

# Group Abstraction for Large-Scale Agent-Based Social Diffusion Models with Unaffected Agents

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**Abstract.** In this paper an approach is proposed to handle complex dynamics of large-scale multi-agents systems modelling social diffusion processes. A particular type of systems is considered, in which some agents (e.g., leaders) are not open to influence by the other agents. Based on local properties characterising the dynamics of individual agents and their interactions, groups and properties of the dynamics of these groups are identified. To determine such dynamic group properties two abstraction methods are proposed: determining group equilibrium states and approximation of group processes by weighted averaging of the interactions within the group. This enables simulation of the group dynamics at a more abstract level by considering groups as single entities substituting a large number of interacting agents. In this way the scalability of large-scale simulation can be improved significantly. Computational properties of the developed approach are addressed in the paper. The approach is illustrated for a collective decision making model with different types of topology, which may occur in social systems.

**Keywords:** group dynamics, model abstraction, social diffusion, large-scale agent-based simulation

## 1 Introduction

Social diffusion models describe spread and changes of states or attitudes in a group or community of agents under the impact of social interaction. Such models have been extensively used to study diverse social processes, such as dynamics of social power [4], polarization of opinions of individuals in a group [8, 3], and spread of innovation [10].

In many existing social diffusion models it is assumed that each agent changes its state (e.g., an opinion) continuously, under influence of other agents. However, in real systems actors may exist, which for some reason are not open to change, for example, because they are not willing or able to change their state. Specific examples of such actors are autocratic leaders or persons with no control over a given state (e.g., related

to ethnicity or gender) [8]. Such agents may affect other agents, by continuously propagating their state to them, and as they are not affected themselves in the end have much effect on the group's state, as happens, for example, for strong leader figures. In this paper social diffusion in large-scale social systems with such unaffected agents is considered from an agent-based perspective.

Although the local behaviors of each agent may be simple, the global patterns that emerge from interaction between the agents in a large-scale social diffusion system are far from trivial. Such patterns are difficult to infer directly from the local dynamic properties of the agents. A high computational complexity of such large-scale multi-agent systems hinders automated analysis of such systems by systematic simulation and verification.

In this paper an approach is proposed to handle complex dynamics of large-scale agent-based social diffusion models by using abstraction methods. The approach is based on identifying groups of interacting agents with similar states (e.g., opinions on an issue) in a society of agents. The idea is that an approximate form of simulation is obtained by using such groups as single entities representing abstractions of large numbers of interacting agents. In such a way the scalability of a large-scale multi-agent simulation can be improved. The obtained abstracted process provides an approximation with a behavioural error that can be estimated.

To determine global emerging properties of groups based on local properties of the group members, two group abstraction methods are proposed. In the first method relative degrees of importance of the agents in a group are determined. The degree of importance of an agent is an estimation of the strength of the agent's influence on the group. The aggregated state of the group is determined as the weighted average of the states of the group members with the weights defined by the relative degrees of importance of the members. In the paper this method is called *abstraction by weighted averaging*. The second abstraction method used is based on identifying an equilibrium state of a group by a standard procedure. In the paper this method is called *equilibrium-based abstraction*.

The proposed group abstraction approaches are illustrated by a case of a collective decision making model for a number of scenarios with two different topologies, which may exist in real social systems. The approximation errors and time complexity of the proposed abstraction methods applied for this case are discussed.

The paper is organized as follows. An agent-based collective decision making model used as a case is described in Section 2. The proposed methods for group abstraction are explained in Section 3. Some simulation results are discussed in Section 4. In Section 5 the proposed abstraction methods applied to the collective decision making model are evaluated. Section 6 concludes the paper.

## 2 A Collective Decision Making Model

In the model collective decision making is specified as the process of social diffusion of opinions of agents on decision options. The agents are assumed to consider two different decision options  $s_1$  and  $s_2$  for one issue (e.g., two exits of a burning building).

In most existing social diffusion models (e.g., [8, 9, 5]), opinions of agents are represented by binary variables, which reflect the opposite attitudes of agents towards an issue. The choice for binary variables is well motivated for models, which focus on attitudes of agents towards highly salient events, for which strong opinions are common (e.g., in voting). However, continuous variables are suited better than binary variables for representing doubts of agents, e.g., when they are situated in uncertain environments with scarce information. Furthermore, the change of the agent's opinion to the opposite one occurs gradually, through a number of phases [7]. This can be captured better by a continuous variable than by a binary variable. In contrast to most existing models, the opinions of an agent in the model used here are described by continuous variables within the range  $[0,1]$ . These variables reflect the degrees of support of an agent for the decision options  $s1$  and  $s2$ .

The initial values of the opinions of the agents on both options are uniformly distributed in the interval  $[0,1]$ . By interaction the agents start to influence each other's opinions. The strength of social influence of an agent  $i$  on another agent  $j$  is determined by parameter  $\gamma_{ij}$  within the range  $[0, 1]$ . This parameter may be refined, e.g., by distinguishing expressiveness of the sender (agent  $i$ ), openness of the receiver (agent  $j$ ), the strength of the channel between  $i$  and  $j$  [6]. This parameter may also refer to a distance between  $i$  and  $j$  in 'social' space. For simplicity  $\gamma_{ij}$  will be used without refinement.

It is assumed that agents interact synchronously with each other: all states of the agents are updated in parallel. As stated in [9] the dynamics of group interaction is captured more accurately when many individuals are allowed to interact simultaneously than by pairwise interaction.

The strength of the social influence on agent  $i$  with respect to decision option  $s$  at time  $t$  is determined by:

$$\begin{aligned} \delta_{s,i}(t) &= \sum_{j \neq i} \gamma_{j,i} (q_{s,j}(t) - q_{s,i}(t)) / \sum_{j \neq i} \gamma_{j,i} && \text{when } \sum_{k \neq i} \gamma_{k,i} \neq 0 \\ \delta_{s,i}(t) &= 0 && \text{when } \sum_{k \neq i} \gamma_{k,i} = 0 \end{aligned}$$

Here  $q_{s,j}(t)$  the strength of support of agent  $j$  of decision option  $s$ . The update of the strength of support of agent  $i$  for  $s$  is determined by:

$$q_{s,i}(t+\Delta t) = q_{s,i}(t) + \eta_i \delta_{s,i}(t) \Delta t$$

Here  $\eta_i$  is an agent-dependent parameter within the range  $[0,1]$ , which determines how fast the agent adjusts to the opinion of other agents.

First an initial consolidation phase takes place during the interval  $[0, t_{end\_init}]$ , in which the agents exchange opinions on the options. After this phase the whole population of agents is divided into two groups  $G1$  and  $G2$  depending on which from two options  $s1$  or  $s2$  is preferred:

$$\begin{aligned} G1 &= \{i \mid q_{s1,i}(t_{end\_init}) \geq q_{s2,i}(t_{end\_init})\} \\ G2 &= \{i \mid q_{s2,i}(t_{end\_init}) > q_{s1,i}(t_{end\_init})\}. \end{aligned}$$

Each group can be viewed as a connected directed graph  $G = \langle V, E \rangle$  with a set of vertices  $V$  representing agents and a set of directed edges  $E$  representing influence

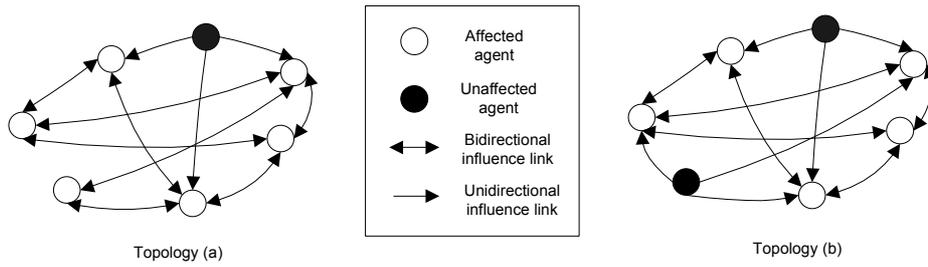
relations between the agents. It is assumed that there are less interactions between members of different groups than between members within a group. This assumption is partially supported by social studies [3].

**Definition**

A subset  $S$  of  $G$  is called *isolated from impact by others* if it is nonempty and not equal to  $G$  and for all agents  $i \in S$  and  $j \notin S$  it holds  $\gamma_{j,i} = 0$ .

In the paper scenarios will be addressed based on the following topologies of groups:

- (a) There is exactly one subset isolated from impact by others, and this is a singleton  $\{i\}$ - an agent, which is not willing or able to change its state (for an example see Fig. 1, left).
- (b) There are exactly two subsets isolated from impact by others, and these are singletons  $\{i\}$  and  $\{j\}$  – two agents with different states, which are not willing or able to change their states; e.g., two conflicting dogmatic leaders (for an example see Fig. 1, right).



**Fig. 1.** Examples of topologies of groups considered in the paper

Every now and then members of a group receive information from diverse external sources via peer-to-peer communication. The degree of influence of external source  $k$  (e.g., an agent from another group) on a group member  $i$  is represented by parameter  $\gamma_{k,i}$ . Based on the states of  $k$  and  $i$  concerning option  $s$ , agent  $i$  updates its state as follows:

$$q_{s,i}(t+\Delta t) = q_{s,i}(t) + \eta_i \gamma_{k,i}(q_{s,k}(t) - q_{s,i}(t))\Delta t$$

If after interaction with an external source, agent  $i$  from group  $G1$  supporting option  $s1$  changes its preference from  $s1$  to  $s2$ , and  $q_{s2,i}(t) - q_{s1,i}(t) > threshold$ , then an agent is considered to leave  $G1$  and become a member of  $G2$  supporting  $s2$ . In the scenarios considered in the paper  $threshold=0.3$ .

### 3 Two Methods for Group Abstraction

To model the dynamics of abstracted states of a group two group abstraction methods are proposed in this section.

#### 3.1 Abstraction by Weighted Averaging

The first method introduced is based on an estimation of an aggregated group state by determining the contributions of each group member to the group state. It is assumed that the contribution of an agent is in proportion to the strength of influence of the agent on the other group members. An agent may influence another agent directly or indirectly through other agents. In this direct case the strength of this (first-order) influence of  $i$  on  $j$  can be estimated by  $\gamma_{i,j}$ . If  $i$  influences  $k$  through  $j$ , the strength of (second-order) indirect influence of  $i$  on  $k$  via  $j$  is estimated as  $\gamma_{i,j} \gamma_{j,k}$ , and in the total second-order influence is estimated as  $\sum_{j \neq i, j \neq k} \gamma_{i,j} \gamma_{j,k}$ . In the general case, the higher order strengths of influence of an agent on any other agent can be calculated recursively.

Thus, for each agent a network of influence can be identified, through which an agent exerts influence and is influenced by other agents. In such a network the degree of importance of an agent  $i$  ( $doi_i$ ) on the group is calculated as follows:

$$doi_i = \frac{\sum_{i \neq j} \gamma_{i,j} (1 + \sum_{k \neq i, k \neq j} \gamma_{j,k} (1 + \dots))}{(1 + \sum_{i \neq j} \gamma_{j,i} (1 + \sum_{k \neq i, k \neq j} \gamma_{k,j} (1 + \dots)))}$$

The denominator contains the term  $1$  to ensure that it is not equal to  $0$  for the agents isolated from impact by others, e.g., as in topologies (b) and (c). The precision of estimation of the group state depends on the number of hops in a network of influence for which indirect influences are calculated. However, the more hops are taken the more intensive computation is required for abstraction by this method. In this paper two hops in a network of influence are used. In the single-hop variant of the method (called *first-order weighted averaging*)  $doi_i$  is calculated as:

$$doi_i = \frac{\sum_{i \neq j} \gamma_{i,j}}{(1 + \sum_{i \neq j} \gamma_{j,i})}$$

The two-hop variant (called *second-order weighted averaging*)  $doi_i$  is:

$$doi_i = \frac{\sum_{i \neq j} \gamma_{i,j} (1 + \sum_{k \neq i, k \neq j} \gamma_{j,k})}{(1 + \sum_{i \neq j} \gamma_{j,i} (1 + \sum_{k \neq i, k \neq j} \gamma_{k,j}))}$$

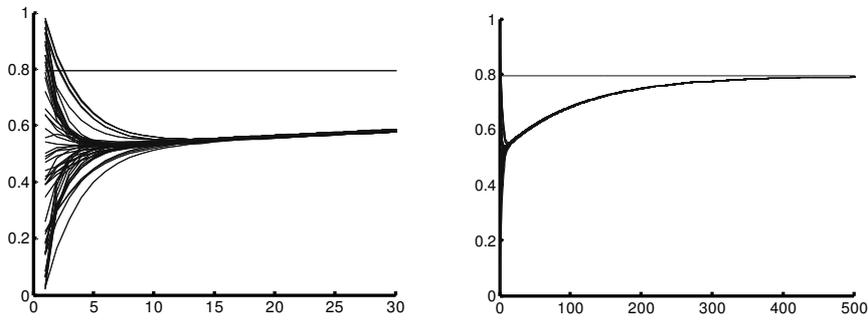
Initially and after each interaction of an agent from group  $G$  with an external agent, the aggregated state of group  $G$  for option  $s$  is estimated by the following weighted average:

$$q_{s,G}(t+\Delta t) = \frac{\sum_{i \in G} doi_i q_{s,i}(t)}{\sum_{i \in G} doi_i}$$

This state represents a common opinion of all agents in the group for decision option  $s$ . It persists until a new interaction with an external agent occurs. Then, the formula for  $q_{s,G}(t+\Delta t)$  is applied again. The weighted averaging method can be used for abstraction of groups with both types of topologies (a) and (b) described in Section 2.

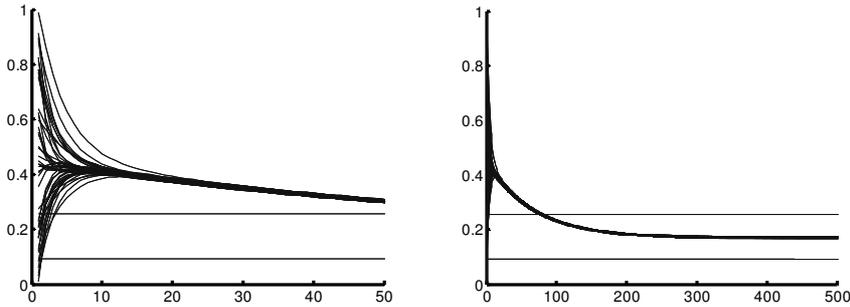
### 3.2 Abstraction by Determining Equilibria

For the cases where one or more of the agents  $i$  are not affected (agents  $i$  with  $\gamma_{j,i} = 0$  for all  $j$ ), the equilibria for other agents do not depend on their initial values and the standard approach (solving the equilibrium equations) using the differential equations can be used. Note that if  $\gamma_{j,i} = 0$  for all  $j$  then the value for  $i$  will be in an equilibrium right from the start, since no impact of other group members occurs. Therefore a group equilibrium with common value can only concern the initial value  $q_{s,i}(0)$  for such an agent  $i$ . For one such an agent in the group this indeed takes place (see Fig. 2).



**Fig. 2.** Convergence of the decision states of 50 agents in a group with topology (a) for the time period  $[0, 30]$  (left) and the time period  $[0, 500]$ ; no external messages are provided to the group

When two or more of such agents occur, with different initial states, then apparently a common equilibrium value is not possible, and the group will end up in a divided equilibrium situation (see Fig. 3). In the simulation, which results are presented in Fig. 2 and 3 the strength of social influence of both unaffected and affected agents on other affected agents was taken from the uniform distribution in the interval  $]0, 1[$ .



**Fig. 3.** Convergence of the decision states of 50 agents in a group with topology (b) for the time period  $[0, 50]$  (left) and the time period  $[0, 500]$ ; no external messages are provided to the group

These cases can be analyzed in a direct, standard manner using the differential equations as follows. Take the set of agents that are not affected:

$$S_0 = \{ i \mid \sum_{k \neq i} \gamma_{k,i} = 0 \}$$

Then the differential equations are

$$q_{s,i}(t+\Delta t) = q_{s,i}(t) + \eta_i \left[ \sum_{j \neq i} \gamma_{j,i} (q_{s,j}(t) - q_{s,i}(t)) / \sum_{k \neq i} \gamma_{k,i} \right] \Delta t \quad \text{for } i \notin S_0$$

$$q_{s,i}(t+\Delta t) = q_{s,i}(t) \quad \text{for } i \in S_0$$

For all agents  $i \in S_0$  the equilibrium value  $q_{s,i}$  is the initial value  $q_{s,i}(0)$ . This value can be used in the equations for the agents  $i \notin S_0$ , thus obtaining for all  $i \notin S_0$ :

$$\sum_{j \in S_0, j \neq i} \gamma_{j,i} (q_{s,j} - q_{s,i}) / \sum_{k \neq i} \gamma_{k,i} + \sum_{j \in S_0} \gamma_{j,i} (q_{s,j}(0) - q_{s,i}) / \sum_{k \neq i} \gamma_{k,i} = 0$$

This can be rewritten into the following system of linear equations in  $q_{s,k}$  for  $k \notin S_0$ :

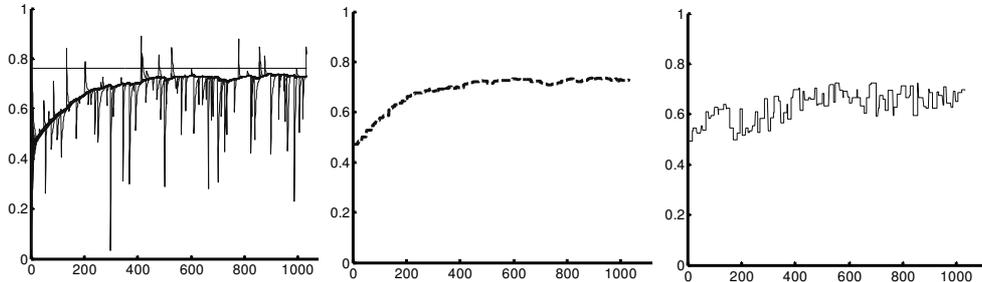
$$q_{s,i} - \sum_{j \in S_0, j \neq i} (\gamma_{j,i} / \sum_{k \neq i} \gamma_{k,i}) q_{s,j} = \sum_{j \in S_0} (\gamma_{j,i} / \sum_{k \neq i} \gamma_{k,i}) q_{s,j}(0)$$

This can be expressed in matrix form as  $\mathbf{B}\mathbf{q} = \mathbf{c}$ , with  $\mathbf{q}$  the vector  $(q_{s,i})_{i \notin S_0}$ , and  $\mathbf{c}$  the vector  $(\sum_{j \in S_0} (\gamma_{j,i} / \sum_{k \neq i} \gamma_{k,i}) q_{s,j}(0))_{i \notin S_0}$ . The matrix  $\mathbf{B}$  has only 1 as diagonal entries, and negative values elsewhere. Taking into account determinant  $\det(\mathbf{B}) \neq 0$ , this system is solvable in a unique manner.

## 4 Simulation

The methods described in Section 3 were implemented in Matlab. Simulation time was 1030 with the initial stabilization interval  $[0, 30]$ . For each simulation setting 50 iterations were executed. The number of agents was varied across simulation runs: 50, 100, 200 and 500. The initial states of each agent for the strengths of support for the two decision options  $s1$  and  $s2$  were uniformly distributed in the interval  $[0, 1]$ .

In addition to the agents external sources were used, which number was 10 times less than the number of agents. The average time between two subsequent messages provided by each external source to a randomly chosen agent was varied across simulation runs: 1, 2, 5, and 10. Each average time value can also be interpreted as a ratio of the time scale of the group's internal dynamics to the time scale of the external dynamics. The impact of these ratios on approximation errors was investigated.

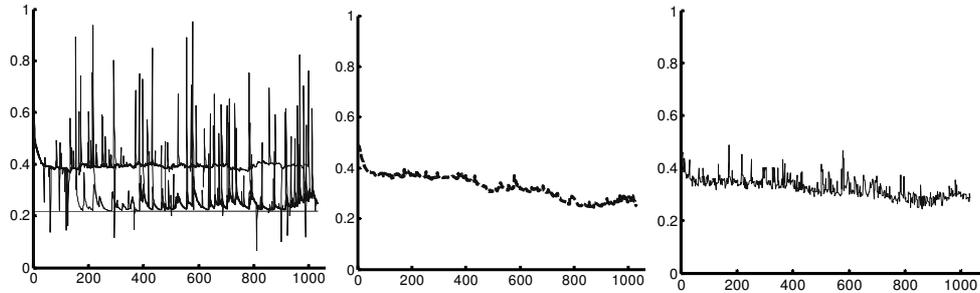


**Fig. 4.** The dynamics of valuation of option 1 by 50 individual agents in a group with topology (a) (left) and the abstraction of the group dynamics obtained by the equilibrium-based method (center) and by the weighted averaging 2 (right); the average time between messages is 10.

The simulation was performed for both types of topology described in Section 2. The parameters  $\gamma$  and  $\eta$  of the agents were taken from the uniform distribution in the interval  $]0,1[$ . For topology (a) all values  $\gamma_{j,i}$  for a randomly chosen agent  $i$  were set to 0. For topology (b) all values of  $\gamma_{j,i}$  and  $\gamma_{m,k}$  for two randomly chosen agents  $i$  and  $k$  were set to 0.

In the simulation the first and second-order weighted averaging methods and the equilibrium-based method were used for abstraction of the model with both types of topologies.

Some of the simulation results for topologies (a) and (b) are presented respectively in Fig. 4 and 5. The peaks in the graphs indicate incoming messages from external sources. As can be seen from the both figures, after receiving each message the group quickly reaches a new stable state.



**Fig. 5.** The dynamics of valuation of option 1 by 50 individual agents in a group with topology (b) (left) and the abstraction of the group dynamics obtained by the equilibrium-based method (center) and by the weighted averaging 2 (right); the average time between messages is 2.

Since a divided equilibrium situation occurs in the group with topology (b) (Fig.5), the abstraction of the group dynamics for this topology is less precise than for topology (a) (Fig.4), in which the group is driven towards a single equilibrium state. A detailed evaluation of efficiency and quality of the proposed abstraction methods is considered in the following Section 5.

## 5 Evaluation of the Two Abstraction Methods

The time complexity and approximation errors of the methods are considered in this section.

### 5.1 Time Complexity Results

The mean time complexity for the original model from Section 2 and for the proposed abstraction methods is provided in Tables 1 and 2. The variances of these results are

very low (of the order of  $10^{-5}$ ). Since the same mechanisms of abstraction by weighted averaging are applied for both topologies (a) and (b), the mean simulation time of the abstracted models based on the weighted averaging methods is the same for all these topologies. The developed abstraction methods increase the computational efficiency of the simulation significantly. The acceleration factor grows with the number of agents: for smaller numbers (around 50) it is of the order 20 to 25, for larger numbers (around 500) it grows to the order of 25 to 33.

**Table 1.** Mean simulation time in seconds for the original and abstracted models for 50 and 100 agents

# of agents	50				100			
Average time between messages	1	2	5	10	1	2	5	10
Original model	5.87	5.46	4.98	4.72	23.67	22.7	20.5	19.25
Equilibrium-based abstraction (a)	0.27	0.24	0.22	0.21	0.9	0.83	0.78	0.76
Equilibrium-based abstraction (b)	0.29	0.25	0.22	0.21	1.02	0.90	0.81	0.78
Abstraction by weighted averaging 1	0.25	0.22	0.20	0.20	0.88	0.83	0.78	0.76
Abstraction by weighted averaging 2	0.27	0.24	0.21	0.20	0.96	0.88	0.80	0.78

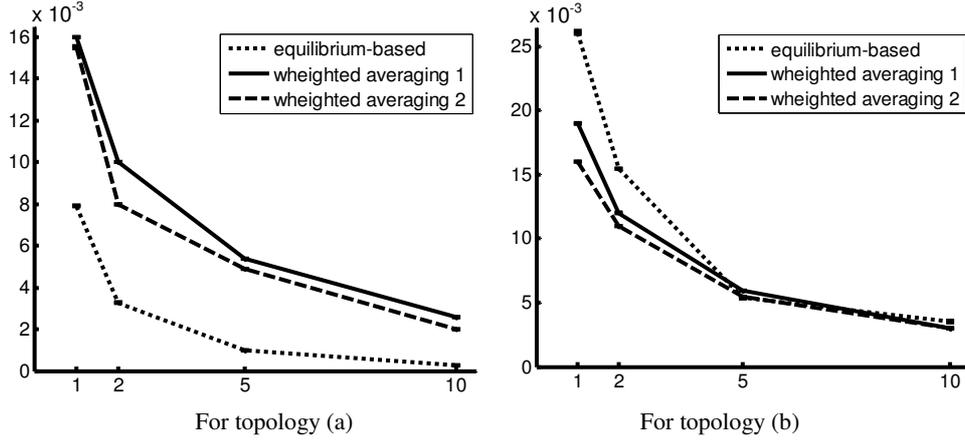
**Table 2.** Mean simulation time in seconds for the original and abstracted models for 200 and 500 agents

# of agents	200				500			
Average time between messages	1	2	5	10	1	2	5	10
Original model	96.4	93.5	87.9	82.7	383.7	383.2	365.3	350.8
Equilibrium-based abstraction (b)	3.60	3.24	3.03	2.96	13.8	13.1	11.7	11.5
Equilibrium-based abstraction (c)	4.08	3.58	3.16	3.05	15.7	14.0	12.1	11.8
Abstraction by weighted averaging 1	3.32	3.18	3.05	3.01	12.9	12.3	11.9	11.1
Abstraction by weighted averaging 2	3.71	3.39	3.14	3.06	14.5	13.4	12.4	12.2

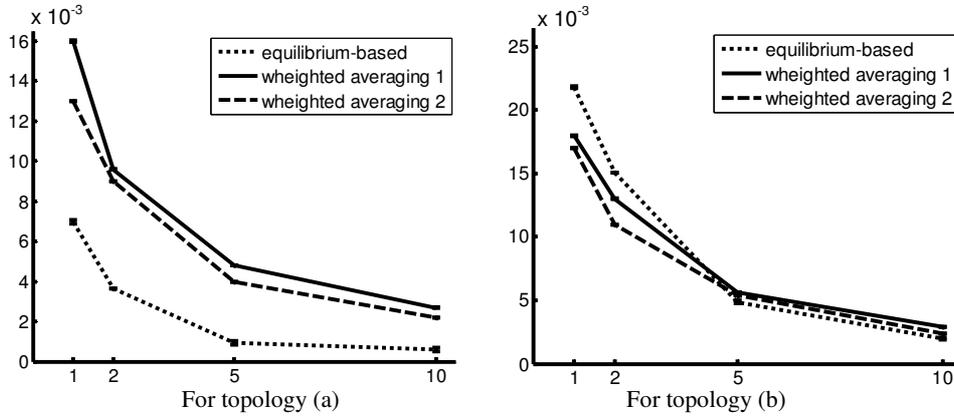
The fastest simulation models are obtained by the abstraction by first-order weighted averaging. The slowest are the models obtained by the equilibrium-based abstraction. However, as one can see from Tables 1 and 2, the ratio of the simulation time of the slowest to the fastest abstraction method is less than 1.3 for all cases. The impact of the number of messages on the simulation time is stronger for the equilibrium-based method than for the weighted averaging methods. This is because (large) systems of linear equations need to be solved in the former methods every time when the structure of a group changes. The greatest decrease of the acceleration

rate for the equilibrium-based method for the settings considered in the paper is of the order of  $1.4$ .

The time ratio between first and second order weighted averaging is at most  $1.1$ . Thus, adding the second level in the network of influence for each agent does not result in a significant increase of simulation time, even for larger numbers of agents.



**Fig. 6.** Mean approximation errors for the proposed abstraction methods for 50 agents; the horizontal axis is the average time between messages.



**Fig. 7.** Mean approximation errors for the proposed abstraction methods for 100 agents; the horizontal axis is the average time between messages.

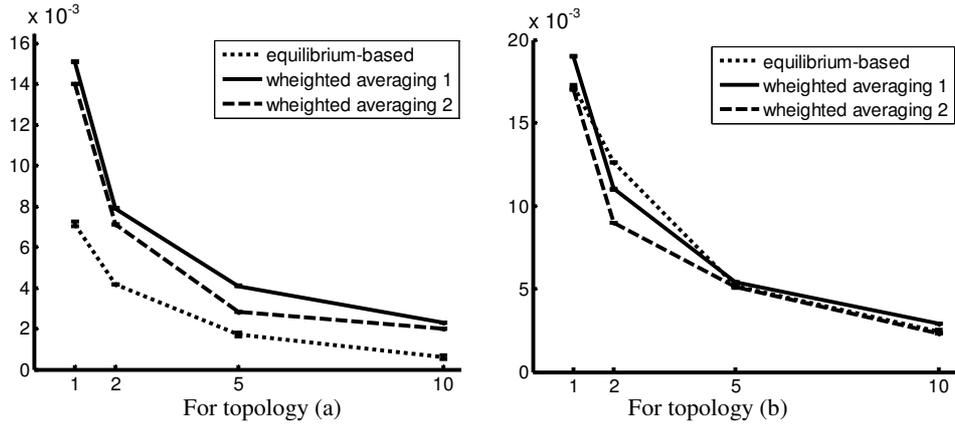
## 5.2 Approximation errors

The error of approximation of the original model by a group abstraction method is defined as

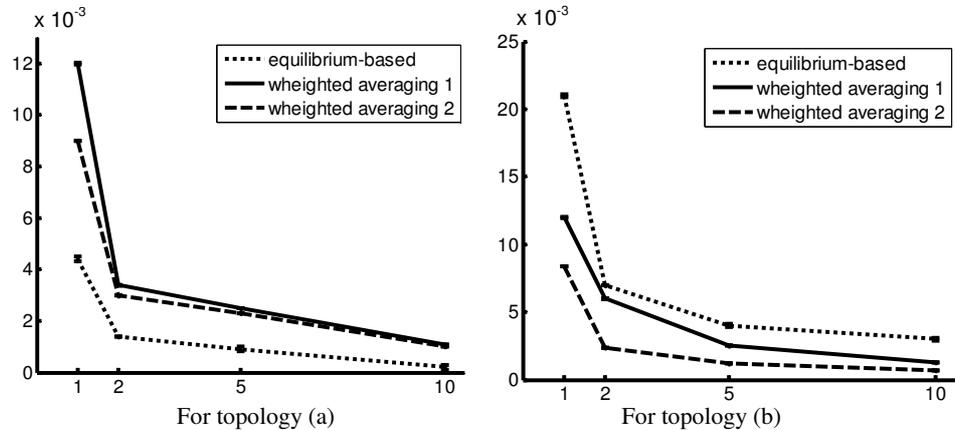
$$\sum_{t \in [31, 1031]} (|GI^{o,t} \cup GI^{a,t}| - |GI^{o,t} \cap GI^{a,t}|) / 1000,$$

where  $GI^{o,t}$  is the group comprising the agents supporting decision option  $sI$  at time point  $t$  according to the original model, and  $GI^{a,t}$  is the group of the agents supporting  $sI$  at time point  $t$  according to the abstracted model.

A comparison of the mean approximation errors for the proposed abstraction methods is provided in Figures 6-9. The variances of the errors are low (of the order  $10^{-6}$ ); they are depicted by small error bars in the figures.



**Fig. 8.** Mean approximation errors for the proposed abstraction methods for 200 agents; the horizontal axis is the average time between messages.



**Fig. 9.** Mean approximation errors for the proposed abstraction methods for 500 agents; the horizontal axis is the average time between messages.

As can be seen from Figures 6-9, both the equilibrium-based and weighted averaging methods are sensitive to the average time between messages from external sources. In particular, for topology (a), when the average time is high (10) the error of the equilibrium-based abstraction is very low: in average less than one agent is placed in a wrong group for 1000 time points. However, when the external world interacts with each group every time point, the error of the equilibrium-based abstraction grows

significantly: 11 times in the worst case for topology (a). Nevertheless, the maximal error of the equilibrium-based abstraction is still rather low:  $7.9 \cdot 10^{-3}$  (meaning that for topology (a) less than 8 agents are placed in a wrong group for 1000 time points). As can be seen from the results, the abstraction methods by weighted averaging perform significantly worse than the equilibrium-based abstraction for the topology (a).

For topology (b) the rate of stabilization of the system is generally slower than for topology (a). Furthermore, the agents of the system do not converge to the same state. For these reasons the approximation errors for (b) are higher than for (a). Surprisingly, although the exact equilibrium state of the system can be determined by the equilibrium-based abstraction method, this method performs comparably to or even worse than the weighted averaging methods for topology (b). However, the greater the average time between the messages, the better the equilibrium-based method performs. This can be explained by the dynamics of convergence of the group to a stable state: the greater the time between the messages, the more closely the group approaches an equilibrium state, thus the smaller the approximation error of the equilibrium-based method. In particular, for the average time 10 the equilibrium-based method performs in average better than the weighted averaging methods.

The weighted averaging methods are less sensitive to the rate of convergence, but also less precise, as they are based on an approximation of the group's emergent property. The approximation error grows with frequency of external messages, since every message results into a decision re-evaluation by the agents, and thus the error accumulates. For topology (b) the group approaches its equilibrium states slowly, thus the equilibrium-based method is less suitable.

The rate of convergence is also the reason why the approximation error of the equilibrium-based method is less when the ratio of the time scale of the external world dynamics to the time scale of the group's internal dynamics is higher. The greater the difference in the scales, the closer a group approaches an equilibrium, which can be calculated precisely using the equilibrium-based method. The error depends on how often an equilibrium state of a group is disturbed by external messages and on how quickly the group reaches a new equilibrium.

As expected, the abstraction by second-order weighted averaging is more precise than the abstraction by first-order weighted averaging. The difference in precision between first- and second-order weighted averaging depends on the density of connections in the topology of a group: in general, the higher the density, the less the error difference between both variants. This is because the density determines how many direct neighbors an agent has, and thus, how many agents are influenced directly by one-hop message propagation of new information. The more densely a graph is connected, the more agents in a group new information reaches by one-hop propagation, and the more fully the new group's state can be captured by first-order weighted averaging. The less the graph's density, the more information about the group dynamics each additional hop provides. In a sparsely connected graph, one-hop propagation reaches only a small number of agents, thus, only partial information about the group dynamics can be extracted by first-order weighted averaging. In this case the difference between first- and second-order weighted averaging may be significant. For the experiments considered in this paper densely connected groups were used. Furthermore, as can be seen from the results, the

abstraction by weighted averaging becomes more precise with the increase of the number of agents.

## 6 Conclusions

In the paper an approach is proposed to handle complex dynamics of large-scale agent-based social diffusion models. On the one hand this approach allows identifying global, emergent properties of groups of agents. On the other hand, it enables a significant increase of the computational efficiency of automated analysis of large-scale social diffusion models (up to 33 times for larger numbers of agents).

The approach comprises two methods dedicated for abstraction of a variety of topologies of social groups. In particular, the equilibrium-based method is well suited for models with topologies of type (a). The higher the ratio of the time scale of the external world dynamics to the time scale of the group's internal dynamics, the less the approximation error of the equilibrium-based method. The high ratio of the scales is also required to reduce the approximation error of the equilibrium-based abstraction of models with topologies of type (b). For low ratios, especially for large groups of agents, the second-order weighting averaging approach is the most suitable.

Note that in many applications the sizes of dynamic groups, which could be numerous, are (much) smaller than the total number of agents. The developed abstraction techniques were applied in a large-scale crowd evacuation study (~10000 agents) [11]. Although the number of agents was significant, the maximal size of emergent dynamic groups was 174.

Social diffusion models have been studied extensively [3, 4, 8, 9]. A common research question of these studies is about the existence of equilibrium states of a model for different topologies. In contrast to the continuous model considered in the paper, most of other studies consider binary, threshold-based models. The authors are not aware of any existing studies focusing on the behavioral abstraction of continuous social diffusion models.

Currently several techniques for abstraction of models based on hybrid automata [1] and differential equations [2] exist. However, such approaches can be applied efficiently for systems described by sparse matrixes. Social diffusion models represent tightly connected systems, which do not allow a significant reduction of the state space using such techniques. In particular, a previous study showed that common model reduction techniques such as balanced truncation [2] do not allow decreasing the rank of the matrix describing the model from Section 2.

In the future it will be investigated whether the developed approach can be applied for abstracting more complex cognitive multi-agent systems, involving interaction between cognitive and affective processes (e.g., collective decision making with emotions and trust).

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