Delft University of Technology
Faculty of Civil Engineering and Geosciences
Structural Mechanics Section

Write your <u>name</u> and <u>study number</u> at the top right-hand of your work.

Exam CIE4143 Shell Analysis
Monday 3 July 2017, 13:30 – 16:30 hours

Problem 1

Explain shortly the meaning of the following eight words in relation to shells structures.

Ellipsoid
Hypar
Theorema Egregium
Orthogonal parameterisation
Sagitta
Tractricoid
Semiloof
Extensional deformation

Problem 2

a Figure 1 shows a part of a shell and edge beam with coordinate system. Draw in this figure the shell loading, membrane forces, moments and shear forces in the positive directions. (You can hand in this page with your answers. You do not need to draw the forces and moments in the edge beam.)

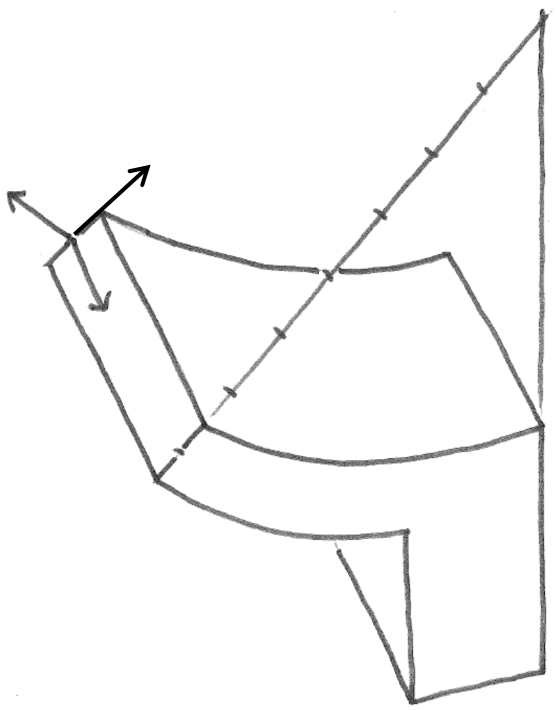


Figure 1. Part of a shell and edge beam

b Suppose we have analysed a shell structure with the finite element method. Subsequently, we rotate the local coordinate system 180° around the *y* axis. Which of the following quantities change sign? $k_{xx}, k_{yy}, k_{xy}, p_x, p_y, p_z, n_{xx}, n_{yy}, n_{xy}, n_{yx}, m_{xx}, m_{yy}, m_{xy}, q_x, q_y$

c There are several coordinate systems used in shell analysis. Which are these?

d Why do we add shape imperfections to a shell finite element model? Do we add the same imperfections for a linear elastic analysis, a linear buckling analysis and a geometrical and physical nonlinear analysis? Explain your answer.

Problem 3

The following deformation is in-extensional when $k_{xy} = 0$ and $\frac{l_x}{l_y} = \sqrt{\frac{-k_{yy}}{k_{xx}}}$

$$u_x = \frac{\pi}{l_x} \cos\frac{\pi x}{l_x} \sin\frac{\pi y}{l_y}$$

$$u_y = -\frac{\pi}{l_y} \sin\frac{\pi x}{l_x} \cos\frac{\pi y}{l_y}$$

$$u_z = -\left(\frac{\pi}{l_x}\right)^2 \frac{1}{k_{xx}} \sin\frac{\pi x}{l_x} \sin\frac{\pi y}{l_y}$$

a What is in-extensional deformation?

b How can be proven that the above deformation is in-extensional indeed?
(A short explanation in words please.)

c Consider a curved panel with a length l_x and a width l_y .
What boundary conditions do not prevent this in-extensional deformation?

d Remarkable about this deformation is that it is in-extensional for any curvature k_{xx} and k_{yy} , however, for positive Gaussian curvatures it is imaginary and cannot be plotted. What conclusion can be drawn from this?

Problem 4

a What are the names of the shapes in which thin shells buckle?

b Why is 1/6 a recommended knockdown factor? Choose A, B, C or D.

A It has been legally agreed upon in the eurocode.
B It corresponds to Yokohama failure.
C It provides a lower limit to 172 tests of aluminium cylinders.
D The solutions to the differential equation shows that this knockdown factor is guaranteed safe.

c What is a good way of adding imperfections to a finite element shell model?
Choose A, B, C or D.

A Randomly displace several nodes perpendicular to the surface over a distance δ.
B Add the first buckling mode to the shell shape with an amplitude δ.
C Subtract the deformation from the shell shape with an amplitude δ.

D Add $\delta \cos(\pi x/l) \cos(\pi y/l)$ to the z coordinate of the shell shape, where l is the buckling length

d Well designed shell structures have very small moments because pressure lines are corrected by hoop forces. How is this called? Choose A, B, C or D.

A in-extensional deformation
B middle third rule
C imperfection sensitivity
D edge disturbance

e Most shell structures have locations where the moments are much larger than elsewhere. How are these called? Choose A, B, C or D.

A in-extensional deformation
B middle third rule
C imperfection sensitivity
D edge disturbance

f The stiffness of negatively and positively curved shell structures to perpendicular point loads is proportional to … Choose A, B, C or D.

A $\frac{1}{2} k_G$
B k_G^2
C $\sqrt{|k_G|}$
D $|k_G|$

Problem 5

A structure consists of a spherical shell and a flat plate roof. The flat plate roof is 200 mm thick. The curved shell is 20 mm thick. The structure is loaded by just self-weight (2500 kg/m^3). The bottom edge is fixed. The principal membrane forces are shown in figure 2. Will this shell buckle? Explain your answer.

span	$l = 8$ m
radius of curvature	$a = 4.5$ m
thickness	$t = 0.020$ m
Young's modulus	$E = 30.5\ 10^9$ N/m^2
mass density	$\rho = 2500$ kg/m^3
gravitational acceleration	$g = 9.8$ m/s^2
theoretical buckling force	$n_{cr} = 0.6\ E\ t^2 / a$
knockdown factor	$C = 1/6$

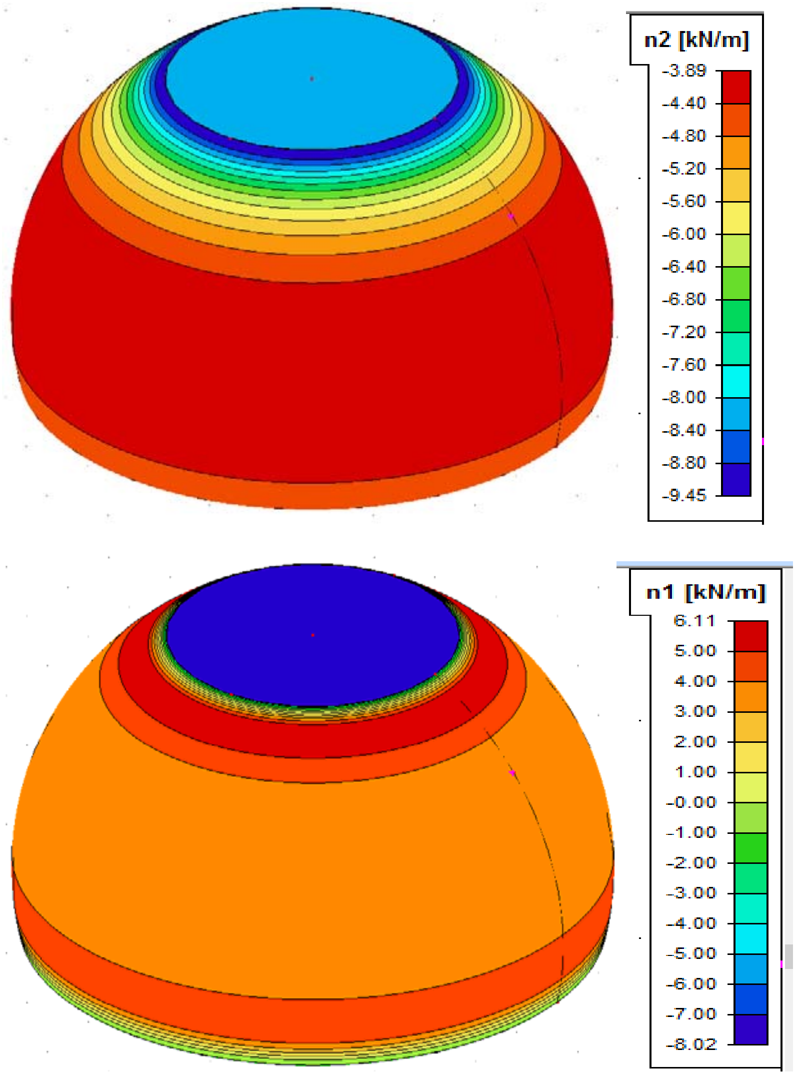


Figure 2. Spherical shell with a circular flat part loaded by self-weight

Problem 6

A timber grid shell consists of curved laths. If the laths are curved too much they would break during construction. Therefore, the curvature of a lath should be less than the following value.

$$|k| \leq \frac{2f}{Eh}$$

where

k is the curvature of the lath,
f is the breaking stress of the timber,
E is the Young's modulus of the wood and
h is the lath thickness.

a Derive this design rule.

For a curved lath in the local x-direction the largest bending stress is σ_{xx}. This lath is can also be twisted and the largest shear stress is σ_{xy}.

$$\sigma_{xx} = \tfrac{1}{2} E\, h\, k_{xx}$$
$$\sigma_{xy} = \tfrac{1}{2} E\, h\, k_{xy}$$

The unity check for the combined stress is

$$(\frac{\sigma_{xx}}{f_b})^2 + (\frac{\sigma_{xy}}{f_s})^2 \le 1$$

We assume that the bending strength f_b is the same as the shear strength f_s.

b For which grid shell shapes is the unity check independent of the direction of the laths? Explain your answer.

Consider two curved laths perpendicular to each other. One lath is in the local x direction; the other lath is in the local y direction. The unity check value of a lath is a measure for the amount of material required. The summation of both unity check values is a measure for the total amount of material required in one point of the grid shell.

$$\mu_{sum} = (\frac{\sigma_{xx}}{f_b})^2 + 2(\frac{\sigma_{xy}}{f_s})^2 + (\frac{\sigma_{yy}}{f_b})^2$$

We still assume that the bending strength f_b is the same as the shear strength f_s.

c For which grid shell shapes is the total amount of material required independent of the direction of the laths? Explain your answer.

Problem 7

Suppose that a finite element analysis has been made of a shell structure. The analysis is repeated two times with smaller elements. The table shows the results.

element size	*deflection*	*computation time*	*hard disk space used*
300 mm	23.0 mm	11 minutes	12 GB
150	26.1	80	45
75	27.6	630	173

a What is an accurate estimate of the exact deflection of this shell. Explain your answer.

b What is the order of the error of the computed deformation of this element type? Explain your answer.

c Is the last computation sufficiently accurate. Explain your answer.

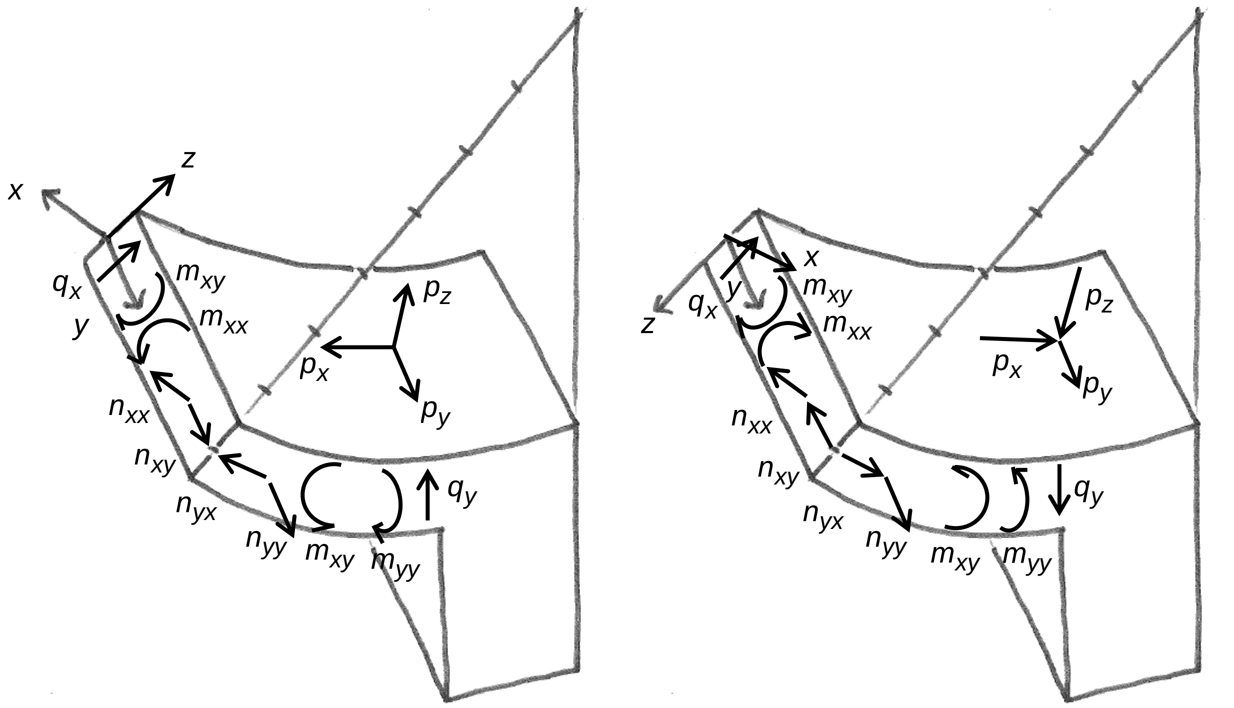
d What can be a practical limitation of obtaining more accuracy?

Answers to problem 1

Ellipsoid …………………………	3D shape	$\bar{x} = a\sqrt{\dfrac{a^2 - B}{a^2 - c^2}}\cos u$ $\quad A = a^2 \sin^2 u + b^2 \cos^2 u$ $\bar{y} = b\cos v \sin u$ $\quad B = b^2 \sin^2 v + c^2 \cos^2 v$ $\bar{z} = c\sqrt{\dfrac{A - c^2}{a^2 - c^2}}\sin v$ $\quad a \ge b \ge c > 0$
Hypar ………………………......	hyperbolical paraboloid; 3D shape	$z = \frac{1}{2}k_{xx}x^2 + k_{xy}xy + \frac{1}{2}k_{yy}y^2$ $k_{xx}k_{yy} - k_{xy}^2 < 0$
Theorema Egregium …………	"If the deformation of a shell is in-extensional then the Gaussian curvature does not change."	
Orthogonal parameterisation …	mathematical representation of a 3D shape; $\bar{x}$, $\bar{y}$ and $\bar{z}$ as function of *u* and *v*. The lines *u* is constant and *v* is constant are perpendicular.	
Sagitta …………………………..	arrow; height of an arch; also used for shells	
Tractricoid …………………….	3D shape	$\bar{x} = a(\cos u + \ln\tan\frac{u}{2})$ $\bar{y} = a\sin u \sin v$ $\bar{z} = a\sin u \cos v$
Semiloof ………………………..	shell finite element developed by Irons	
Extensional deformation ………	deformation in which the middle surface of a shell stretches; either ε_{xx}, ε_{yy} or γ_{xy} are not equal to zero.	

Answers to problem 2

a



b $k_{xx}, k_{yy}, p_x, p_z, n_{xy}, n_{yx}, m_{xx}, m_{yy}, q_y$

c - global coordinate system $\bar{x}, \bar{y}, \bar{z}$
- local coordinated system x, y, z
- curved coordinate system u, v, z
- finite element coordinate system
- integration point coordinate system

d Because shell buckling is very sensitive to shape imperfections.
Linear elastic analyses do not need shape imperfections. They would not make a difference.
Linear buckling analyses do not need shape imperfections. They would make some difference but not enough. The reason is that imperfections grow when the load is incremented and linear buckling analysis does not account for this.
Only in geometrical nonlinear analyses shape imperfections are introduced.

Answers to problem 3

a Strains in the middle surface are zero. $\varepsilon_{xx} = \varepsilon_{yy} = \gamma_{xy} = 0$

b Substitution in Sanders-Koiter equations 13, 14 and 15.

c $x = 0$ or $x = l_x \Rightarrow u_z = u_y = n_{xx} = m_{xx} = 0$
$y = 0$ or $y = l_y \Rightarrow u_z = u_x = n_{yy} = m_{yy} = 0$
These are called diaphragm boundary conditions.

d Apparently, this in-extensional deformation does not exist for positive Gaussian curvatures. Actually, for $k_G = 0$ this deformation is not physical either ($l_x / l_y = \infty$ or 0). Nevertheless, experience shows that positive Gaussian curvatures do have in-extensional deformations. Perhaps with a small adaptation the present formulation can be made real for positive Gaussian curvatures.

Answers to problem 4

a ring pattern -> elephant foot; chess board pattern -> Yoshimura pattern
b C
c B
d B
e D
f C

Answer to problem 5

$$n_{cr} = -0.6 \frac{Et^2}{a} = -0.6 \frac{30.5\,10^9 \times 0.020^2}{4.5} = -1.627\,10^6 \text{ N/m}$$

$$n_{ult} = \tfrac{1}{6} n_{cr} = -271000 \text{ N/m} = -271 \text{ kN/m} \ll \text{-9.45 kN/m}$$

So, it does not buckle.

Answers to problem 6

a $\sigma = \frac{M}{W} = \frac{EI\kappa}{W} = \frac{E\frac{1}{12}bh^3k}{\frac{1}{6}bh^2} = \frac{Ehk}{2} \le f \Rightarrow k \le \frac{2f}{Eh}$

Curvature can be negative or positive so

$|k| \le \frac{2f}{Eh}$ Q.E.D.

b $(\frac{\sigma_{xx}}{f_b})^2 + (\frac{\sigma_{xy}}{f_s})^2 \le 1$

$(\frac{\frac{1}{2}E\,h\,k_{xx}}{f})^2 + (\frac{\frac{1}{2}E\,h\,k_{xy}}{f})^2 \le 1$

$(\frac{Eh}{2f})^2(k_{xx}^2 + k_{xy}^2) \le 1$

Consider a <u>sphere</u>. So $k_{xy} = 0$, $k_{xx} = k_{yy} = k$ for any direction.

$k_{xx}^2 + k_{xy}^2 = k^2 = k_G$

Since k_G does not change for rotations of the local coordinate system (invariant) the unity check is invariant for spheres.

Consider a <u>hypar</u>. So $k_{xx} = -k_{yy}$ for any direction.

$k_{xx}^2 + k_{xy}^2 = -k_{yy}k_{xx} + k_{xy}^2 = -k_G$

Since k_G is invariant the unity check is invariant for hypars.

c $\mu_{sum} = (\frac{\sigma_{xx}}{f_b})^2 + 2(\frac{\sigma_{xy}}{f_s})^2 + (\frac{\sigma_{yy}}{f_b})^2$

$\mu_{sum} = (\frac{\frac{1}{2}E\,h\,k_{xx}}{f})^2 + 2(\frac{\frac{1}{2}E\,h\,k_{xy}}{f})^2 + (\frac{\frac{1}{2}E\,h\,k_{yy}}{f})^2$

$\mu_{sum} = (\frac{\frac{1}{2}E\,h}{f})^2 \left(k_{xx}^2 + 2k_{xy}^2 + k_{yy}^2\right)$

Since $k_{xx}^2 + 2k_{xy}^2 + k_{yy}^2$ is invariant (see reader p. 24) μ_{sum} is invariant for <u>any grid shell shape</u>.

Answers to problem 7

a $27.6 + (27.6 - 26.1) = 29.1$ mm

b O(h) because $\log_2 \frac{26.1 - 23.0}{27.6 - 26.1} = 1.04$

c Any good argument for 1.5 mm being large or small.

d Computation time or the available hard disk space.