Two-dimensional controlled-source electromagnetic interferometry by multidimensional deconvolution: spatial sampling aspects

Jürg Hunziker1*, Evert Slob1, Yuanzhong Fan3, Roel Snieder2 and Kees Wapenaar1

1Department of Geoscience and Engineering, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands, 2Center for Wave Phenomena, Colorado School of Mines, Golden CO, USA, and 3formerly Colorado School of Mines, now at Shell International Exploration and Production, Houston TX, USA

Received August 2010, revision accepted August 2011

ABSTRACT

We use numerically modelled data sets to investigate the sensitivity of electromagnetic interferometry by multidimensional deconvolution to spatial receiver sampling. Interferometry by multidimensional deconvolution retrieves the reflection response below the receivers after decomposition of the fields into upward and downward decaying fields and deconvolving the upward decaying field by the downward decaying field. Thereby the medium above the receiver level is replaced with a homogeneous half-space, the sources are redatumed to the receiver level and the direct field is removed. Consequently, in a marine setting the retrieved reflection response is independent of any effect of the water layer and the air above. A drawback of interferometry by multidimensional deconvolution is a possibly unstable matrix inversion, which is necessary to retrieve the reflection response. Additionally, in order to correctly separate the upward and the downward decaying fields, the electromagnetic fields need to be sampled properly. We show that the largest possible receiver spacing depends on two parameters: the vertical distance between the source and the receivers and the length of the source. The receiver spacing should not exceed the larger of these two parameters. Besides these two parameters, the presence of inhomogeneities close to the receivers may also require a dense receiver sampling. We show that by using the synthetic aperture concept, an elongated source can be created from conventionally acquired data in order to overcome these strict sampling criteria. Finally, we show that interferometry may work under real-world conditions with random noise and receiver orientation and positioning errors.

Key words: Electromagnetics, Numerical study.

1 INTRODUCTION

In geophysical exploration, seismic interferometry is known as the process of cross-correlating recordings at two receiver positions to retrieve the Green’s function between these two receivers. The theory has been derived by various authors for lossless (Schuster et al. 2004; Wapenaar 2004) and dissipative media (Snieder 2006), and it has been applied in passive (Draganov et al. 2006) as well as in controlled-source seismics (Bakulin and Calvert 2006). A more complete overview on seismic interferometry can be found in Wapenaar et al. (2010a,b) or Schuster (2009). The concept of Green’s function retrieval by cross-correlation has been extended also to electromagnetic waves (Slob et al. 2007a) and is called electromagnetic interferometry. For electromagnetic diffusive fields, interferometry by cross-correlation requires sources to
be present throughout the whole volume of interest (Snieder 2006), which is not practical. Therefore, another approach for interferometry with diffusive fields is needed.

It has been shown that the process of cross-correlation (CC) can be replaced by a multidimensional deconvolution (MDD) in both the controlled-source case (Wapenaar et al. 2008a) and the passive case (Wapenaar et al. 2008b). The advantages of multidimensional deconvolution include elimination of the source signature, improved radiation characteristics of the retrieved source, and relaxation of the assumption of a lossless medium. Furthermore, interferometry by multidimensional deconvolution works with one-sided illumination, whereas interferometry by cross-correlation requires a closed surface of sources. On the other hand, multidimensional deconvolution is more expensive and the involved matrix inversion may be unstable. Furthermore, in most cases a decomposition of the measured fields into up- and downgoing fields is necessary, which requires acquisition with multi-component receivers on a sufficiently dense grid. Interferometry by multidimensional deconvolution also requires an array or a network of sensors and cannot be done with two receivers only, in contrast to interferometry by cross-correlation. A similar approach to eliminate the overburden from multicomponent seismic data was presented by Holvik and Amundsen (2005).

Controlled-Source Electromagnetic (CSEM) data are often collected in a marine environment for oil exploration. In this technique a horizontal electric-dipole source is towed behind a boat emitting a low-frequency electric field, which diffuses through the water and through the subsurface. The field that diffuses above the source in the sea decays in the upward direction, while the field that diffuses below the source in the sea decays in the downward direction. When the diffusing field experiences changes in the electric conductivity or magnetic permeability, the reflected diffusive field decays in the direction opposite to the corresponding incident diffusing field. This means that after some time at any level in the subsurface diffusive fields occur that decay in the upward direction and in the downward direction. The resulting EM-field is recorded at the ocean bottom by horizontal multicomponent receivers as a function of offset (Constable and Snoka 2007). Unfortunately, not only subsurface responses are recorded but also the direct field in the water and interactions of the electric field with the sea surface are registered.

One strong event is the so-called airwave. It consists of that part of the electromagnetic field which diffuses vertically upward from the source to the sea surface, continues in the air as a propagating wave with the speed of light along the sea surface, refracts continuously back into the sea as a vertically downward diffusing field, and eventually is recorded at the receivers (Um and Alumbaugh 2007). The field amplitude decays exponentially with distance in the sea and in the earth, whereas in the air the amplitude decays proportional to the inverse of distance to the power 3 (Wait 1959; Bannister 1984; King et al. 1992; Amundsen et al. 2006; Slob et al. 2010). The direct field contributions at the receiver decay therefore exponentially with offset, while the airwave contribution decays exponentially as a function of two-way depth from source to water surface and back to the receiver. Therefore, at horizontal source-receiver offsets that are small compared to the two-way source-receiver depth, the recorded field is dominated by the direct field and reflections from the sea surface. On the other hand, in shallow seas and at relatively large offsets, the airwave is strong (Amundsen et al. 2006).

Since the airwave does not contain any information about the subsurface, one aims to minimize the effect of the airwave. Various approaches like modelling and adjacent subtraction (Nordskag and Amundsen 2007), weighted differences between pairs of receivers or sources (Chen and Alumbaugh 2009; Loseth et al. 2010), taking the frequency derivative (Chen and Alumbaugh 2009; Maaø and Nguyen 2010) or a difference between fields at two frequencies that are further apart (Wirianto et al. 2011), or magnetotelluric impedance stripping (Chen and Alumbaugh 2009) exist. Assuming that only the upward decaying field at the receiver level bears information about the subsurface and that the field which interacted with the air-water interface is downward decaying at the receiver level, Amundsen et al. (2006) proposed to decompose the diffusive fields into upward and downward decaying fields in order to remove the effects related to the air-water interface. They demonstrated this approach on computed data and on measured data. However, all interactions across the receiver level, e.g., the diffusive electric field interacting first with the air-water interface and subsequently with the reservoir, are not removed with this approach, which can be a problem in shallow water. Another approach to deal with the airwave is to carry out CSEM measurements in the time-domain instead of the frequency-domain, where the airwave is easier to separate from the subsurface response (Weiss 2007; Andrés and MacGregor 2007).

Wapenaar et al. (2008a) and Nordskag et al. (2009) proposed to deconvolve the upward decaying field by the downward decaying field, both just below the receiver level. This can be applied to wave fields in dissipative media as well as to diffusive fields. This field decomposition followed by deconvolution was coined interferometry by multidimensional deconvolution (multidimensional deconvolution) by

© 2011 European Association of Geoscientists & Engineers, Geophysical Prospecting, 60, 974–994
Wapenaar et al. (2008a) and Lorentz water-layer elimination by Nordskag et al. (2009). We use here the term interferometry by multidimensional deconvolution. The retrieved response is a response of the scattered field, where scattered field should be interpreted as the field that would exist if the medium above the receiver level were a homogeneous half-space and had the same properties as the medium just below the receiver level where the decomposition has been carried out. For this reason, all the interactions of the electromagnetic field with the air-water interface are removed from the recorded data, including the effects of the wave action of the surface sea waves. Consequently, also the in shallow-marine CSEM so problematic airwave is removed. This includes the above described interactions across the receiver level which are not removed with the approach of Amundsen et al. (2006).

Another advantage of multidimensional deconvolution, besides the removal of the airwave, is that the locations and orientations of the actual sources do not need to be known. The imprint of the actual sources is removed from the data, and the receivers act as virtual sources. Consequently, it does not matter anymore if the source is towed close to the receivers at the ocean bottom or further above, which allows a more flexible survey design. In time-lapse CSEM for monitoring purposes, it is feasible to position the receivers at the same location in a second survey but it is almost impossible to position the source at the same location. This problem of repositioning the source at the same location can be circumvented by interferometry by multidimensional deconvolution, which simply redatums the source from its unknown position to a receiver location.

Hence, by applying interferometry by multidimensional deconvolution to CSEM data, the source is redatums to the receiver level, the direct field is eliminated, and the water layer is replaced by a homogeneous half-space that has the same properties as the medium directly below the receiver level (Wapenaar et al. 2008a). Note that interferometry by multidimensional deconvolution is a data driven method. Only the material parameters just below the sea bottom, i.e., in most situations at an infinitesimal small step below the sea-sediment interface, are necessary for the decomposition. However, when there is a very thin layer below the receivers followed by a thicker layer, it can be advantageous to use the parameters of the thicker layer. In any case, no knowledge of the water layer is required. Drawbacks of this technique are the possibly unstable matrix inversion and the requirement for properly sampled data, i.e., regularly and dense enough sampled at all offsets. A numerical example of interferometry by multidimensional deconvolution applied to CSEM data was given in Slob et al. (2007b), where all sea surface related effects were removed from the data resulting in a much clearer response of the target reservoir layer. A similar approach to remove the airwave from CSEM data is proposed by van den Berg et al. (2008) but in their algorithm the decomposition is carried out in the water layer. Consequently, the water layer is not removed, only the air-water interface is eliminated.

The retrieved reflection response can, for example, be used as a direct hydrocarbon indicator by normalizing it with a reflection response based on data acquired above a region without a reservoir. When a reservoir is present in one of the datasets, the resulting ratio is, for a laterally invariant reservoir, larger than one at all offsets. For standard CSEM data, this is only the case at intermediate offsets because at short offsets the data are dominated by the direct field and at large offsets by the airwave. A proper sampling is not necessary if the CSEM data is used as a direct hydrocarbon indicator but is most useful in a scheme for monitoring purposes, it is feasible to position the receivers at the same location in a second survey but it is almost impossible to position the source at the same location. This problem of repositioning the source at the same location can be circumvented by interferometry by multidimensional deconvolution, which simply redatums the source from its unknown position to a receiver location.

Hence, by applying interferometry by multidimensional deconvolution to CSEM data, the source is redatums to the receiver level, the direct field is eliminated, and the water layer is replaced by a homogeneous half-space that has the same properties as the medium directly below the receiver level (Wapenaar et al. 2008a). Note that interferometry by multidimensional deconvolution is a data driven method. Only the material parameters just below the sea bottom, i.e., in most situations at an infinitesimal small step below the sea-sediment interface, are necessary for the decomposition. However, when there is a very thin layer below the receivers followed by a thicker layer, it can be advantageous to use the parameters of the thicker layer. In any case, no knowledge of the water layer is required. Drawbacks of this technique are the possibly unstable matrix inversion and the requirement for properly sampled data, i.e., regularly and dense enough sampled at all offsets. A numerical example of interferometry by multidimensional deconvolution applied to CSEM data was given in Slob et al. (2007b), where all sea surface related effects were removed from the data resulting in a much clearer response of the target reservoir layer. A similar approach to remove the airwave from CSEM data is proposed by van den Berg et al. (2008) but in their algorithm the decomposition is carried out in the water layer. Consequently, the water layer is not removed, only the air-water interface is eliminated.

The retrieved reflection response can, for example, be used as a direct hydrocarbon indicator by normalizing it with a reflection response based on data acquired above a region without a reservoir. When a reservoir is present in one of the datasets, the resulting ratio is, for a laterally invariant reservoir, larger than one at all offsets. For standard CSEM data, this is only the case at intermediate offsets because at short offsets the data are dominated by the direct field and at large offsets by the airwave. A proper sampling is not necessary if the CSEM data is used as a direct hydrocarbon indicator but is most useful in a scheme for monitoring purposes, it is feasible to position the receivers at the same location in a second survey but it is almost impossible to position the source at the same location. This problem of repositioning the source at the same location can be circumvented by interferometry by multidimensional deconvolution, which simply redatums the source from its unknown position to a receiver location.
size of sources and receivers. The reservoir response is smooth in the space domain and contains few high-wavenumber components because the reservoir is at a relatively large vertical distance from the receivers and the field is diffusive in nature, for which reason field components at large wavenumbers attenuate faster than those at small wavenumbers. For practical reasons, receiver stations need to be sparsely used, hence the vertical source-receiver distance and the source length are of interest because these can be chosen within practical limits.

In this paper we investigate the required receiver spacing for a successful up-down decomposition and subsequent multidimensional deconvolution using numerical examples for the conventional situation, where the source is vertically close to the receivers, and for the alternative situation, where the source is vertically far away from the receivers. We then present how synthetically elongated sources can be created in order to apply interferometry by multidimensional deconvolution to datasets acquired with current acquisition techniques. Finally, noise issues are discussed to show the potential applicability of interferometry by multidimensional deconvolution under real conditions.

2 THEORY

Ampère’s law and Faraday’s law from Maxwell’s equations are given in the space-frequency domain as

\[
jo\varepsilon E_i + \partial_i H_k = -\hat{J}^e, \tag{1}
\]

\[
jo\mu H_k + \partial_i E_i = -\hat{J}^m, \tag{2}
\]

where \( E_i = E_i(x, \omega) \) is the electric field strength, \( H_k = H_k(x, \omega) \) the magnetic field strength, and \( \hat{J}^e = \hat{J}^e(x, \omega) \) and \( \hat{J}^m = \hat{J}^m(x, \omega) \) are external electric and magnetic source functions, respectively. The vector \( \mathbf{x} \) represents the three spatial coordinates. The material parameters electric permittivity, magnetic permeability, and conductivity are given by \( \varepsilon = \varepsilon(x, \omega), \mu = \mu(x, \omega), \) and \( \sigma = \sigma(x, \omega), \) respectively, \( \omega \) is the angular frequency and \( j \) the imaginary unit. The circumflex denotes the space-frequency domain. The Levi-Civita tensor \( \varepsilon_{ijk} \) is equal to 1 for \( \varepsilon_{123}, \varepsilon_{312}, \) and \( \varepsilon_{231}, \) equal to \(-1\) for \( \varepsilon_{213}, \varepsilon_{132}, \) and \( \varepsilon_{321}, \) and 0 otherwise. The spatial derivatives \( \partial_i \) are written in a short form as \( \partial_i \). Latin subscripts can have the values 1, 2, or 3, representing the three spatial coordinates. Greek subscripts, on the other hand, can only have the values 1 or 2, representing the two horizontal coordinates. Einstein’s summation convention is used for repeated subscripts.

Since we are dealing with diffusive fields, we neglect the term \( jo\varepsilon E_i \) in equation (1). Next, the horizontal field components are separated from the vertical ones. At the receiver level, the resulting set of equations is combined such that the vertical components \( \hat{E}_3 \) and \( \hat{H}_3 \) are eliminated. This leads to the following matrix equation:

\[
\partial_t \hat{Q} = \hat{A}\hat{Q} + \hat{D}, \tag{3}
\]

where \( \hat{Q} = (\hat{E}_1, \hat{E}_2, \hat{H}_2, -\hat{H}_1)^T \) contains the horizontal field components. The superscript \( ^T \) indicates transposition. The operator matrix \( \hat{A} \) is defined as

\[
\hat{A} = \begin{pmatrix}
0_2 & \hat{A}_{12} \\
\hat{A}_{21} & 0_2
\end{pmatrix}, \tag{4}
\]

where \( 0_2 \) is a 2 by 2 null matrix and the matrices \( \hat{A}_{12} \) and \( \hat{A}_{21} \) are given by

\[
\hat{A}_{12} = \begin{pmatrix}
-j\omega\hat{\mu} + \hat{\sigma}_1 \left( \frac{1}{\varepsilon_1} \partial_1 \right) & \hat{\sigma}_1 \left( \frac{1}{\varepsilon_1} \partial_1 \right) \\
\hat{\sigma}_1 \left( \frac{1}{\varepsilon_1} \partial_1 \right) & -j\omega\hat{\mu} + \hat{\sigma}_2 \left( \frac{1}{\varepsilon_2} \partial_2 \right)
\end{pmatrix}, \tag{5}
\]

\[
\hat{A}_{21} = \begin{pmatrix}
-j\hat{\sigma} + \hat{\sigma}_2 \left( \frac{1}{\mu_1} \partial_2 \right) & \hat{\sigma}_2 \left( \frac{1}{\mu_1} \partial_2 \right) \\
\hat{\sigma}_2 \left( \frac{1}{\mu_1} \partial_2 \right) & -j\hat{\sigma} + \hat{\sigma}_1 \left( \frac{1}{\mu_2} \partial_1 \right)
\end{pmatrix}. \tag{6}
\]

The dot in each element of \( \hat{A}_{12} \) and \( \hat{A}_{21} \) represents a placeholder for the quantity on which the operator is applied. The source vector \( \hat{D} \) is defined as

\[
\hat{D} = \begin{pmatrix}
-j\hat{J}^m - \hat{\sigma}_1 \frac{1}{\varepsilon_1} \hat{J}^e \\
\hat{J}^m - \hat{\sigma}_2 \frac{1}{\varepsilon_2} \hat{J}^e \\
\hat{J}^m - \hat{\sigma}_2 \frac{1}{\mu_1} \hat{J}^e \\
-j\hat{J}^m + \hat{\sigma}_1 \frac{1}{\mu_2} \hat{J}^e
\end{pmatrix}. \tag{7}
\]

Equation (3) is transformed to the horizontal wavenumber domain using a 2D spatial Fourier Transformation \( \hat{u}(k_1, k_2, x_1, x_2) = \int_{-\infty}^{\infty} \exp(ik_1x_1 + ik_2x_2)\hat{u}(x_1, x_2, x_3, \omega) \) \( dx_1dx_2, \) which gives

\[
\partial_t \hat{Q} = \hat{A}\hat{Q} + \hat{D}, \tag{8}
\]

where the tilde denotes the horizontal wavenumber-frequency domain. The radial wavenumber is defined as \( k_r = 2\pi \kappa \omega \) where \( \kappa \) is the natural wavenumber. We have assumed that the material parameters do not change laterally at the receiver level. Subsequently, an eigenvalue decomposition of the system matrix \( \hat{A} = \hat{L}\hat{\Gamma}\hat{L}^{-1} \) is used to get

\[
\hat{L}^{-1}\partial_t \hat{Q} = \hat{\Gamma}\hat{L}^{-1}\hat{Q} + \hat{L}^{-1}\hat{D}. \tag{9}
\]

The matrix \( \hat{\Gamma} \) is a diagonal matrix containing the eigenvalues of \( \hat{A} \), whereas \( \hat{L} \) consists of the eigenvectors. Equation (9)
suggests to introduce $\hat{P}$ as
\begin{equation}
\hat{P} = \tilde{L}^{-1}\hat{Q} \quad \text{or} \quad \hat{Q} = \tilde{L}\hat{P}.
\end{equation}

After some algebraic manipulation we obtain
\begin{equation}
\alpha_0\hat{P} = (\tilde{F} - \tilde{L}^{-1}\alpha_0\tilde{L})\hat{P} + \tilde{L}^{-1}\tilde{D}.
\end{equation}

If at the decomposition level the vertical derivative of the material parameters, and hence of $\tilde{L}$, vanishes and if there are no sources present at this level, equation (11) simplifies to a set of equations wherein the components of $\hat{P}$ are decoupled. The solution gives two downward decaying modes and two upward decaying modes. With the choices for the eigenvectors given in the appendix, this decomposition algorithm separates Transverse Electric (TE) and Transverse Magnetic (TM) modes at the receiver level (Slob 2009). Furthermore, the decomposed field vector $\hat{P}$ is flux-normalized.

Consequently, we introduce the decomposed field vector $\tilde{P} = (\tilde{P}_{TM+}, \tilde{P}_{TE+}, \tilde{P}_{TM-}, \tilde{P}_{TE-})^T$, where the superscript $+$ and the superscript $-$ indicate downward and upward decaying components, respectively. The matrices $\tilde{L}$ and $\tilde{F}$ are given in the appendix.

In order to obtain $\hat{P}$ in 3D, the two horizontal electric field components and the two horizontal magnetic field components are required on a grid. In the 2D TM case, this requirement reduces to the horizontal inline electric field component and the horizontal crossline magnetic field component along a line. In this case, the decomposed field vector $\tilde{P}$ consists of only two components, one upward decaying and one downward decaying TM component. The up-down decomposition can be done at any depth level where no sources are present. The implementation used here assumes the material parameters to be laterally constant at the depth level of decomposition but this is not a condition for the following theory (Wapenaar and Grimbergen 1996; Grimbergen et al. 1998). Additionally, it is necessary that the field is sampled properly. In other words, a data gap or clipped amplitudes in the vicinity of the source can not be handled by this decomposition algorithm. The presence of gaps can be mitigated using interpolation methods. Clipped amplitudes in the vicinity of the source, which are common in CSEM data, can be substituted by a modelled direct field. Furthermore, clipping can be avoided by using future receivers that automatically adapt their dynamic range as a function of the measured amplitude. Consequently, we ignore these limitations.

In the space-frequency domain, assuming the receivers are located at a constant depth level below the source, in 3D one downward decaying component of $\hat{P}$, either TE or TM, is related with one upward decaying component of $\hat{P}$, again either TE or TM, through the reflection response $\hat{R}_o^0(x_R, x_R')$ (Wapenaar et al. 2008a):
\begin{equation}
\hat{P}^-(x_R, x_S) = \int_{\partial D_R} \hat{R}_o^0(x_R, x_R')\hat{P}^+(x_R', x_S)dS_R,
\end{equation}
where the array of receivers is represented by $x_R$ and the source coordinates by $x_S$. The integration is taken over all receivers. The superscript $-$ in the reflection response indicates that its origin is a downward decaying field and the subscript $0$ represents the absence of heterogeneities above the receiver level in the reflection response. In other words, $\hat{R}_o^0(x_R, x_R')$ is the scattered field response of the subsurface below the receivers due to an impulsive downward radiating source, i.e. a scattered Green's function. Equation (12) can be rewritten in matrix notation (Berkhout 1982) as
\begin{equation}
\hat{P}^- = \hat{R}_o^0\hat{P}^+.
\end{equation}

Each column of the matrices contains various receiver positions for a fixed source position, while for the rows the situation is reversed. Interferometry by multidimensional deconvolution solves equation (13) for $\hat{R}_o^0$, for example, by least-squares (LSQR) inversion:
\begin{equation}
\hat{R}_o^0 = \hat{P}^- (\hat{P}^+)\left[\hat{P}^+(\hat{P}^+)\right]^{-1} + \varepsilon^2 f_{LSQR}\mathbf{I}^{-1}.
\end{equation}

The superscript $\dagger$ denotes complex-conjugation and transposition and $\mathbf{I}$ is the identity matrix. The stabilization parameter $\varepsilon^2$ prevents instabilities in the inversion. It is scaled with a factor $f_{LSQR}$, which is based on the downward decaying field. The scaling factor $f_{LSQR}$ is taken as the mean of the diagonal elements of the correlation of the downward decaying field. In this way, the stabilization parameter becomes comparable for different datasets.

Assuming that the receivers are at the ocean bottom and that the decomposition into downward and upward decaying fields is done with the medium parameters of the first layer below the ocean bottom, the multidimensional deconvolution result of equation (14) is the reflection response that would be measured at the ocean bottom if the water layer were replaced with a homogeneous half-space consisting of the same material as the first layer below the water.

When the stabilization parameter $\varepsilon^2$ is chosen very large, $\hat{P}^+(\hat{P}^+)\dagger$ is much smaller than $\varepsilon^2 f_{LSQR}\mathbf{I}$ and can therefore be ignored. The term in square brackets becomes then a scaled identity matrix and the multidimensional deconvolution-equation (equation (14)) can be simplified to the cross-correlation-equation
\begin{equation}
\hat{R}_o^0 = \hat{P}^- (\hat{P}^+)\dagger,
\end{equation}

© 2011 European Association of Geoscientists & Engineers, *Geophysical Prospecting*, 60, 974–994
which basically retrieves $\hat{R}_0^+$ by cross-correlation. This part can be carried out for each receiver pair separately in contrast to equation (14), where the whole array of receivers is necessary to solve the equation.

The choice of the stabilization parameter $\varepsilon^2$ can be judged by computing an error using the following error function (van der Neut et al. 2008):

$$\text{Error}(\varepsilon) = \frac{\sum_{\text{samples}} |\hat{A}_{\text{MDD}}(\varepsilon) - \hat{A}_{\text{ref}}|}{\sum_{\text{samples}} |\hat{A}_{\text{CC}} - \hat{A}_{\text{ref}}|},$$

(16)

where $\hat{A}$ is the amplitude of the reflection response retrieved by multidimensional deconvolution, cross-correlation or by direct modelling, indicated with the subscripts multidimensionaldeconvolution, cross-correlation or ref, respectively. The summation is carried out over all samples of the reflection response for the redatumed source location at the centre of the receiver array in the space-frequency domain. In other words, the error function compares the retrieved reflection response by multidimensional deconvolution (equation (14)) with the directly modelled reflection response. This result is then normalized by the comparison of the retrieved reflection response by cross-correlation (equation (15)) with the directly modelled reflection response. The consequence of the normalization is that the Error converges to 1 for large values for the stabilization parameter because the multidimensional deconvolution result approaches the cross-correlation solution for large values of $\varepsilon^2$. Solving the cross-correlation equation (equation (15)) retrieves the reflection response with far too low amplitudes and with phase errors. Consequently, if the values of the error function are smaller than 1, the retrieved reflection response is better than the reflection response retrieved by cross-correlation. On the other hand, if the values of the error function are larger than 1, multidimensional deconvolution is unstable and no useful results can be expected for this $\varepsilon^2$. The best result lies at the minimum of the error function.

A necessary condition to compute an error function is the availability of a directly modelled reflection response. It allows one to find the perfect stabilization parameter to investigate what is possible at best. In practice, this is not possible and therefore the error functions can only be used for theoretical studies. Minimizing the misfit $\sum_{i=1}^{N} \sum_{j=1}^{M} |\hat{P}_r - \hat{R}_0^+(e)\hat{P}_r|_1$ can give an indication about a good value of the stabilization parameter in practical situations.

### 3 ACQUISITION ASPECTS

As stated in section 2, the decomposition algorithm needs properly sampled fields in order to produce correct results. Wapenaar et al. (2008a) use in their paper a sampling distance of 40 m, which is quite dense for CSEM. Unlike for wavefields, where proper sampling is achieved with two points per wavelength, there is no sampling rule available for diffusive electromagnetic fields. It is the goal of this section to find such a rule.

We expect that a denser sampling is necessary for a source vertically close to the receivers. Therefore, two different 2D Transverse Magnetic CSEM-datasets are modelled, a conventional situation with the source 25 m above the receivers, and an alternative case, in which the source is located 825 m above the receivers. In the following, the vertical distance between source and receivers is indicated as $x_{3,sr}$, where the subscript $s$ represents the source and the subscript $r$, the receiver. The length of the source antenna is in both situations 80 m. The datasets are obtained in a configuration shown in Fig. 1 that consists from top to bottom of a half-space of air, a water layer that has a thickness of 1000 m, a 200 m thick sedimentary layer, and a half-space of sediments. The lower half-space is intersected by a reservoir layer 1000 m below the sea floor. An electric source, with antenna orientation parallel to the line of receivers, generates a monochromatic signal at a frequency of 0.5 Hz. The resulting earth response is recorded by $E_1$ and $H_2$ receivers. The subscript $1$ indicates inline receivers, whereas the subscript $2$ represents crossline receivers.

---

© 2011 European Association of Geoscientists & Engineers, Geophysical Prospecting, 60, 974–994
The magnitude of the two electromagnetic field components $H_2$ and $E_1$ are shown in Fig. 2 in a semi logarithmic plot. The conventional case ($x_{3, sr} = 25$ m) is plotted with a red line and the alternative case ($x_{3, sr} = 825$ m) with a blue line. The amplitude is generally lower in the alternative case than in the conventional case because the source is further away from the receivers. For the same reason, the slope of the curve representing the conventional case is steeper at small offsets for both field components than in the alternative case. When the magnetic and the electric fields are compared for the conventional case, it can be seen that the electric field decays over five orders of magnitude whereas the magnetic field decays only over three orders of magnitude. This difference is less pronounced in the alternative situation because of the generally smaller dynamic range.

The same electromagnetic fields are shown in the wavenumber domain in Fig. 3. Both, the electric and magnetic fields, are stronger at large wavenumbers in the conventional case than in the alternative case. In other words, the data have a larger bandwidth in the conventional situation than in the alternative situation. The reason for this can be found in the steep field gradients in the space domain for the conventional situation around zero offset. These steep gradients are translated to a higher field strength at large wavenumbers in the wavenumber domain.
The spatial Nyquist criterion defines the largest wavenumber $\kappa_{1,\text{max}}$ that can be reproduced with a specific receiver sampling $dx_1$:

$$\kappa_{1,\text{max}} = \frac{1}{2dx_1}. \quad (17)$$

According to equation (17), increasing the spacing means in the wavenumber domain a limitation of the range of the wavenumbers. Consequently, a larger spacing introduces aliasing for the large wavenumbers and can alter the data significantly. In our example, the large wavenumbers found in the conventional situation can only be recorded if $dx_1$ is accordingly small. When the data need to be properly sampled up to the maximum wavenumber of 0.2 m$^{-1}$ as shown in Fig. 3, a spacing of $dx_1 = 2.5$ m becomes necessary. The bandwidth of the conventional and the alternative datasets justify our expectation that a denser sampling will be necessary in the conventional case compared to the alternative case.

The different lobes visible in the conventional dataset are the imprint of the source with a length of 80 m. If the sampling distance is increased to half the length of the source, in this case 40 m, the bandwidth is limited such that only the central lobe remains. The signal of the lobes at larger wavenumbers are aliased and therefore folded to smaller wavenumbers. Due to the exponential decay of the electromagnetic fields, the lobes at larger wavenumbers are much smaller in amplitude than the central lobe and since at the edge of each lobe the field is smoothly going to zero, the resulting artefacts are negligible. In other words, the sampling distance can be increased. If the source length is increased, the lobes become narrower. Consequently, increasing the source length allows an increase of the sampling distance because more and more large wavenumbers can be eliminated.

If the bandwidth of the complete data is narrowed such that it is smaller than the bandwidth of the reservoir response, the response from the reservoir is altered. Therefore the question arises, how large is the bandwidth of the response from the reservoir? To answer this question a dataset without the reservoir layer was modelled for the conventional and for the alternative situation. Subtraction of the dataset with a reservoir from the corresponding dataset without a reservoir in the wavenumber domain gives the bandwidth of the reservoir response (Fig. 4). The bandwidth of the reservoir is much smaller than the bandwidth of the complete data in the conventional case (Note that the horizontal axis has been scaled by a factor of 100 compared to Fig. 3.). Also the limited bandwidth of the alternative dataset does not affect the information about the reservoir. With a bandwidth of 0.001 m$^{-1}$ of the reservoir response, the spatial sampling distance can be as large as 500 m to still properly sample the reservoir response. Consequently, the large bandwidth in Fig. 3 does not originate in the reservoir response but in the direct field. Besides the direct field, also reflectors and inhomogeneities close to the receivers can cause a large bandwidth of the data.

### 4 PROCESSING

After modelling, which was carried out in the wavenumber domain, the data were multiplied with a sinc function, which models an elongated source with constant current density. This multiplication corresponds to a convolution with a boxcar function in the space domain. Next, the datasets are inverse-Fourier transformed to the space domain, where samples are deleted to create more sparse datasets. Then the datasets are decomposed into upward and downward decaying fields in the wavenumber domain. Equation (14) is solved efficiently in the wavenumber domain. The inverse-Fourier transformed result is equivalent to $\hat{R}_j$ retrieved by multidimensional deconvolution. The complete processing flow is shown in Fig. 5.

The data for one source in the wavenumber domain correspond to the following setup of sources and receivers in the space domain: If the receiver array consists of $N$ receivers, then there are $2N - 1$ sources in the complete setup because each receiver is illuminated by $N$ sources. The remaining
J. Hunziker et al.

Figure 5 Processing flow: The dashed box labelled Realistic Flow contains all the steps that would be applicable to a real dataset. The letters x and k on black background indicate space and wavenumber domain, respectively.

\[ N - 1 \] sources are out of range of this receiver. In Fig. 6a the layout is plotted for a setup with, for illustration purposes, \( N \) equal to four receivers and seven \( (2N - 1) \) sources. In the space domain, the same data for such an example correspond to a matrix like the one shown in Fig. 6b, where each column contains data from a common source and each row from a common receiver. Black squares indicate data and white squares mean no data. Since we are dealing here with a 1D subsurface structure and since we are not adding new offsets (Fig. 6), multiple sources do not actually add new data but the amount of sources influences the stability of the inversion in the space domain.

In the wavenumber domain, the inversion of the downward decaying field, which would be stored in the space domain in a matrix as shown in Fig. 6b, becomes a simple division and therefore equation (14) can be solved efficiently. A necessary condition to solve equation (14) in the wavenumber domain is the presence of horizontally constant medium parameters. The scaling factor \( f_{LSQR} \) is changed to the mean of the complete correlation matrix of the downward decaying field. Consequently, the stabilization parameter \( \varepsilon^2 \) in the wavenumber domain cannot be compared with the stabilization parameter in space domain.

From solving equation (14) in the wavenumber domain it follows that when the receiver sampling distance is increased, also the source sampling distance increases. Because the up-down decomposition of the fields is independent of the source sampling and since the inversion problem remains over-determined in any case, the coupling of the receiver-sparsening with the source-sparsening has no influence. In other words, if equation (14) were solved in the space domain with the described source-receiver setup and with only sparsened receivers but with dense sources, the same results as with the more efficient wavenumber domain solution would be produced.

5 RESULTS: PHYSICAL SOURCE

The reflection response \( \hat{R}_0^+ \), which is retrieved after decomposition into upward and downward decaying fields and multidimensional deconvolution, is shown in the wavenumber domain. In the wavenumber domain, the data for one source in wavenumber domain corresponds to the plotted layout of sources (stars) and receivers (triangles) in the space domain. For \( N \) receivers, there are \( 2N - 1 \) sources and the distance between sources is equal to the distance between receivers. In this case \( N \) is equal to 4. In the space domain, the data could be stored in a matrix as depicted here. Each column contains data from one specific source, each row from a specific receiver. Black squares indicate receiver positions within the range of the corresponding source, whereas white squares indicate receiver positions outside the range of the actual source. For example, receiver 1 is illuminated by sources 1 to 4 (dashed source gather in Subfigure a). Equivalently, in the first row of the datamatrix in Subfigure b), the first four datapoints are indicated in black corresponding to sources 1 to 4. Analogous, the third receiver is illuminated by sources 3 to 6 (solid source gather in Subfigure a). Consequently in the third row of the datamatrix, those datapoints are in black.

\( \odot \) 2011 European Association of Geoscientists & Engineers, *Geophysical Prospecting*, 60, 974–994
Figure 7 Reflection responses $\hat{R}_0^+$ in the wavenumber domain (left column) and in the space domain (right column) as a function of offset for a) b) a receiver spacing $dx_1$ of 2.5 m, c) d) 80 m, e) f) 160 m and g) h) 320 m. The error between the retrieved and the directly modelled reflection response normalized with the maximum amplitude of the directly modelled reflection response is shown below each graph. Note that the plots zoom into the important part of the data and do not show the complete wavenumber axis nor the complete axis of offsets. The spacing $dx_1$ and the amount of data-points $N$ are given in the figure captions. For clarity the symmetry of the reflection responses with respect to the zero-offset position is used. Therefore, the conventional case (red) is plotted for negative wavenumbers or for negative offsets only, whereas the alternative case (blue) is plotted for positive wavenumbers or for positive offsets only. The axes span over the same range in all plots.
domain and in the space domain in Fig. 7 for different receiver spacings $dx_1$. The conventional case (dashed red line) is plotted only for negative wavenumbers or for negative offsets, whereas the alternative case (dashed blue line) is plotted only for positive wavenumbers or for positive offsets in order to achieve a clear comparison. In this way no information is lost because the retrieved reflection responses are symmetric with respect to the zero-offset position. With increasing receiver spacing $dx_1$ the number of receiver positions $N$ decreases because the total offset is kept constant. In the wavenumber domain as well as in the space domain, only a limited range of wavenumbers or offsets are plotted. In this section, all retrieved reflection responses are due to a redatumed source at the centre of the receiver array. Since multidimensional deconvolution replaces the water layer by a homogeneous half-space and redatums the source position to a receiver position, the reflection responses for the conventional and the alternative case should be identical (in case of sufficiently dense sampling). The retrieved reflection responses are compared with a directly modelled reflection response (light-grey solid line), which is the exact response when the water layer is replaced by a homogeneous half-space, the direct field is removed and the source is at a receiver position. Below each plot, the error between the retrieved and the directly modelled reflection response is shown normalized with the maximum amplitude of the directly modelled reflection response.

In Fig. 7a and b the receiver spacing $dx_1$ is equal to 2.5 m. The two retrieved reflection responses and the directly modelled reflection response show exactly the same shape in both domains, thus verifying that multidimensional deconvolution was successful in removing the imprint of the water layer for both cases. The error is in the wavenumber domain as well as in the space domain zero, verifying the perfect retrieval. This is also true for larger spacings up to $dx_1 = 20$ m (not shown). A spacing of $dx_1 = 40$ m exhibits some artefacts around zero offset in the space domain in the conventional case (not shown). These artefacts are confined to an area around zero offset and do not affect larger offsets. We get a similar picture if the spacing is equal to the length of the source, in this case $dx_1 = 80$ m (Fig. 7c and d). These oscillations lead to an increased error in the conventional case for small offsets. In the wavenumber domain (Fig. 7c), these artefacts are visible as increased amplitude for high wavenumbers. They can be significantly damped by applying a lowpass filter in the wavenumber domain. Because of this and because the overall shape and amplitude of the reflection response are retrieved properly, we neglect these artefacts and consider the reflection response as sufficiently well retrieved. If the spacing is further increased to $dx_1 = 160$ m, as shown in Fig. 7e and f, the reflection response for the conventional situation is now also for intermediate offsets not retrieved correctly. The red curve does not overlay the grey curve at all. Consequently, the error is, especially in the wavenumber domain, over a large area increased. The reason is, that the original data are aliased at this spacing. Consequently, information at high wavenumbers is folded back to small wavenumbers, leading to completely altered data at all wavenumbers. On the other hand, the alternative case $\hat{R}^c$ could be retrieved well. Increasing the spacing to 320 m (Fig. 7g and h) does not change the picture. The reflection response is still well retrieved in the alternative case, whereas a retrieval is not possible in the conventional case.

The retrieved reflection response for the densest sampling distance of 2.5 m (Fig. 7a and b) is shown for a larger span of wavenumbers in Fig. 8. Note that both axes are different than in the previous plots in the wavenumber domain. The bandwidth of the retrieved reflection response is rather small in both the conventional (dashed red line) and the alternative (dashed blue line) situation although the original fields show huge differences in bandwidth (Fig. 3). The bandwidth of the response from the reservoir shown in Fig. 4 is narrower than the bandwidth of the reflection response of Fig. 8 because the latter also includes the interface 200 m below the water bottom. In the case of the traditional situation, the retrieved reflection response overlays perfectly the directly modelled reflection response for the complete range of wavenumbers. The reflection response retrieved in the alternative situation coincides with the directly modelled reflection response only
for small wavenumbers but deviates from the modelled reflection response at larger wavenumbers. Due to the large source-receiver distance, data at large wavenumbers are filtered out. Consequently, only information at smaller wavenumbers remains. Since the affected large wavenumbers have a relatively small amplitude, this effect is not visible in the space domain (Fig. 7b).

6 STABILITY ANALYSIS

A larger receiver spacing does not only affect the decomposition algorithm but also the stability of the inversion. Fig. 9a shows three error curves (equation (16)) for the conventional case for three different spacings. The densest spacing of $dx_1 = 2.5$ m is shown by a solid line. The inversion is stable for very small values of $\varepsilon^2$. For larger values of $\varepsilon^2$ the error function converges to 1 as described in the theory section. The dashed line shows the error function for a spacing of $dx_1 = 80$ m. Also in this case the inversion is stable for very small values of $\varepsilon^2$. Note that in this case even the smallest error is larger than the error for a spacing of 2.5 m. The reason is that at this spacing, the retrieved reflection response is in error around zero offset (Fig. 7d), which is detected by the error function. Finally, the dotted line stands for the error function for a spacing of $dx_1 = 320$ m. The minimum has now become a clearly defined trough. For too small values of the stabilization parameter the function becomes unstable (values larger than 1). The minimum error is larger than in the two other curves because at this spacing the retrieved reflection responses is suffering significantly from the large spacing.

The error curves for the alternative situation is shown in Fig. 9b. Since the bandwidth is limited in this case, a quite large spacing can be handled by the decomposition and multidimensional deconvolution. All three curves show a broad minimum and are therefore stabilized rather easily. An error of 1 is reached for smaller values for $\varepsilon^2$ with increasing sampling distance. Note that in the alternative situation the smallest value of the error function at a spacing of $dx_1 = 80$ m is smaller than for the same spacing in the conventional case. The reason is that there are almost no artefacts around zero offset at a spacing of 80 m in the alternative case, whereas the amplitude is underestimated in the conventional case. At a spacing of $dx_1 = 320$ m some artefacts are introduced as mentioned for the conventional situation. Consequently, the minimum of the error curve rises compared to the other two curves.

Fig 7 and 9 show that the alternative dataset can be sampled sparser than the conventional CSEM dataset because the source is vertically further away from the receivers in the alternative case. Hunziker et al. (2009) showed an empirical rule to define the proper sampling distance $dx_1$ as a function of the vertical source-receiver distance $x_{3, sr}$. We modify this rule with respect to two parameters: Firstly, we are able to relax this rule because in this study the stabilization parameter $\varepsilon^2$ is not fixed anymore for the different spacings. Secondly, we add another parameter which is the length of the source antenna $l$. Our findings for the largest possible receiver sampling distance for different vertical source-receiver distances (source heights) and different lengths of the source antenna (source lengths) are summarized in Fig. 10. It can be seen

![Figure 9](image-url) Error as a function of the stabilization parameter $\varepsilon^2$ for three different sampling distances $dx_1$. a) for the conventional case; b) for the alternative case.
that it is possible to sample the electromagnetic fields sparser with increasing source height or increasing source length. We found that the largest possible receiver sampling distance has to be equal to or smaller than the larger of the two parameters, source length or source height

$$dx_1 \leq \max\{x_{3_{sr}}, l\}. \quad (18)$$

Note that this rule assumes that instabilities around zero offset are not problematic.

### 7 RESULTS: SYNTHETIC APERTURE SOURCE

If it is not possible to pull the source antenna further away from the sea bottom (e.g. in a shallow marine situation) or if it is not desired (e.g. in order to keep a large signal strength), according to equation 18 the only option left is the usage of a long source antenna. To have a physically long source may not be practical. But, there is also the option of using a synthetically long source as proposed by Fan et al. (2010). They show for modelled and field data, that electromagnetic fields can be weighted and summed over many source positions to create one long source antenna. Since the source is towed by a boat over the array of receivers, there is an almost continuous record of source positions available. We create a synthetic aperture source by summing the electromagnetic field over a specific length of source positions. We give weights to the sources that are given by a Gaussian distribution function

$$f(x) = \frac{1}{\sqrt{2\pi} \gamma^2} \exp\left(-\frac{(x - x_0)^2}{2\gamma^2}\right), \quad (19)$$

where $x_0$ is the centre of the synthetic aperture source. The parameter $\gamma$ is chosen to be one eighth of the length of the synthetic aperture source. The parameter $x$ represents the source location in inline direction.

In Fig. 11 the reflection responses retrieved from data from a synthetic elongated source of a length of 5 km are shown in the wavenumber domain and in the space domain. The figure is set up in the same way as Fig. 7 but for different receiver spacings. In fact, the smallest receiver spacing of Fig. 11 is 320 m, which is the largest receiver spacing of Fig. 7. For all three receiver spacing distances (i.e. 320 m, 640 m and 1280 m) the reflection response is retrieved perfectly within a specific bandwidth of wavenumbers. By creating a synthetically elongated source, the high wavenumbers, with respect to the complete wavenumber spectrum, are damped but not eliminated. Consequently, the high wavenumbers can be retrieved using interferometry by multidimensional deconvolution if they remain above the noise floor in the original data. However, retrieving damped high wavenumbers leads in the deconvolution step to some instabilities seen in the space domain. Note, that the retrieved bandwidth is determined by the receiver sampling distance and not by the length of the synthetic aperture source. Thus, with increasing receiver separation, the bandwidth decreases as indicated with the vertical black lines in the wavenumber domain plots representing the spatial Nyquist frequency at $1/(2dx_1)$. The directly modelled reflection response has a much wider bandwidth than the retrieved one, because it is computed using a much denser receiver spacing. This leads to a different shape in the space domain. Due to this difference in bandwidth between the directly modelled and retrieved reflection response, the error in the space domain is increasing with increasing receiver sampling distance. Still, also in the space domain, the reflection responses are retrieved well also for large receiver sampling distances within the bandwidth defined by the receiver spacing.

This shows that a physical long source can be emulated by a synthetically elongated source to overcome the strict sampling rule given in equation 18. In this way, large receiver sampling distances can be achieved, while keeping the current acquisition geometry. Consequently, for processing a dense receiver sampling is not necessary but one should be aware that a too large receiver sampling distance will lead to a loss of

![Figure 10: The largest sampling distance which still retrieves the reflection response well, as a function of vertical source receiver distance (source height) and length of the source antenna (source length).](image-url)
Figure 11 Reflection responses $\hat{R}_0^+$ retrieved from data using the synthetic aperture source concept. The Reflection responses are shown in the wavenumber domain (left column) and in the space domain (right column) as a function of offset for a)b) a receiver spacing $dx_1$ of 320 m, c)d) 640 m and e)f) 1280 m. The black vertical lines in c) and e) indicate the spatial Nyquist frequency at $1/(2dx_1)$. Otherwise the same as Fig. 7.
information about the reservoir response, which in turn will have a negative effect on possible inversion results.

So far we have considered data for a situation with a deep ocean in order to have the option to move the source vertically in the water layer. Next we show an example of a very shallow marine situation. The model is the same as in Fig. 1, but the water layer is only 50 m thick. The source is 25 m above the receivers at the ocean bottom. The retrieved reflection responses for three receiver spacing distances is shown in the wavenumber domain and in the space domain in Fig. 12. The results are almost exactly the same as for the deep sea case (Fig. 11), because the retrieved reflection responses only depend on the subsurface, which is the same in both models. The water-layer is replaced by a homogeneous halfspace and has therefore no impact on the reflection response.

As mentioned earlier, retrieving damped high wavenumbers in the deconvolution process may lead to instabilities. By creating a smaller synthetic aperture source, less high wavenumbers are damped and, consequently, less instabilities occur. This is illustrated in Fig. 13a and b, which are the same as Fig. 11a and b, but for a synthetic aperture source of only 2500 m length. In the wavenumber domain more high wavenumbers are retrieved properly in the conventional as well as in the alternative situation and, therefore, less artefacts appear in the space domain. Decreasing the length of the synthetic aperture source further to 1250 m, introduces slight aliasing at high wavenumbers in the conventional case because not enough high wavenumbers are damped (Fig. 13c and d). A length of the synthetic aperture source of 630 m is in our situation definitely too short. All wavenumbers are now affected by the aliased high wavenumbers in the conventional situation. A correct retrieval of the reflection response is no longer possible in the conventional situation (Fig. 13e and f). The reflection response is still retrieved very well in the alternative situation, because the large vertical distance between the source and the receivers filters the high wavenumbers.

8 NOISE ANALYSIS

To test the applicability of interferometry by multidimensional deconvolution in combination with the synthetic aperture concept under realistic conditions, we contaminated the data for the deep sea case with three different types of noise, which are random noise, receiver orientation errors and receiver positioning errors. As already mentioned, interferometry by multidimensional deconvolution does not suffer from uncertainties in source orientation and location. Therefore we do not consider these uncertainties.

Figure 14a and b) show the reflection response retrieved from a dataset which was contaminated with random noise. The noise floor of the electric field is $10^{-11}$ Vs/m and the one of the magnetic field is $10^{-8}$ As/m. Normalized with the source dipole moment we get noise levels of approximately $10^{-13}$ Vs/(Am²) and $10^{-10}$ As/(Am²). These noise levels are two orders of magnitude larger than noise levels of current receivers (Constable 2010). The various datasets with different receiver spacings were affected by the noise in a similar way. Here we show only the dataset with a receiver spacing of 320 m because a relatively dense sampled curve illustrates the effects of the noise better than a curve consisting only of a few datapoints. Comparing the retrieved reflection responses based on noisy data (Fig. 14a and b) with the reflection response retrieved from clean data (Fig 11a and b) shows that even with these high noise levels, the reflection response is retrieved well. Due to the noise, the bandwidth of the alternative case is narrowed leading to a larger error at high wavenumbers. However, the average error of the retrieved reflection response increases only by 0.01% with respect to a noise free dataset. This means, that the uncertainty increases very little by adding noise.

Random receiver orientation errors of up to 10 degrees for the antenna measuring the electric field are simulated. The reflection response retrieved from that dataset (Fig. 14c and d) is affected significantly by this kind of noise. The average error changes from 1.24% of a noise free data to 2.84%. In other words, the uncertainty of the retrieved reflection response doubles if the data are contaminated by orientation errors of up to 10 degrees. Still, within a limited bandwidth the reflection response is retrieved well.

Finally, random receiver positioning errors are modelled. In this dataset, the receivers are misplaced up to ± 50 m. Similar to the receiver orientation errors, mispositioning of the receivers limits the bandwidth within which the reflection response can be retrieved properly (Fig. 14e and f). For this rather large mispositioning errors, the uncertainty doubles. Still, the reflection response can be retrieved well.

We have also modelled combined receiver positioning and orientation errors. The effect of both errors add up what results in a decrease of the properly retrieved bandwidth in the wavenumber domain or in a deteriorated reflection response in the space domain. Still, with random receiver positioning errors of up to ±50 m and random receiver orientation errors of up to 10 degrees, the reflection response could be retrieved properly (not shown).
Figure 12 Reflection responses $R_0^+$ retrieved from shallow marine data using the synthetic aperture source concept. Otherwise the same as Fig. 11.
Figure 13  Reflection responses $\hat{R}_0^+$ retrieved from data sampled with a receiver spacing of 320 m for different synthetic aperture source lengths: a) 2500 m, b) 2500 m, c) 1250 m and d) 630 m. Otherwise the same as Fig. 11.
Figure 14 Reflection responses $\hat{R}_0^+$ retrieved from data sampled with a receiver spacing of 320 m using the synthetic aperture source concept. The reflection responses are shown in the wavenumber domain (left column) and in the space domain (right column). Below them, the error is plotted. In a) and b) are the original data contaminated with random noise with a noise floor of $10^{-11}$ Vs/m for the electric field and a noise floor of $10^{-8}$ As/m for the magnetic field. In c) and d) random orientation errors of up to 10 degrees for the receiver antenna measuring the electric field are incorporated. In e) and f) receiver positioning errors of up to ±50 m are included.

© 2011 European Association of Geoscientists & Engineers, *Geophysical Prospecting*, 60, 974–994
Another problem, not discussed so far, is bathymetry. The decomposition assumes that the receivers are located on a flat surface. Thus rapid, small scale variations in vertical receiver position lead to noise, which is expected to be comparable to noise from receiver positioning errors. Smooth receiver variations due to large scale changes in the topography may be handled by a decomposition algorithm for curved surfaces. Such an algorithm could be based on recently published oneway wave fields in curvilinear coordinate systems (Frijlink and Wapenaar 2010). The situation, where the receivers are on a flat, but tilted surface, is ongoing research but it is assumed that decomposition works properly. In that case the decomposed fields are not purely upward or downward decaying but contain also horizontal components, such that the decomposed fields decay in a direction orthogonal to the tilted surface. When receiver height variations are large and erratic, this method will not work.

9 CONCLUSIONS

We modelled CSEM data in a marine environment for a conventional situation of a source vertically close to the receivers and an alternative situation where the source is vertically far away from the receivers for an 80 m long source. We decomposed the electromagnetic fields into upward and downward decaying fields and applied interferometry by multidimensional deconvolution, which redatums the sources to the receiver level, replaces everything above the receivers, i.e. the water layer and the air, with a homogeneous half-space and eliminates the direct field. The retrieved reflection response $\mathbf{R}^d_0$ contains only information about the subsurface below the sea bottom and is independent of the water layer and the position and orientation of the source within the water layer. Consequently $\mathbf{R}^d_0$ is the same for the conventional and the alternative situation.

A denser receiver sampling is required for the conventional case to prevent aliasing and, therefore, problems in the up-down decomposition of the fields because the bandwidth of the data is much larger when the source is vertically close to the receivers. Consequently, moving the source vertically away from the receivers allows for sparser receiver sampling. The bandwidth of the data depends also on the length of the source. Therefore, if a longer source is used, the sampling distance can be increased as well. It was found that the larger of the following two parameters, vertical distance between source and receivers and the length of the source antenna, defines the largest possible receiver spacing. The spacing should be equal to or smaller than the larger of the two parameters.

To realize a large source in reality may be difficult. An alternative is the application of synthetic aperture methods to conventionally acquired datasets. We tested interferometry by multidimensional deconvolution in combination with a synthetic aperture source for receiver spacings up to 1280 m successfully. Theoretically, even larger receiver spacings are possible but then the bandwidth of the retrieved reflection response gets smaller than the bandwidth of the reservoir response. The synthetic aperture source limits the bandwidth of the data and allows in this way to retrieve the reflection response for severely aliased data. Is the synthetic aperture source very large, small oscillations occur in the retrieved reflection response because too many high wavenumbers are damped. On the other hand, a too short synthetic aperture source leads to improperly retrieved reflection responses due to aliasing caused by insufficient damping of high wavenumbers. Interferometry by multidimensional deconvolution in combination with a synthetic aperture source has also been tested successfully for datasets contaminated with significant levels of random noise, receiver orientation errors and receiver positioning errors. The results of this numerical study look promising but further development may be necessary to apply this method on field data.

ACKNOWLEDGEMENTS

This research is supported by the Dutch Technology Foundation STW, applied science division of NWO and the Technology Program of the Ministry of Economic Affairs. We thank associate editor Oliver Ritter and three anonymous reviewers for their constructive comments. We would also like to thank Joost van der Neut, Deyan Draganov and Karel van Dalen for constructive discussions and valuable input to this research.

REFERENCES


has only 2 different eigenvalues, the first one on its diagonal.

\[ \Lambda = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \]

where \( \sigma_1, \sigma_2, \sigma_3 \) are the eigenvalues, \( \Lambda = \Lambda^T \).
corresponds to the downward decaying fields, whereas the second one corresponds to the upward decaying fields.

The composition matrix $\tilde{L}$ contains the eigenvectors of $\tilde{A}$ in its columns. In this case it is set up as follows

$$\tilde{L} = \begin{pmatrix} \tilde{L}_1 & \tilde{L}_2 \\ \tilde{L}_2 & -\tilde{L}_2 \end{pmatrix},$$  \hspace{1cm} (A2)

with the 2 by 2 submatrices

$$\tilde{L}_1 = \frac{1}{\sqrt{2(k_1^2 + k_2^2)}} \begin{pmatrix} jk_1 \sqrt{\frac{\tau}{\bar{\eta}}} & -jk_2 \sqrt{\frac{\tau}{\bar{\eta}}} \\ jk_2 \sqrt{\frac{\tau}{\bar{\eta}}} & jk_1 \sqrt{\frac{\tau}{\bar{\eta}}} \end{pmatrix},$$  \hspace{1cm} (A3)

$$\tilde{L}_2 = \frac{1}{\sqrt{2(k_1^2 + k_2^2)}} \begin{pmatrix} jk_1 \sqrt{\frac{\tau}{\bar{\eta}}} & -jk_2 \sqrt{\frac{\tau}{\bar{\eta}}} \\ jk_2 \sqrt{\frac{\tau}{\bar{\eta}}} & jk_1 \sqrt{\frac{\tau}{\bar{\eta}}} \end{pmatrix}.$$  \hspace{1cm} (A4)

The inverse composition matrix, i.e. the decomposition matrix, $\tilde{L}^{-1}$ is given by

$$\tilde{L}^{-1} = \frac{1}{2} \begin{pmatrix} \tilde{L}_1^{-1} & \tilde{L}_2^{-1} \\ \tilde{L}_2^{-1} & -\tilde{L}_2^{-1} \end{pmatrix},$$  \hspace{1cm} (A5)

where the inverses of the submatrices are given by $\tilde{L}_1^{-1} = -2\tilde{L}_1^T$ and $\tilde{L}_2^{-1} = -2\tilde{L}_2^T$. 

© 2011 European Association of Geoscientists & Engineers, *Geophysical Prospecting*, 60, 974–994