t=1..end\_time} ((q^*_{1,G}(t) - q_{1,G}(t))^2 + (q^*_{2,G}(t) - q_{2,G}(t))^2) / end\_time)^{1/2}$$

In Algorithm 1 the precision of abstraction method *M3* depends on the number of iterations. In the scale-free networks the decrease of the error with an increase of the number of iterations is gradual (see Fig.6, left), whereas in the regular and random networks the error drops only during a few first iterations and remains almost constant afterwards.

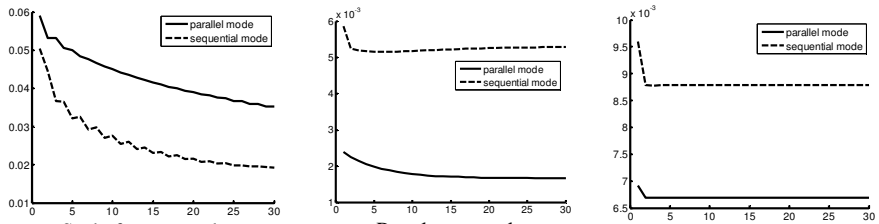

Fig. 6. Change of the root-mean-square error during the first 30 iterations (averaged over 1000 simulation trials) in networks with 100 agents

This may be explained by that regular and random networks are more densely connected than scale-free networks. Thus, in the former networks indirect influence of agents reaches every agent in a fewer number of iterations than in the latter networks.

In the next section the abstraction methods proposed will be evaluated by measuring RMSE and simulation time for the network topologies under consideration.

4 Evaluation of the Model Abstraction Methods

To evaluate the model abstraction methods described in Section 3, 72 simulation settings were introduced. Each setting is characterized by the mode (parallel, sequential), network topology type (scale-free, regular, random), the number of agents (100, 500, 1000), and the average time between messages (1, 2, 5, 10 for the parallel mode and 10, 50, 100, 500 for the sequential mode). In addition, the number of information source agents was 10 times less than the number of normal agents. The initial opinion states of the agents $q_{s,i}(1)$ were uniformly distributed in the interval [0,1]. The parameters $\gamma_{i,j}$ and η_i were taken from the uniform distribution in the

interval]0,1]. Moreover, $\Delta t = 1$. The simulation time was 1000 for the parallel mode and 10000 for the sequential mode. For method *M3* in Algoritm1 30 iterations were performed for the scale-free networks and 10 iterations were performed for the regular and random networks. Each simulation setting was simulated 1000 times for each abstraction method, and RMSE and simulation time was determined and averaged over all simulations runs for each setting. The results obtained for three abstraction methods proposed were also compared to the results of the invariant-based abstraction method, which showed the best performance in [12]. The RMSE obtained for the four abstraction methods for networks with 500 and 1000 agents are provided in Figures 7 and 8. In the following the results obtained are discussed.

It was established in Section 2.2 that the speed of convergence of the model to equilibrium is higher for random and regular networks than for scale-free networks in both sequential and parallel modes. Because of a relatively slow convergence of scale-free networks, the invariant-based abstraction method, which relies on a quick convergence to an equilibrium state, performed poorly in the parallel mode in comparison to the methods proposed in this paper (Fig.7). Also, in the sequential mode, *M3* outperformed the invariant-based method (Fig.8).

From the three methods proposed *M3* provides the best approximation results for the scale-free network topology. In the parallel interaction mode *M1* slightly outperforms *M2*, again because *M2* approximates the equilibrium state of the network slightly better than *M1*.

In the sequential mode, the scale-free network has more time to reach an equilibrium state, due to longer periods between subsequent messages. Thus, the averaged opinion states of the network are closer to the equilibrium states. Therefore, the invariant-based method outperforms methods *M1* and *M2*, however, still worse than *M3*. As *M2* produces approximations that are closer to the equilibrium states than approximations obtained by *M1*, *M2* shows better results than *M1* for the scale-free networks in the sequential mode.

For the regular network, in the parallel mode the invariant-based method outperforms greatly all other methods. This can be partially explained by a relatively fast convergence of regular networks to equilibrium in the parallel mode. In sequential mode, all the methods show similar performance for both regular and random networks (Fig.8).

For the random networks in the parallel mode the invariant-based method and *M2* show the best approximation results. Partially this can be explained by fast convergence of random networks to equilibrium. As both the invariant-based and *M2* methods provide a close approximation of an equilibrium state, these methods perform the best for random networks.

Note that for all methods the error decreases with the increase of the number of agents. This result was also observed in [12]. In many cases the error grows with the increase of the average time between messages from information source agents. This is because the update of the approximated states occurs only after interaction with information source agents. The longer periods between updates are, the more error accumulates. However, frequent messages disturb the process of convergence and, thus, also contribute to the error growth. The errors for the regular and random networks are lower than the errors for the scale-free networks. Furthermore, the errors in the sequential mode on average lower than the errors in the parallel mode.

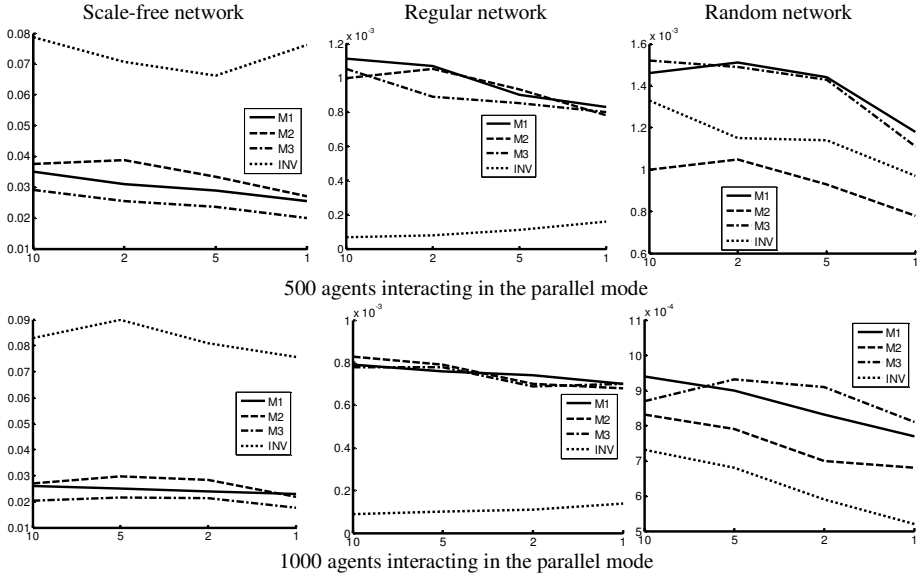

Fig. 7. Mean RMSE for the proposed abstraction methods *M1-M3* and the invariant-based method (*INV*) in the parallel mode; the horizontal axis is the average time between messages.

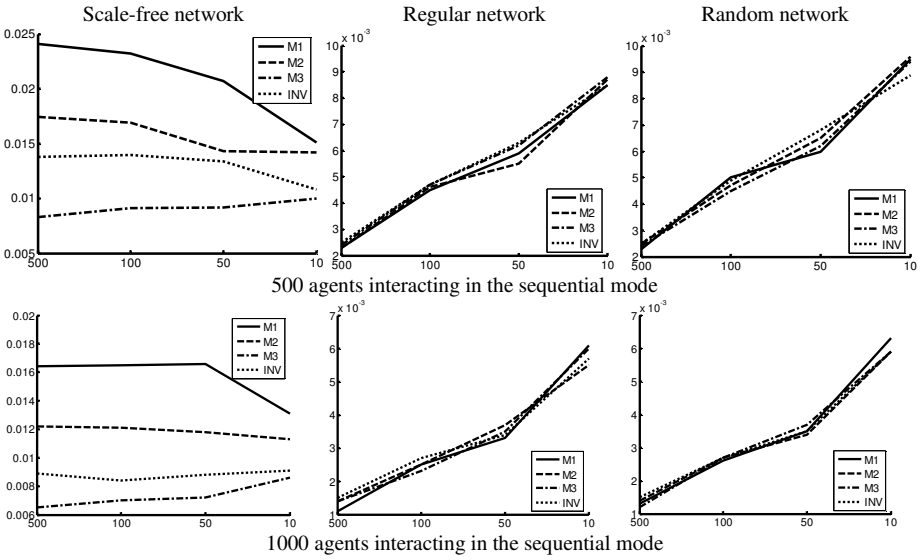

Fig. 8. Mean RMSE errors for the proposed abstraction methods *M1-M3* and the invariant-based method (*INV*) in the sequential mode; the horizontal axis is the average time between messages.

The mean time complexity for the original model from Section 2 and for the proposed abstraction methods is provided in Table 1. The variances of these results are very low (of the order of 10^{-5}).

The developed abstraction methods increase the computational efficiency of the original model in the parallel mode significantly. The fastest simulation models are obtained by abstraction method *M1*, whereas abstraction by method *M3* is the slowest. However, the time difference between *M1* and *M3* is small (less than 1.5 in the worst case). The fastest simulation model is obtained for the scale-free networks, the slowest – for the random networks. Thus, the simulation time depends on the density of a network. The largest acceleration factor due to abstraction is obtained for the regular networks (up to 40); for scale-free networks – up to 25, and for random networks – up to 30. With the increase of the number of messages, the simulation time increases. This is because of the update of the approximated states after each interaction with information source agents.

Note that the simulation time of the model in the sequential mode is low, and depends weakly on the number of agents. This is because the states of only one pair of agents are updated at each time point.

Table 1. Mean simulation time in seconds for the original and abstracted models; the average time between messages is 1 for the parallel mode and 10 for the sequential mode.

Network type	**Scale-free**			**Regular**			**Random**		
# of agents	**100**	**500**	**1000**	**100**	**500**	**1000**	**100**	**500**	**1000**
Parallel mode									
Original model	0.51	9.9	41	0.73	15.9	54.2	1	25.1	99.48
Method *M1*	0.02	0.05	2.7	0.03	0.74	4.42	0.05	0.92	4.62
Method *M2*	0.02	0.05	2.7	0.03	0.75	4.51	0.05	0.97	4.83
Method *M3*	0.03	0.07	3.6	0.03	0.93	5.3	0.05	0.99	5.8
Sequential mode									
Original model	0.56	3.3	9.4	0.55	3.5	10.9	0.59	3.8	11.7
Method *M1*	0.31	2.1	6.8	0.3	2.2	8.3	0.33	2.4	8.6
Method *M2*	0.31	2.1	6.8	0.3	2.2	8.3	0.33	2.4	8.6
Method *M3*	0.31	2.3	7.8	0.31	2.4	9.2	0.36	2.7	9.5

5 Conclusions and Discussion

Model abstraction methods, as proposed in the paper, provide information about global dynamics of a system by approximating global system states. Furthermore, these methods allow tackling computational efficiency problems of large-scale simulations. As demonstrated in the paper, the precision and efficiency of model abstraction methods depends greatly on the network type underlying the model. The equilibrium-based methods are well-suited for random and regular networks, which allow a fast convergence of the model to equilibrium. At the same time, the methods proposed in this paper are much better than the equilibrium-based methods for models with scale-free networks. In contrast to the existing model abstraction techniques, our methods do not rely on the system reaching equilibrium, which is a desirable property for slow converging scale-free networks.

As demonstrated in the paper, the developed abstraction methods increase the computational efficiency of the original model in the parallel mode significantly (up to 40 times for the regular topology).

Currently several techniques for abstraction of models based on hybrid automata [1] and differential equations [2] exist. However, such approaches can be applied efficiently only for systems described by sparse matrixes. Social diffusion models represent tightly connected systems, which do not allow a significant reduction of the state space using such techniques. In particular, a previous study showed that common model reduction techniques such as balanced truncation [2] do not allow decreasing the rank of the matrix describing the model from Section 2.

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