Marchenko scheme based internal multiple reflection elimination in acoustic wavefield

Lele Zhang*, Myrna Staring

Delft University of Technology, 2628CN, Delft, The Netherlands

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A B S T R A C T

Marchenko imaging is a methodology to image the subsurface with two important properties: (1) accurate amplitude and (2) free from free-surface and internal multiple artefacts. It requires an estimate of the first arrival of the focusing function which is commonly obtained from a macro velocity model. Inspired by this limitation, a projected Marchenko scheme has been introduced from which an internal multiple reflection elimination scheme has been derived. This internal multiple reflection elimination scheme requires an estimate of the two-way travel time surface of a selected horizon in the subsurface instead of a macro model. In order to make it totally model free we have rewritten the scheme by replacing the estimate of the two-way travel time surface by a fixed truncation for all traces. The output of the current scheme contains primary reflections without contamination from internal multiple reflections. We apply this scheme to a 2D numerical example to illustrate the procedure of this method and show how the internal multiple reflection eliminated data set can be retrieved and the migration image is improved.

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1. Introduction

Strong artefacts due to internal multiple reflections can occur in migration of images produced from marine (Hadidi and Verschuur, 1997; Van Borselen, 2002) and land data (Kelamis et al., 2006). Several schemes have been developed to predict and subtract internal multiple reflections from measured data, such as internal multiple elimination (IME) (Berkhout and Verschuur, 2005) and inverse scattering series (ISS) (Weglein et al., 1997). For IME, the identification of the internal multiple reflection generators in the input data is required. The subtraction of the predicted internal multiple reflections is applied by a least-square matching filter with a minimum-energy criterion. For ISS, the internal multiple reflections can be predicted with approximated amplitude (Löer et al., 2016). Ten Kroode et al. (2002) derive a multiple reflection elimination scheme from ISS and source-receiver interferometry with specific truncation. However, global or local matching filter is usually required to subtract the predicted internal multiple reflections from measured data (de Melo et al., 2014).

Recently, Marchenko imaging has been introduced to deal with internal multiple reflections (Slob et al., 2014; Wapenaar et al., 2014a; da Costa Filho et al., 2015). For this scheme, up- and downgoing focusing functions with focal point at arbitrary position in the subsurface can be retrieved by solving the coupled Marchenko equations and up- and downgoing Green's functions can be solved from Green's function representations by using the solved focusing functions as inputs. By deconvolving the retrieved upgoing Green's function with the downgoing Green's function, a virtual reflection response with virtual source and virtual receivers in the subsurface can be obtained. The virtual reflection response forms the basis for obtaining an artefact-free image when the zero-time-lag crosscorrelation between the retrieved up- and downgoing virtual responses is extracted at any image point (Wapenaar et al., 2014b; Broginni et al., 2014). Based on Marchenko redatuming and convolutional interferometry, an approximate primary reflection retrieval scheme has been proposed (Meles et al., 2016). Van der Neut and Wapenaar (2016) rewrite the coupled Marchenko equations by projecting them to the acquisition surface. Based on the revised Marchenko scheme an adaptive overburden elimination approach is introduced. All orders of internal multiple reflections above a specified horizon can be removed without having to remove internal multiple reflections from shallower horizons.

Based on the revised Marchenko equations, van der Neut and Wapenaar (2016) present a scheme to eliminate internal multiple reflections from the measured acoustic wave field and apply it to a 1D numerical example. In order to make it totally model free we have rewritten the scheme by replacing the estimate of the two-way travel time surface by a fixed truncation for all traces in this paper. The rewritten scheme entirely obviates the requirement of estimating the first arrival in the focusing wave field from one-sided Marchenko equations (Slob et al., 2014; Wapenaar et al., 2014b) to eliminate internal multiple reflections from the measured acoustic wave field. One-sided reflection data is required as input. Because of the elimination of internal multiple...
reflections, the retrieved data set contains only primary reflections. The obtained data set is more suitable for subsequent AVO or AVA analysis and as input for velocity model building. The updated model can subsequently be used to image the medium, it can also be used to image the medium without artefacts due to internal multiple reflections. We present a 2D numerical example to illustrate how a dataset free from internal multiple reflections can be retrieved using the current scheme leading to improved quality in the resulting migrated image.

2. Theory

We indicate time as $t$ and the position as $\mathbf{x} = (x,y,z)$, where $z$ denotes depth and $(x,y)$ denotes the horizontal coordinates. The acoustically transparent acquisition boundary $\partial D_0$ is defined as $z_0 = 0$. For convenience, the coordinates at $\partial D_0$ are denoted as $\mathbf{x}_0 = (x_0, y_0, z_0)$, with $\mathbf{x}_0 = (x, y)$. Similarly, the position at an arbitrary depth level $\partial D_0$ is denoted as $\mathbf{x}_0 = (x_0, y_0, z)$, where $z$ denotes the depth of $\partial D_0$. We express the acoustic impulse reflection response as $R(\mathbf{x}_0', \mathbf{x}_0, t)$, where $\mathbf{x}_0$ denotes the source position and $\mathbf{x}_0'$ denotes the receiver position, both located at the acquisition surface $\partial D_0$. In practice, it means that free-surface related multiple reflections should first be removed from the measured reflection response, in which step the source locations are redatumed to the receiver depth level and the source wavelet should be recovered and accurately deconvolved from the data. The focusing wave field $f_1(t, \mathbf{x}_0)$ is the solution of the homogeneous wave equation in a truncated medium and focuses at the focal point $\mathbf{x}_0$. We define the truncated domain as $\partial D_0 < z < \partial D$ with $z_0 < z < z_f$. Inside the truncated domain, the properties of the medium are equal to the properties of the physical medium. Outside the truncated domain, the medium is reflection-free. The Green's function $G(\mathbf{x}_0', \mathbf{x}_0, t)$ is defined for an impulse source that is excited at $\mathbf{x}_0$ and a receiver is positioned at the focal point $\mathbf{x}_0$. The Green's function is defined in the same physical medium as the measured data. The focusing and Green's functions can be partitioned into up- and downgoing constituents and for this we use power-flux normalized quantities (Wapenaar et al., 2014a).

We start with the 3D versions of one-way reciprocity theorems for flux-normalized wave fields and use them for the depth levels $z_0$ and $z_f$. When the medium above the acquisition level $z_0$ is reflection-free, we have the Green's function representations (Wapenaar et al., 2014a),

$$G^{-}(\mathbf{x}, \mathbf{x}_0, -t) = \int_{\partial D_0} d\mathbf{x}_0 \int_{-\infty}^{0} R(\mathbf{x}_0', \mathbf{x}_0, t') f_1^{+}(\mathbf{x}_0', \mathbf{x}, t-t') dt', f_1^{-}(\mathbf{x}_0', \mathbf{x}, t),$$

Superscripts $+$ and $-$ stand for downgoing and upgoing fields, respectively. The downgoing component of the focusing function $f_1^{-}(\mathbf{x}_0, \mathbf{x}_0, t)$ is the inverse of the transmission response in the truncated medium. We can write both the focusing function and the transmission response as the sum of a direct part and a coda

$$f_1^{-}(\mathbf{x}_0, \mathbf{x}_0, t) = f_1^{\text{id}}(\mathbf{x}_0, \mathbf{x}_0, t) + f_1^{\text{m}}(\mathbf{x}_0, \mathbf{x}_0, t),$$

$$T(\mathbf{x}_0, \mathbf{x}_0, t) = T_d(\mathbf{x}_0, \mathbf{x}_0, t) + T_m(\mathbf{x}_0, \mathbf{x}_0, t),$$

where $f_1^{\text{id}}$ and $T_d$ indicate the direct part, whereas $f_1^{\text{m}}$ and $T_m$ indicate the following coda. Wapenaar et al. (2014b) show that

$$\int_{\partial D_0} d\mathbf{x}_0 \int_{0}^{t} T_m(\mathbf{x}_0, \mathbf{x}_0', t') f_1^{\text{id}}(\mathbf{x}_0, \mathbf{x}_0, t-t') dt' = \delta(\mathbf{x}_0' - \mathbf{x}_0) \delta(t),$$

where $\delta(\mathbf{x}_0)$ is a spatially band-limited 2D delta function in space and $\delta(t)$ is a delta function in time. Eq.(5) means that $T_d$ is the inverse of $f_1^{\text{id}}$. Following van der Neut and Wapenaar (2016), we apply multidimensional convolution with $T_d$ as shown in Eq.(5) to Eqs.(1) and (2) to find

$$U^{-}(\mathbf{x}_0, \mathbf{x}_0', t) + v^{-}(\mathbf{x}_0, \mathbf{x}_0', t) = \int_{\partial D_0} d\mathbf{x}_0 \int_{0}^{t} R(\mathbf{x}_0', \mathbf{x}_0, t') \left\{ \delta(t-t') \delta(\mathbf{x}_0' - \mathbf{x}_0) + v_m^{-}(\mathbf{x}_0', \mathbf{x}_0, t-t') \right\} dt',$$

$$U^{-}(\mathbf{x}_0, \mathbf{x}_0', t) + v^{-}(\mathbf{x}_0, \mathbf{x}_0', t) = \int_{\partial D_0} d\mathbf{x}_0 \int_{0}^{t} R(\mathbf{x}_0', \mathbf{x}_0, t') v^{-}(\mathbf{x}_0, \mathbf{x}_0', t-t') dt',$$

where $v_m^{-}$ is convolved version of $f_1^{\text{m}} - U^{-}$ and $U^{+}$ are convolved versions of $G^{-}$ and $G^{+}$, similar as is shown in Eq.(8) for $f_1^{-}$. Based on the fact that the convolved Green's and focusing functions in Eqs.(6) and (7) are separated in time except for the first event in the convolved downgoing focusing function and last event in the convolved time-reversed downgoing Green's function in Eq.(7) (both of them are delta functions after the convolution) that coincide with each other. We rewrite Eqs.(6) and (7) as

$$v^{-}(\mathbf{x}_0, \mathbf{x}_0', t) = \int_{\partial D_0} d\mathbf{x}_0 \int_{0}^{t} R(\mathbf{x}_0', \mathbf{x}_0, t') \left\{ \delta(t-t') \delta(\mathbf{x}_0' - \mathbf{x}_0) + v_m^{-}(\mathbf{x}_0', \mathbf{x}_0, t-t') \right\} dt',$$

for $t < t_2 - \varepsilon$

$$v_m^{-}(\mathbf{x}_0, \mathbf{x}_0', t) = \int_{\partial D_0} d\mathbf{x}_0 \int_{0}^{t} R(\mathbf{x}_0', \mathbf{x}_0, t-t') v^{-}(\mathbf{x}_0, \mathbf{x}_0', t-t') dt',$$

for $t < t_2 - \varepsilon$

where $t_2$ denotes the two-way travel time from a surface point $\mathbf{x}_0'$ to the focusing level $z_f$ and back to the surface point $\mathbf{x}_0$ and $\varepsilon$ is a positive value to account for the finite bandwidth. Then we give Eqs.(9) and (10) in the operator form as

$$v^{-}(\mathbf{x}_0, \mathbf{x}_0', t) = (\Theta(\mathbf{x}_0' - \mathbf{x}_0) + \Theta(\mathbf{x}_0 - \mathbf{x}_0') R_v^{-}(\mathbf{x}_0')) v_m^{-}(\mathbf{x}_0, \mathbf{x}_0', t),$$

where $R_v^{-}(\mathbf{x}_0')$ is a positive value
Fig. 2. (a) The modelled reflection response. (b) The retrieved primary reflections with $k = 2$ estimated from Eq. (16). (c) The retrieved primary reflections with $k = 5$ estimated from Eq. (16). (d) The retrieved primary reflections with $k = 10$ estimated from Eq. (16). (e) The retrieved primary reflections with $k = 15$ estimated from Eq. (16). (f) The retrieved primary reflections with $k = 20$ estimated from Eq. (16).
as a Neumann series to give the equation as

\[ v_m^\prime(x_0, x_0, t) = (\Theta_k^{z,z-} R^\prime \Theta_k^{z,z-}) (x_0, x_0, t), \]

(12)

where R indicates a convolution integral operator of the measured data \( R \) with any wave field and \( R^\prime \) a correlation integral operator. \( \Theta_k^{z,z-} \) is a time window to exclude values outside of the interval \((c, t_2 - c)\). Then we substitute Eq.(11) into Eq.(12) to get the final equation for \( v_m^\prime \) as

\[ (1-\Theta_k^{z,z-} R^\prime \Theta_k^{z,z-}) v_m^\prime(x_0, x_0, t) = (\Theta_k^{z,z-} R^\prime \Theta_k^{z,z-} R_0) (x_0, x_0, t). \]

(13)

Following (Van der Neut and Wapenaar, 2016) we expand Eq.(13) as a Neumann series to give the equation as

\[ v_m^\prime(x_0, x_0, t) = \sum_{k=1}^{\infty} (\Theta_k^{z,z-} R^\prime \Theta_k^{z,z-} R)^k \delta (x_0, x_0, t). \]

(14)

We substitute Eq.(14) into Eq.(6) to retrieve the projected \( U^\prime \) as

\[ U^\prime(x_0, x_0, t) = \Theta_k^{z,z-} R(x_0, x_0, t) \]

\[ + \sum_{k=1}^{\infty} \Theta_k^{z,z-} R(\Theta_k^{z,z-} R^\prime \Theta_k^{z,z-} R)^k \delta (x_0, x_0, t). \]

(15)

Eq.(15) shows that \( U^\prime \) can be evaluated for a single pair of surface points at a single time instant, which means that the time window after the sum of repeated correlations and convolutions can be taken with constant values \((c \text{ and } t_2 - c)\) for each source-receiver pair instead of a curve line corresponding to a horizontal subsurface level. The \( t_2 \) in Eq.(15) describes a fictitious focusing level in the subsurface where we have focused to and projected back from. When the focusing level coincides with an actual subsurface reflector, the first event in \( U^\prime \) at time instant \( t_2 \) will be the primary reflection of that reflector with two-way travel time \( t_2 \). Otherwise, the value in \( U^\prime \) at time instant \( t_2 \) will be zero. This means that \( U^\prime \) can be evaluated and its first event can be picked to represent a possible primary reflection event of the medium. The time instant \( t_2 \) can be chosen as \( t \) and we collect the value of \( U^\prime \) for each value of \( t \) and store it in a new function containing only primary reflections. We can write it as

\[ \mathcal{R}_f(x_0^*, x_0^*, t) = \mathcal{R}(x_0^*, x_0^*, t) + \sum_{k=1}^{\infty} \mathcal{R}(\Theta_k^{z,z-} R^\prime \Theta_k^{z,z-} R)^k \delta (x_0^*, x_0^*, t). \]

(16)

where \( \mathcal{R}_f \) denotes the retrieved primary reflections. The hat indicates that quantities have been convolved with source wavelet, \( \tau \) is the half width of the source wavelet to account for the finite bandwidth of the measured data.

Eq.(16) can be estimated for \( \mathcal{R}_f \). The retrieved data set is more suitable for velocity model estimation and standard imaging as presented by (Dokter et al., 2017). Moreover, we can see the operator as a mechanism to determine which parts of the data are predictable from the parts that are not. The non-predictable parts are retained in this expression. These include the primary reflections and forward scattered waves. The predictable parts are removed from this expression. These include all orders internal multiple reflections. The processing can be performed without knowledge of the model.

3. Example

The aim of the current method is to retrieve the primary reflections by removing the internal multiple reflections given the measured reflection response at one side of the medium. To illustrate the method, we use a 2D example of a numerical acoustic experiment. Fig. 1 shows the values for the acoustic impedance as a function of depth and horizontal position. The source emits a Ricker wavelet with 20Hz centre frequency. We have computed the single-sided reflection responses with 601 sources and 601 receivers on a fixed spread with a spacing of 10m at the top of the model. Absorbing boundary conditions are applied at the top of the model and the direct wave has been removed. One of the computed single-sided reflection responses \( \mathcal{R}(x_0, x_0^*, t) \) is shown in Fig. 2(a). Note that internal multiple reflections occur at later times. This reflection response is used as input to solve Eq.(16) for \( \mathcal{R}_f \). Fig. 2(b), (c), (d), (e) and (f) show the retrieved \( \mathcal{R}_f \) with \( k = 2, 5, 10, 15, 20 \) estimated from Eq.(16). Note that internal multiple reflections have been partially suppressed in Fig. 2(b), (c) and (d) with visible

![Fig. 3. The comparison of zero-offset traces from original and eliminated gather. The blue solid line (ORIG) indicates the zero-offset trace from original gather and red dotted line (ELIM) indicates the zero-offset trace from eliminated gather.](image)

![Fig. 4. (a) Image of modelled reflection responses and (b) image of retrieved primary reflections.](image)
4. Conclusions

We have rewritten the scheme in which the one-sided reflection response can be used as its own filter to remove internal multiple reflections by a fixed truncation for all traces. No model information is required and the intermediate time window is applied after each convolution or correlation. The 2D acoustic numerical example shows that the method effectively removes internal multiple reflections. We expect that the current method can be used in seismological reflection imaging and monitoring of structures and processes in the Earth’s interior. In all of these applications, the medium under investigation needs to be accessible only from one side.

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References


