Constructing only the primary reflections in seismic data - without multiple removal

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Summary

State of the art methods to image the Earth’s subsurface using active-source seismic reflection data involve reverse-time migration (RTM). This, and other standard seismic processing methods such as velocity analysis, provide best results only when all waves in the data set are primaries (waves reflected only once). A variety of methods are therefore deployed as pre-processing to predict multiples (waves reflected several times); however, accurate removal of those predicted multiples from recorded data using adaptive subtraction techniques proves challenging, even in cases where they can be predicted with reasonable accuracy. We describe a new, alternative strategy: we construct a parallel data set consisting of only primaries, which is calculated directly from recorded data. This obviates the need for both multiple prediction and removal methods. Primaries are constructed using convolutional interferometry to combine first arriving events of up-going and direct-wave down-going Green’s functions to virtual receivers in the subsurface. The required up-going wavefields to virtual receivers are constructed by Marchenko redatuming. Crucially, this is possible without detailed models of the Earth’s subsurface velocity structure. The method is shown both to be particularly robust against errors in the reference velocity model used, and to improve migrated images substantially.
Introduction

Advanced seismic data processing methods such as full-waveform inversion can properly take into account data that includes multiply scattered waves. However, many current standard processing steps are based on the so-called Born approximation which states that waves have only scattered once from heterogeneities in the medium. This requires that data only include primaries (singly scattered waves) as multiples represent a source of coherent noise and must be suppressed to avoid artefacts. Multiples related to reflections from the Earth’s free surface particularly impact on images resulting from seismic marine data, and much effort has been devoted to their removal (see review by Dragoset et al., 2010). By contrast, internal multiples affect both marine and land data, and relatively fewer techniques exist to predict and remove them from reflection data. We propose a new method to predict primaries directly, based on seismic interferometry and Marchenko redatuming; this substantially improves test images.

Theory

Seismic interferometry techniques synthesize Green’s functions between source (or receiver) locations by integrating cross-correlations or convolutions of wavefields recorded by receivers (or emanating from sources) located elsewhere (Wapenaar and Fokkema, 2006). With these methods, one of the sources (or receivers) is essentially turned into a virtual receiver (or source). Marchenko redatuming on the other hand, estimates up- and down-going components of Green’s functions between an arbitrary location in the Earth’s subsurface where no sources (or receivers) are placed, and real receivers (or sources) located at the surface (Broggini et al., 2014). Similarly to standard linear migration methods, Marchenko focusing requires an estimate of the direct wave from the virtual source (or to the virtual receiver), illumination from only one side of the medium, and no physical sources (or receivers) inside the medium. We now show how these methods may be combined to predict internal multiples.

Convolutional interferometry uses acoustic reciprocity theorems to express the Green’s function between two locations as (Wapenaar and Berkhout, 1989)

\[
G(x_2, x_1) \approx \int_S \frac{2j\omega}{c(x)\rho(x)} \{G^-(x, x_2)G^+(x, x_1) - G^+(x, x_2)G^-(x, x_1)\} dS
\]

where \(c(x)\) indicates wave speed, and \(G^{\pm}\) represent in/out-going Green’s function components across closed surface \(S\). The main contributions to interferometric surface integrals come from neighborhoods of points where the phase of the integrand is stationary (Snieder et al., 2006). Figure 1 illustrates how primary reflections are reconstructed in convolutional interferometry; equations (1) essentially pieces together and integrates up/down-going wavefields around each stationary point on horizontal sections of surface \(S\), to calculate wavefields that would travel along each full wave path between \(x_1\) and \(x_2\).

Figure 1 Geometrical configuration that constructs primaries from convolutional interferometry. Stars are sources at \(x_1\) and \(x_2\), dashed line is an ideal receiver boundary \(S\). (a) Circles indicate stationary points associated with primary reflections between \(x_1\) and \(x_2\). Around each such point, convolutional interferometry connects a direct and a first-order scattering event to create a primary wave between \(x_1\) and \(x_2\). Filled circles indicate stationary points \(x\) connecting direct waves \(G_D^D(x, x_2)\) and the first arriving reflection in \(G^-(x, x_2)\), or \(G_D^D(x, x_2)\) and the first arriving reflection in \(G^-(x, x_1)\). The unfilled circle indicates a stationary point \(x\) not connecting \(G_D^D(x, x_1)\) and a first arriving reflection in \(G^-(x_2, x)\).

Meles et al. (2015) noted that the number of reflections undergone by an event in \(G(x_2, x_1)\) (its scattering order) is equal to the sum of the number of reflections undergone by its constitutive components, \(G(x, x_1)\) and \(G(x, x_2)\), and used that property to synthetize only multiple reflections. By contrast, in the
current paper we predict primaries directly, based on the observation that primaries may be constructed by convolving down-going direct waves with first-arriving up-going first-order scattered waves.

Following the standard decomposition of Green’s functions into direct and scattered waves (e.g., $G(x,x') = G_D(x,x') + G_S(x,x')$, where $G_D(x,x')$ represents the component of $G(x,x')$ that does not undergo any reflection), direct waves are uniquely defined for any source-receiver pairs $G_D^D(x,x_2)$ or $G_D^S(x,x_2)$. By contrast, up-going Green’s functions $G^U$ comprise many first order scattering events (in addition to multiples). This is illustrated in Figure 1, which discriminates between the construction of two different primaries. Filled circles indicate points at which direct waves are pieced together with first-arriving events of scattered up-going Green’s functions on surface $S$. The unfilled circle indicates a point where this does not apply: for that point the associated primary reflection $G^U(x,x_2)$ is not the first scattered arrival. Thus for arbitrary boundaries $S$, the components associated with primaries do not necessarily involve direct waves and first arriving events of up-going Green’s functions.

![Figure 1](image1.png)

In Figure 2 different partial boundaries (comprising only horizontal lines) are used to construct primaries. Filled circles and solid rays indicate points at which direct waves and first-arriving events of up-going Green’s functions are pieced together at a stationary point to construct the corresponding primary. Unfilled circles and dashed rays indicate points at which a later-arriving singly-scattered event of the up-going Green’s function must be used. Note that the reflection generated by reflector B is associated with later-arriving and first-arriving events in the up-going Green’s functions when using boundaries $S_1$ and $S_2$, respectively (Figure 2(a) and 2(b)). Keeping in mind the above observations and the limitations concerning performance of the method for different boundaries summarized in Figure 2, if we assume that the first arriving energy of any up-going Green’s function $G^U(x_1,x)$ is associated with a singly-scattered event, then we can reconstruct primaries by combining such events with direct waves. More precisely, we postulate that primaries, and primaries only, are reconstructed when first-arriving up-going events are convolved with direct down-going Green’s functions, and that for every primary there is always at least one surface on which this is true. We therefore propose the following approximate representation for primaries:

$$G_p(x_2,x_1) \approx \sum_i \int_{S_i} \frac{j \omega}{\epsilon(x)\rho(x)} \{G_F^U(x,x_2)G_D^D(x,x_1)+G_F^U(x,x_2)G_F^U(x,x_1)\}dS \quad (2)$$

where $G_p$ stands for the primary arrivals in a Green’s function, $G_D^D$ for the direct down-going wave, $G_F^U$ for the first-arriving events of up-going components of Green’s functions (which in our examples are created using Marchenko redatuming), and $S_i$ is a partial (non-closed), horizontal boundary ($i=1,2,\ldots$).

We distil this method into the following algorithm:

1) Choose a horizontal boundary $S_i$ in the subsurface. Locate virtual receivers at regularly-sampled locations $x$ along $S_i$, and use Marchenko redatuming to compute corresponding up-going Green’s function $G_F^U(x,x_p)$, where source locations $x_p$ span the surface array.
2) Mute events occurring before the direct waves in the up-going Green’s functions $G_F^U(x,x_p)$ to remove possible Marchenko artefacts (Thorbecke et al., 2013).
3) Pick first-arriving event $G_F^U(x,x_p)$ in the muted up-going Green’s function $G_M^U(x,x_p)$.
4) Apply equation 2 to predict primaries $G_p(x_j,x_k)$ for all $x_j,x_k$ in the surface array.
5) Repeat steps 1 to 4 using $S_i$ located at different depths to predict different primaries, then sum the results as specified in equation 2.

**Numerical Example**

We test the algorithm using a 2-dimensional varying density-velocity synclinal model (Figure 3). We compute synthetic surface seismic data with a finite-difference time domain modelling code and a Ricker source wavelet with central frequency 20 Hz, using absorbing boundaries on all sides (thus assuming that surface-related multiples have been removed from recorded data), between 201 co-located sources and receivers equally spaced along the surface of the model shown in Figure 5, with inter-source spacing of 12 m. Partial boundaries consist of horizontal lines $S_i$ to $S_t$ in Figure 3. Up-going Green’s functions $G^-(x, x_p)$ are estimated at a set of 121 points $x$ along each boundary using Marchenko redatuming. We estimate direct waves $G^+(x, x_q)$ using a smooth velocity model. First arriving events of up-going Green’s functions are then picked automatically and windowed. Despite inaccuracies in these wavefields and the consequent errors in picking, primaries, including the triplication associated with the synclinal interface, were relatively well reconstructed through application of equation 2, with only small, low amplitude artefacts (Figure 4). We then apply reverse time migration (RTM) to both the observed data and the estimated primaries using the smoothed reference velocity model. Resulting images are shown in Figure 5. Linear migration of internal multiples results in many multiple-related artefacts contaminating the conventional image (as indicated by red arrows in Figure 5(a)). RTM of only primaries provides a much cleaner image, with only a few artefacts below the top reflector (Figure 5(b)).

**Conclusions**

We present a new method to predict primary reflections based on Marchenko redatuming and convolutional interferometry. The method was demonstrated on acoustic data and proved to be stable with respect to inaccuracies in the redatumed Green’s functions. The synthesized primaries were used
to produce images almost free of multiple-related artefacts via linear reverse-time migration. For simplicity, the method was tested on a dataset free of surface-related multiples, recorded for collocated sources and receivers. Extensions to datasets collected in standard acquisition setups and including ghosts and surface related multiples will be the topic of future research. Applications connected to other methods such as full-waveform inversion and velocity analysis will also be investigated.

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