Seismic interferometry for passive and exploration data

Kees Wapenaar

Department of Geotechnology, Delft University of Technology (C.P.A.Wapenaar@TUDelft.NL).

ABSTRACT

Exact Green’s function representations for seismic interferometry are based on the assumption that the receivers used in the correlation process are surrounded by a closed surface of sources. We investigate two situations for which this condition is not fulfilled: sources only in the subsurface, as in passive seismics, and sources only at the surface, as in exploration seismics. We show that in both cases the full Green’s function, including primary and multiply scattered waves, can be reconstructed.

KEY WORDS: Green’s function, cross-correlation, multiple scattering

INTRODUCTION

Seismic interferometry is the process of generating new seismic responses by crosscorrelating seismic observations at different receiver locations. A first version of this principle was derived in 1968 by Claerbout (1968), who showed that the reflection response of a horizontally layered medium can be synthesized from the autocorrelation of its transmission response. Between 1968 and 2000 the research on this subject has been quite sporadic. Some highlights are Scherbaum (1987a), (1987b), Duvall et al. (1993), Daneshvar et al. (1995) and Rickett and Claerbout (1999). This changed dramatically since the beginning of the new millennium, when several research groups around the world in different scientific disciplines (ultrasoundics, exploration geophysics, seismology, oceanography) independently discovered the high potential of retrieving new responses by cross-correlating measurements at different receivers [Weaver and Lobkis (2001), Schuster (2001), Wapenaar et al., (2002), Campillo and Paul (2003), Roux et al. (2004), Shapiro et al. (2005)]. To date seismic interferometry (and its counterparts in ultrasoundics, oceanography etc.) has grown to a new branch of research, practiced by many research groups around the world. The main reason for this broad interest is the potential of retrieving new information from noise without requiring knowledge about the sources. However, also for controlled-source experiments, like in seismic exploration, interferometry has interesting applications [Schuster et al., (2004), Bakulin and Calvert (2004), Wapenaar (2006b)].

Various theories have been developed for the interferometric principle, ranging from diffusion theory for enclosures [Lobkis and Weaver (2002)], multiple scattering theory and stationary-phase theory for random media [van Tiggelen (2003), Malcolm et al. (2004), Snieder (2004)] and reciprocity theory for deterministic and random media (non-moving or moving) [Wapenaar et al. (2002), (2004), (2006a), Weaver and Lobkis (2004), van Manen et al. (2005)].

The derivations based on reciprocity theory yield exact representations of Green’s functions in arbitrary inhomogeneous lossless media. Hence, the reconstructed Green’s functions do not only contain the direct wavefield between the two receiver points but all primary and multiply scattered waves. This requires, however, that the two receivers are surrounded by sources on an arbitrarily shaped closed surface. In reality this condition is seldom fulfilled. In this paper we discuss two distinct situations for which the surface containing the sources is not closed and we discuss the conditions that are needed in order to reconstruct the exact Green’s function, including the internal multiples. The first situation we consider corresponds to passive seismic data, for which we usually assume a distribution of natural noise sources along an open surface in the Earth’s subsurface. The free surface acts as a mirror, which obviates the need of having sources on a closed surface. This situation has been extensively discussed in the literature, but we include it for completeness. The second situation is that of seismic exploration, with sources at the Earth’s surface only. Seismic interferometry for exploration data has been extensively discussed by Schuster et al. (2001), (2004) and Bakulin and Calvert (2004). Also the work of Verschuur and Berkhout (2005) has interesting connections with seismic interferometry (the latter authors apply weighted cross-correlations). In the mentioned references, impressive results have been obtained for primaries and free surface multiples, but internal multiples have not received much attention. In this paper we reconsider seismic interferometry for exploration data and show how, in theory, all internal multiples can be correctly retrieved. The receivers involved in the correlation may be located at the surface or in the subsurface, for example at the ocean bottom or in a vertical or horizontal borehole.

GREEN’S FUNCTION REPRESENTATION

We consider an arbitrary inhomogeneous lossless medium in which we define an arbitrarily shaped
closed surface $\partial \mathbb{D}$ with outward pointing normal vector $n = (n_1, n_2, n_3)$. Inside this surface we define two points $x_A$ and $x_B$. In the frequency domain, the Green’s function between these two points, $\hat{G}(x_A, x_B, \omega)$, can be represented as [Wapenaar et al. (2004), (2006), van Manen et al. (2005)]

$$2\Re \{ \hat{G}(x_A, x_B, \omega) \} =$$

$$\iint_{\partial \mathbb{D}} \frac{1}{j\omega \rho(x)} \left( \hat{G}^*(x_A, x, \omega) \partial_t \hat{G}(x_B, x, \omega) - (\partial_t \hat{G}^*(x_A, x, \omega)) \hat{G}(x_B, x, \omega) \right) n_3 d^2x,$$

where $\Re$ denotes the real part, $\omega$ the angular frequency, $j$ the imaginary unit and $\rho$ the mass density. The terms $\hat{G}$ and $\partial_t \hat{G}n_3$ under the integral in the right-hand side of equation 1 represent responses of monopole and dipole sources at $x$ on $\partial \mathbb{D}$. The products $\hat{G}^* \partial_t \hat{G}n_3$ etc. correspond to crosscorrelations in the time domain. Hence, the right-hand side can be interpreted as the integral of the Fourier transform of crosscorrelations of observed wavefields at $x_A$ and $x_B$, respectively, due to impulsive sources at $x$ on $\partial \mathbb{D}$; the integration takes place along the source coordinate $x$. The left-hand side of equation 1 is the Fourier transform of $G(x_A, x_B, t) + G(x_A, x_B, -t)$, which is the superposition of the response at $x_A$ due to an impulsive source at $x_B$ and its time-reversed version. This reconstructed Green’s function is exact and contains, apart from the direct wave between $x_B$ and $x_A$, all scattering contributions (primaries and multiples) from inhomogeneities inside as well as outside $\partial \mathbb{D}$. When the medium outside $\partial \mathbb{D}$ is homogeneous, equation 1 can be approximated by

$$2\Re \{ \hat{G}(x_A, x_B, \omega) \} \approx$$

$$\frac{2}{\rho c} \iint_{\partial \mathbb{D}} \hat{G}^*(x_A, x, \omega) \hat{G}(x_B, x, \omega) d^2x,$$

where $c$ is the propagation velocity. Since the right-hand side contains one crosscorrelation product of monopole responses only, this representation is better suited for seismic interferometry than equation 1. For a detailed analysis of the approximations in equation 2, see Wapenaar and Fokkema (2006). Evaluation of either equation 1 or 2 requires that sources are available on a closed surface $\partial \mathbb{D}$ around the observation points $x_A$ and $x_B$. In the following sections we discuss two situations for which $\partial \mathbb{D}$ is not closed.

**CONFIGURATION FOR PASSIVE DATA**

For the situation of passive data we assume that natural sources are available in the subsurface and that the responses of these sources are measured by receivers at or below the free surface. We divide the closed surface $\partial \mathbb{D}$ into a part $\partial \mathbb{D}_0$ coinciding with the free surface and a part $\partial \mathbb{D}_1$ containing the sources in the subsurface, see Figure 1. For this situation equation 1 needs to be evaluated over $\partial \mathbb{D}_1$ only. This is exact as long as $\partial \mathbb{D}_0$ and $\partial \mathbb{D}_1$ together form a closed surface. Hence, the direct wave as well as the primaries and multiples in $\hat{G}(x_A, x_B, \omega)$ are correctly reconstructed by the integral along the sources on $\partial \mathbb{D}_1$. A more intuitive explanation is that the free surface $\partial \mathbb{D}_0$ acts as a mirror which obviates the need of having sources on a closed surface.

We illustrate equation 2 with a 2-D example for a configuration with a free surface at $x_3 = 0$. We consider a single diffractor at $(x_1, x_3) = (0, 600)$ m in a homogeneous medium with propagation velocity $c = 2000$ m/s, see Figure 2, in which C denotes the diffractor. Further, we define $x_A = (-500, 100)$ m and $x_B = (500, 100)$ m, denoted by A and B in Figure 2. The surface $\partial \mathbb{D}_1$ is a semi-circle with its center at the origin and a radius of 800 m. The solid arrows in Figure 2 denote the Green’s function $G(x_A, x_B, t)$. For the Green’s functions in equation 2 we use analytical expressions, based on the Born approximation (hence, the contrast at the point diffractor is assumed to be small). To be consistent with the Born approximation, in the cross-correlations we also consider only the zeroth and first order terms. Figure 3a shows the time-domain representation of the integrand of equation 2 (convolved with a wavelet with a
central frequency of 50 Hz). Each trace corresponds to a fixed source position \( x \) on \( \partial \Omega_1 \); the source position in polar coordinates is \((\phi, r = 800)\). The sum of all these traces (multiplied by \( r d\phi \)) is shown in Figure 3b. This result accurately matches the time-domain version of the left-hand side of equation 2, i.e., \( G(x_A, x_B, t) + G(x_A, x_B, -t) \), convolved with a wavelet. Figure 3 clearly shows that the main contributions come from Fresnel zones around the stationary points of the integral. The causal contributions come from the indicated stationary points in Figure 2 between \( \phi = 0^\circ \) and \( 45^\circ \), the anticausal contributions from the indicated points between \( \phi = 135^\circ \) and \( 180^\circ \). The contributions from the indicated stationary points around \( \phi = 90^\circ \) cancel each other.

Note that when the sources at \( \partial \Omega_1 \) are uncorrelated noise sources, the right-hand side of equation 2 reduces to a direct crosscorrelation of the observed wavefields at \( x_A \) and \( x_B \), see Draganov et al. (2006) for a real data example.

**CONFIGURATION FOR EXPLORATION DATA**

For the situation of exploration data, sources are only available at the acquisition surface. Again we divide the closed surface \( \partial \Omega \) into two parts, this time a part \( \partial \Omega_0 \) coinciding with the acquisition surface and an arbitrarily chosen source-free part \( \partial \Omega_1 \) in the subsurface, see Figure 4. In the following we assume that source related multiples have been eliminated, hence the acquisition surface \( \partial \Omega_0 \) is assumed non-reflecting (if we would assume a free surface there would be no integral left to be evaluated). Assuming the responses of the sources at \( \partial \Omega_0 \) are measured by receivers at \( x_A \) and \( x_B \) in the subsurface (for example in a VSP, a vertical array, a horizontal well, or at the ocean bottom), crosscorrelation and integration along the sources on \( \partial \Omega_0 \) yields an approximation of the Green’s function \( \tilde{G}(x_A, x_B, \omega) \). The fact that the integral over \( \partial \Omega_1 \) cannot be evaluated due to the absence of sources in the subsurface means that not only the amplitudes of the direct wave and primary reflections may be erroneously reconstructed, but also that the internal multiples are incorrectly handled and that spurious multiples may occur. The occurrence of spurious multiples is extensively discussed by Snieter et al. (2006). Here we illustrate it with a simple plane-wave experiment.

Consider a horizontally layered medium, consisting of 25 layers with a thickness of 20 m each, with random velocities around an average velocity of 2000 m/s, represented by the first 500 m of the velocity profile in Figure 5. We define \( x_A = (0, 100) \)m and \( x_B = (0, 300) \)m (as in a vertical seismic profile). A vertically downward travelling plane wave is incident to this configuration at \( x = (0, 0) \). The responses at \( x_A \) and \( x_B \) are shown in Figure 6. We use equation 2 to reconstruct the Green’s functions \( \tilde{G}(x_A, x_B, t) + \tilde{G}(x_A, x_B, -t) \). For this plane-wave experiment we replace the integral over \( \partial \Omega_0 \) by a direct crosscorrelation \( \tilde{G}(x_A, x_B, t) \). The result is shown in Figure 7. Note the asymmetry and the occurrence of spurious multiples around \( t = 0 \) in the correlation result due to negligence of the contribution from \( \partial \Omega_1 \). This result could be made exact by adding a similar contribution from a vertically upward propagating wave illuminating the layered medium from below.

**ANALYSIS OF THE NEGLECTED BOUNDARY INTEGRAL**

Consider again the configuration of Figure 4. The
fact that the closed boundary integral of equation 1 or 2 needs to be replaced by an open boundary integral over \( \partial \mathcal{D}_0 \) implies a number of approximations, as we have seen above. Let us have a closer look at the neglected part of the integral over \( \partial \mathcal{D}_1 \) in the exact representation of equation 1. Let \( \partial \mathcal{D}_1 \) be a half-sphere with radius \( r_\text{B} \). If we take \( r_\text{B} \to \infty \) and assume that the medium is homogeneous outside some finite domain \( \mathcal{D}_0 \), then the Green’s functions under the integral are \( O(1/r_\text{B}) \) and each of the correlation products is \( O(1/r_\text{B}^2) \). In the corresponding convolution-type theorem the two terms of \( O(1/r_\text{B}) \) cancel each other, making the integrand \( O(1/r_\text{B}^2) \). However, in the correlation-type theorem of equation 1 this cancellation does not take place, which means that the integrand is \( O(1/r_\text{B}) \). Since the surface area of the integration boundary \( \partial \mathcal{D}_1 \) increases with \( r_\text{B}^2 \), the integral over \( \partial \mathcal{D}_1 \) in equation 1 (and also in equation 2) is \( O(1) \). In other words, the boundary integral over \( \partial \mathcal{D}_1 \) does not vanish when \( r_\text{B} \to \infty \). For a plane-wave experiment, as in the example above, we arrive at a similar conclusion. For \( z \to \infty \) the Green’s functions are \( O(1) \) and so are the crosscorrelations. Again no cancellation takes place and, since the integral is omitted, the end result is also \( O(1) \).

For a plane-wave experiment in a horizontally layered medium we find, following the same reasoning as in the previous section, that the contribution from the lower boundary decreases with \( T^2(z) \) and vanishes for \( z \to \infty \). An example of \( T^2(z) \) is shown in Figures 8 and 9. Figures 10 and 11 show that the energy that is lost in the transmission response is transferred to the reflection response (bear in mind we assumed a lossless medium from the very start). Similar as the free surface acted as a mirror in the situation for sources in the subsurface, the inhomogeneous medium acts as a ‘mirror’ for sources at the surface (but this mirror has a very complex phase behavior).

We illustrate the reconstruction of \( G(x_A, x_B, t) + G(x_A, x_B, -t) \) for a plane-wave experiment in a horizontally layered medium. This time we consider the complete velocity profile of Figure 5. We saw already in Figures 8 and 9 that the transmitted energy vanishes, hence, we expect a good reconstruction of the Green’s function. Again a vertically downward travelling plane wave is incident to this configuration at \( x = (0,0) \). The responses at \( x_A = (0,100) \text{m} \) and \( x_B = (3,300) \text{m} \) are shown in Figure 12. The main difference with the responses in Figure 6 is the longer coda. The crosscorrelation result is shown in Figure 13. Note that this response is perfectly symmetric and that the spurious events around \( t = 0 \) have disappeared. Apparently the crosscorrelation of the long codas have contributed to the improved

**Fig. 6: Responses at \( x_A \) and \( x_B \).**

**MODIFIED EXTINCTION CONDITION**

The integral over \( \partial \mathcal{D}_1 \) does not vanish when \( r_\text{B} \to \infty \), assuming a homogeneous medium outside some finite domain, but what happens when the medium is inhomogeneous throughout \( \mathcal{D} \) (i.e., the domain enclosed by \( \partial \mathcal{D} \))? Due to internal multiple scattering, the integrand will be \( O(T^2(r_\text{B})/r_\text{B}^2) \), where \( T^2(r_\text{B}) \) is a decaying function accounting for transmission loss. The precise behavior of \( T^2(r_\text{B}) \) depends on the type and distribution of the inhomogeneities, but what matters is that it will vanish for \( r_\text{B} \to \infty \). The integration surface area increases again with \( r_\text{B}^2 \), hence, the integrand is now \( O(T^2(r_\text{B})) \), which means that it vanishes for \( r_\text{B} \to \infty \). Hence, equation 1, with closed surface \( \partial \mathcal{D} \) replaced by the acquisition surface \( \partial \mathcal{D}_0 \) (see Figure 4), is exact when the medium is inhomogeneous throughout the lower half-space (and equation 2 is a good approximation for the same situation).

This is a quite remarkable result. Usually one starts with deriving a modelling or processing scheme for a simplified situation; complications only arise when it is applied to a more realistic situation. Here we see the opposite happening. For the situation of a simple subsurface configuration embedded in a homogeneous medium, equation 1 or 2 (with \( \partial \mathcal{D} \) replaced by \( \partial \mathcal{D}_0 \)) involves erroneous amplitudes and spurious multiples. For the more realistic situation of an inhomogeneous subsurface, equation 1 (with \( \partial \mathcal{D} \) replaced by \( \partial \mathcal{D}_0 \)) becomes exact.

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reconstruction of the response and to the suppression of spurious events at early times.

CONCLUDING REMARKS

From the theory discussed in this paper as well as from the numerical example it follows that seismic interferometry applied to exploration data (Figure 4) benefits from the fact that the Earth’s subsurface is inhomogeneous. Errors that would occur in the reconstructed Green’s function when the response of only a few layers would be available are suppressed by crosscorrelating the full response of the inhomogeneous subsurface. This leads to the recommendation that much longer traces should be recorded than the usual four seconds in seismic exploration. To avoid long time intervals between the shots, the data could be recorded in a continuous mode, yielding a superposition of time-shifted shot records [see for example Shiraishi et al. (2005)]. Correlating the superposed traces replaces the integral along the sources but also introduces undesired cross-terms. This needs further investigation.

The reconstruction of the Green’s function is the result of a complex interference of crosscorrelated primaries and multiply scattered events, present in the coda of the response. It has been observed before that coda waves are surprisingly stable [Fink (1997), Snieder and Scales (1998)], hence, we expect that this is not a limiting factor for practical applications. Note that, despite the complexity of the coda, this reconstruction process is fully deterministic and thus does not rely on diffusivity and equipartitioning assumptions, as in some of the references mentioned in the introduction.

Throughout the paper we have assumed that the medium is lossless. Investigations by Slob et al. (2006) for electromagnetic passive data indicate that when the losses are small, interferometry yields Green’s functions with correct traveltimes and approximate amplitudes. It remains to be investigated how anelastic losses will degrade the Green’s function reconstruction for exploration data, as discussed in this paper.

References


