Adaptive double-focusing method for source-receiver Marchenko redatuming on field data
Myrna Staring (Delft University of Technology), Roberto Pereira (CGG), Huub Douma, Joost van der Neut (Delft University of Technology) and Kees Wapenaar (Delft University of Technology)

SUMMARY
We present an adaptive double-focusing method for applying source-receiver Marchenko redatuming to field data. Receiver redatuming is achieved by a first focusing step, where the coupled Marchenko equations are iteratively solved for the one-way Green’s functions. Next, source redatuming is typically performed by a multi-dimensional deconvolution of these Green’s functions. Instead, we propose a second focusing step for source Marchenko redatuming, using the upgoing Green’s function and the downgoing focusing function to obtain a redatumed reflection response in the physical medium. This method makes adaptive processing more straightforward, making it less sensitive to imperfections in the data and the acquisition geometry and more suitable for the application to field data. In addition, it is cheaper and can be parallelized by pair of focal points.

INTRODUCTION
The Santos basin offshore Brazil contains pre-salt reservoirs below a highly reflective salt structure (Cypriano et al. (2015)). This salt structure generates internal multiples that interfere with the primary reflections in the target area (figure 1a). Since current imaging techniques assume that the recorded wavefields have been reflected only once, internal multiples appear as phantom reflectors in the image (figure 1b). In order to obtain an image of the reservoir that is free from artefacts due to internal multiples, interactions with a complex overburden have to be accurately removed from the reflection response. This can be achieved using Marchenko redatuming, a data-driven method that recovers the redatumed reflection response at any depth level, without needing physical sources and receivers inside the medium (Broggini et al. (2012); Wapenaar et al. (2014)). This is a two-step process (see figure 2), where receiver redatuming is achieved first by iteratively solving the coupled Marchenko equations. This results in Green’s functions that travel from a source at the acquisition surface to a virtual receiver at the redatuming level, correctly accounting for all orders of internal multiples. The Marchenko method constructs these multiples using both convolutions and cross-correlations, comparable to other internal multiple removal methods (Weglein et al. (1997); Jakubowicz (1998); Hung and Wang (2012)). However, unlike related methods, the strength of the Marchenko method is that it in principle retrieves all orders of internal multiples at any desired depth level, without the need to resolve the overlying layers first. When using the retrieved one-way Green’s functions for redatuming, a reflection response that is free of artefacts due to multiple scattering in the overburden will result.

In this paper, we focus on the second redatuming step: source redatuming using the wavefields retrieved from the coupled Marchenko equations. This is typically done using a multi-dimensional deconvolution (MDD), which uses the one-way Green’s functions (Wapenaar et al. (2014)). The result is a redatumed reflection response in a truncated medium, where all interactions with the overburden have been removed (see figure 3). However, performing the MDD is equal to solving an inverse problem (when solving $\tilde{G} = \int R \ast \tilde{G}_{+}$, inversion is required to find our redatumed reflection response $R$ that resides inside the integrand) and comes with the accessory limitations. The ill-posed inversion has to be stabilized and artefacts can appear when illumination is incomplete (van der Neut et al. (2011)). Therefore, it is sensitive to imperfections in the acquisition geometry and the data. While we can ensure that synthetic data does not have these imperfections, this poses a problem for the field data application (Ravasi et al. (2016)). In addition, this processing step is computationally expensive. Hence, we desire an alternative that is cheaper and less sensitive to imperfections in the recorded data and the acquisition geometry.

Figure 1: a) RTM image of a 2D model from the Santos basin, b) same image but with the model being homogeneous below the base of salt, such that only the multiples generated in the overburden are visible.

Figure 2: Illustration of a) the reflection response as measured at the acquisition surface, b) receiver redatuming c) source-receiver redatuming. The seismic image in the background is the same in all sub-figures and only meant to illustrate an inhomogeneous medium.

Figure 3: a) a virtual receiver for the MDD, b) the redatumed response for the MDD.
Adaptive double-focusing method for source-receiver Marchenko redatuming on field data

AN ALTERNATIVE METHOD

First, receiver redatuming is achieved by iteratively solving the coupled Marchenko equations, following Wapenaar et al. (2014). As input, we need an accurate reflection response at the surface and a smooth velocity model. This results in one-way focusing functions and one-way Green’s functions at specified focal points. Second, we perform source redatuming using the output wavefields from the Marchenko method. While the upgoing and downgoing Green’s functions were used for the MDD, we now select the upgoing Green’s function and the downgoing focusing function. Using these wavefields for redatuming, we have replaced the multi-dimensional deconvolution step by a second focusing step, creating a ‘source Marchenko redatuming’ that seamlessly complements the already achieved ‘receiver Marchenko redatuming’. We convolve the downgoing focusing function at a virtual source location to create downward-radiating virtual sources and upward-measuring virtual receivers at the redatuming level (Wapenaar et al. [2016]: Singh et al. [2016]):

\[ \hat{G}^{-+}(x_{fr}, x_{rs}, \omega) = \int_{D_0} \hat{G}^{-}(x_{fr}, x, \omega) \hat{f}^+(x, x_{rs}, \omega) \, d^2 x \]  

(1)

Here, \( x_{fr} \) and \( x_{fr} \) represent virtual source and receiver locations at the redatuming level, while \( x \) indicates positions at the acquisition level. The band-limitation of the Green’s function and focusing function is indicated by the \( \hat{\cdot} \) symbol. Application of this equation results in the wavefield \( \hat{G}^{-+} \), the upgoing Green’s function measured by a virtual receiver at the redatuming level due to a downgoing virtual source at the redatuming level. This response is different from the wavefield \( R \) retrieved by MDD: we have now redatumed in the physical medium instead of in the truncated medium (see figure [3]). Therefore, waves that propagate from the virtual source downwards into the target, back up into the overburden, back down into the target, and then again up to the virtual receiver will not be removed (see figure [3]). However, we do not expect these interactions to interfere with the primary reflections from the reservoir in the geological settings of the Santos basin. In addition, note that we integrate over the acquisition surface, while the multi-dimensional deconvolution requires integration over the redatuming level (Wapenaar et al. [2014]). As a consequence, the MDD requires an array of focal points as input. In contrast, the proposed method provides the flexibility to apply source-receiver Marchenko redatuming to a single pair of focal points only, which is useful for parallelization of the algorithm. Also, the proposed method is computationally much cheaper than MDD.

Furthermore, both the upgoing Green’s function and the downgoing focusing function possess properties that are useful for us. van der Neut and Wapenaar [2016] observed that the initial estimate of the upgoing Green’s function \( \hat{G}^{-} \) already contains all correct physical arrivals. However, it also contains artefacts that we would like to see removed from this Green’s function. The first update \( \hat{G}^{-} \) contains all the necessary counter-events to take care of these artefacts, just with the wrong amplitudes. Following updates only correct the amplitudes of these counter-events until they match and completely eliminate the artefacts. The story is similar for the downgoing focusing function, where the first estimate \( \hat{f}^{+} \) already contains all physical information, while its first update \( \hat{f}^{+} \) takes care of the artefacts due to internal multiples. Again, consecutive updates will only alter the amplitudes. Based on these dynamics, the selected wavefields are perfectly suitable for adaptive subtraction. We would only need the initial terms and their first updates, and substitute the amplitude corrections in next updates by an adaptive filter. In addition, we expect this method to be less sensitive to imperfections in the data and the medium assumptions, since the adaptive filter can correct for the amplitude mismatch of the updates. Note that adaptive subtraction can also be applied to the MDD (when writing it as a series [van der Neut and Wapenaar [2016]]), but this is less straightforward.

Since iterative substitution of the coupled Marchenko equations is equal to solving a Fredholm equation of the second kind, we can directly express the retrieval of our desired wavefields as a Neumann series (van der Neut et al. [2015]):

\[ \hat{G}^{-}(x_{fr}, x, t) = \sum_{i=0}^{m} \hat{G}^{-}_i = \Psi R \sum_{j=0}^{m} \Omega \hat{f}^{+}_j, \]  

(2)

and

\[ \hat{f}^{+}(x, x_{rs}, t) = \sum_{j=0}^{m} \hat{f}^{+}_j = \sum_{j=0}^{m} \Omega \hat{G}^{-}_j. \]  

(3)

Here \( \hat{G}^{-} \) and \( \hat{f}^{+} \) represent updates of the upgoing Green’s function and the downgoing focusing function respectively, where \( i \) and \( j \) indicate the number of iterations. The scheme is initiated with the direct wave of the downgoing focusing function \( \hat{f}^{+}_0 \), which can be obtained from a smooth velocity model. The reflection response \( R \) is assumed to be free of source signature, noise and surface-related multiples. The symbol \( \Omega = \theta R^T \theta R \) represents an operator that applies first a convolution and then a cross-correlation with the reflection response \( R \) to \( \hat{f}^{+}_0 \). After every convolution or cross-correlation, a time-symmetric window \( \theta \) is applied to the result to separate the focusing function from the Green’s function. Applica-
Adaptive double-focusing method for source-receiver Marchenko redatuming on field data

tion of the window function $\theta$ results in the focusing function, while the window $\Psi = I - \theta$ is applied to obtain the Green’s function. The properties of the upgoing Green’s function and the downgoing focusing function allow us to write this equation as a series, using equations 2 and 3:

$$\hat{G}^{++}(x_{vr}, x_{vs}, \omega) =$$

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \int_{D_0} \hat{G}(x_{vr}, x_{t}) \hat{f}_i^{+}(x, x_{vs}, t) d^2x$$

$$\approx \int_{D_0} \hat{G}_0(x_{vr}, x_{t}) \hat{f}_0^{+}(x, x_{vs}, t) d^2x$$

$$+ \int_{D_0} \hat{G}_1(x_{vr}, x_{t}) \hat{f}_0^{+}(x, x_{vs}, t) d^2x$$

$$+ \int_{D_0} \hat{G}_0(x_{vr}, x_{t}) \hat{f}_1^{+}(x, x_{vs}, t) d^2x.$$ (4)

These terms use the fields $\hat{G}_0$, $\hat{G}_1$, $\hat{f}_0^+$ and $\hat{f}_1^+$ that include all the events needed for source Marchenko redatuming, except with the wrong amplitudes. Note that this approximation only includes terms for which the data has been correlated no more than twice, thus excluding all higher-order terms. Correlating the data with itself rapidly degrades the quality of the updates, especially when the data is incomplete or contains a band-limitation. The first term on the right-hand side of equation 4 contains the result of conventional redatuming (using the direct wave $f_0^+$) including both primaries and internal multiples, while the second and third terms contain the first-order predictions of multiples at the receiver and source sides respectively, with opposite polarity compared to the first term. In order to avoid needing the amplitude updates from the higher order terms, we add the three terms with an adaptive filter. Throughout this work, we have used an adaptive subtraction in the curvelet domain (e.g., Wu and Hung (2015)), because curvelets provide extra flexibility when multiples coincide with primaries in time and space, but not in slope.

**COMPARISON OF METHODS ON 2D SYNTHETIC DATA**

To illustrate the workings of the proposed method, a 2D synthetic dataset from the Santos basin is used (see figure 1 for the RTM image). Synthetics were generated in a model obtained from an acoustic inversion of field data. As such it can be considered a realistic model that generates realistic internal multiples that would be observed on field data from this area. The reflection response was generated on a line with 601 co-located sources and receivers with a spacing of 25 m, and a band-limitation in the form of an Ormsby wavelet with a central frequency of 35 Hz was imposed. After two iterations of solving the coupled Marchenko equations, convolving the individual updates of $\hat{G}^-$ and $\hat{f}^+$ with each other, and only keeping the terms that have been convolved no more than twice, the three terms of equation (4) result. An example of these terms is shown in Figure 4 for a virtual source location in the middle of the array. It can clearly be seen that the second and third terms contain counter-events for the artefacts in the first term. Figure 5 shows a comparison between source-receiver redatuming using MDD and the adaptive double-focusing method. On the left is the result of modeling a reflection response in a medium with a homogeneous overburden above the redatuming level. As such it can be used as a guide to see how well both methods work. MDD uses the upgoing and downgoing one-way Green’s functions that result from two iterations of solving the coupled Marchenko equations. We apply a mute
Adaptive double-focusing method for source-receiver Marchenko redatuming on field data

Figure 7: Images resulting from the application of adaptive source-receiver Marchenko redatuming to 2D field data.

(indicated by the white lines in figure 5) to both the MDD and the adaptive double-focusing results to remove the acausal parts. When comparing the two approaches to the modeled result, it is clear that the adaptive double-focusing method is capable of producing an improved result over MDD, even though a medium truncation is not achieved. This implies that multiples due to remaining interactions between the overburden and the target area are negligible in this example.

Figure 6 shows the images obtained after RTM. Figure 6a shows the result before Marchenko redatuming to allow comparison with the image obtained when migrating the data including all internal multiples from the surface. Both the MDD and the adaptive double-focusing method in Figures 6c and 6d remove multiples well (cf. Figure 6a), while the proposed method produces a somewhat improved result that compares better to the modeled result in Figure 6b. For convenience of the reader, we have indicated the multiples in Figure 6a by arrows. See also Figure 1b for an example of what the artefacts due to internal multiples look like in the image domain for this synthetic example. In addition, the circles and arrows in Figures 6c and 6d highlight a few areas with subtle differences. We refer the reader to Staring et al. (2017) for two examples that demonstrate that the proposed method is indeed less sensitive to imperfections in the data and the acquisition geometry.

2D FIELD DATA RESULTS

After the concept of this method was demonstrated on 2D synthetic data, we tested the method on 2D field data. We used data of the Santos basin that was acquired in the same region as covered by our synthetic tests. The acquisition consisted of 6 streamers with 6000 m cable length and 150 m cable spacing. We regularized shots and receivers on the same line. The processing prior to regularization included de-noise, de-signature, de-ghosting and surface related multiple attenuation, in order to satisfy the assumptions that underlie the Marchenko scheme. Figure 7 shows the comparison of the initial term $G_0 f_0^+$ containing all primaries and the artefacts due to internal multiples, and the final result of the source-receiver Marchenko redatuming using adaptive subtraction in the curvelet domain. Red circles and arrows were placed to highlight the effect of the proposed method on the internal multiples in the data. Especially note the change indicated in the blue circle, where our method has made a difference for the interpretation. Based on this result, we can conclude that the adaptive double-focusing method for source-receiver Marchenko redatuming has successfully removed internal multiples from 2D field data. Since field data is never 2D in reality, we are missing out-of-plane interactions in this example. Therefore, we expect our method to perform even better on 3D field data.

CONCLUSION

A method to apply adaptive source-receiver Marchenko redatuming was presented and tested on 2D synthetic data and 2D field data. By replacing the multi-dimensional deconvolution step by a second focusing step, we have obtained a method that is more suitable for adaptive subtraction. This results in less sensitivity to imperfections in the data and the acquisition geometry, which is required for a successful application to field data. In addition, the method is much cheaper than MDD and can be parallelized by focal point. A disadvantage is that the redatumed response exists in the physical medium, such that some interactions with the overburden remain. Comparison with the MDD on 2D synthetic data has shown that the adaptive double-focusing method manages to obtain a cleaner redatumed reflection response using the same amount of iterations. Moreover, application to 2D field data was successful and has even improved interpretation, despite an imperfect acquisition geometry and imperfect data. Therefore, we conclude that the adaptive double-focusing method for applying source-receiver Marchenko redatuming is preferred over the MDD, particularly for field data.

ACKNOWLEDGMENTS

This research has been performed in the framework of the project "Marchenko imaging and monitoring of geophysical reflection data", financially supported by the Dutch Technology Foundation STW, applied science division of NWO and the Technology Program of the Ministry of Economic Affairs. We thank CGG Rio de Janeiro for a fruitful collaboration, and Manuel Peiro (CGG Rio) for preparing the model used to create the synthetic 2D data for the tests.