Accelerating the Convergence of MIP-based Unit Commitment Problems

The Impact of High Quality MIP Formulations

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Optimization on Friday
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Outline

1. Short-term Planning (UC)
   - Unit Commitment
   - Why MIP?
   - Improving MIP Formulations
2. Case Studies
   - Deterministic Self-UC
   - Stochastic UCs: Different Solvers
   - Stochastic UCs: IEEE-118 Bus System
3. Conclusions & Collaboration?

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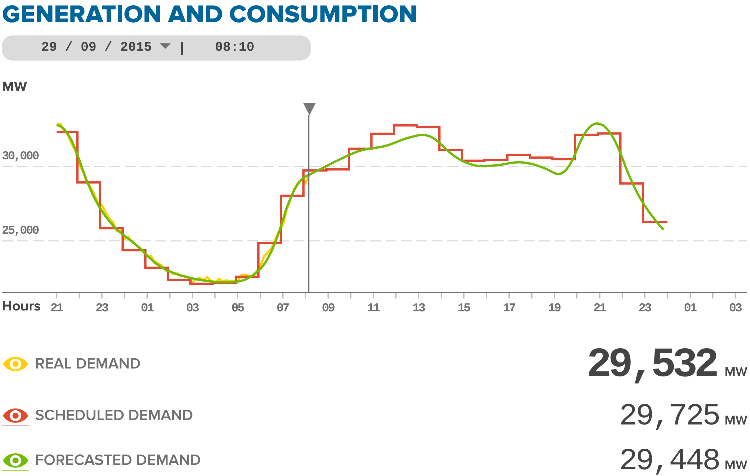
Day-ahead Planning



The system **has to** respond to instantaneous variations of the demand

Unit Commit for Short-term Planning

- Objective:
  - To schedule generating units (thermal, nuclear, hydro, etc.) at **minimum cost** (bids) over the study period subject to:
    - Meet system demand for electricity
    - Satisfy the system constraints (e.g., technical, environmental)
    - Provide a level of flexibility (reserves) to accommodate RES energy

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- Planning:
  - the physical operation
    - To make **Startup** and **shutdown** decisions
    - To obtain hourly schedules (production) for all generating units
  - the economic issues
    - **Give market signals** (prices)
    - Forecast operational cost

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- **MIP: Global optimum or within a tolerance**

Solving MIPs: Perceptions end 1980s

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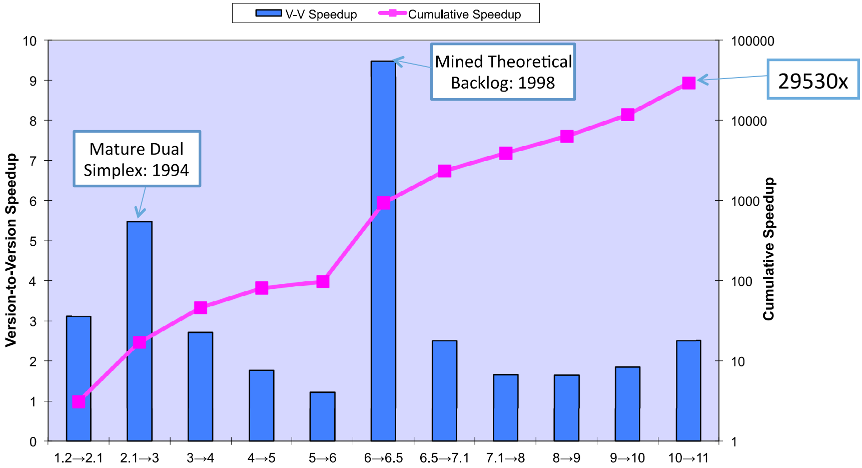

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- California 7-day model (on a desktop PC)
  - CPLEX 6.5 (1999): 22 minutes, optimal
  - CPLEX 11.0 (**2007**): 71 seconds, optimal

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- ∼ **3x** faster year-to-year

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- So, what is left to do?

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  - **Improving the MIP-Based UC formulation** $\Rightarrow \downarrow$ **solving times**

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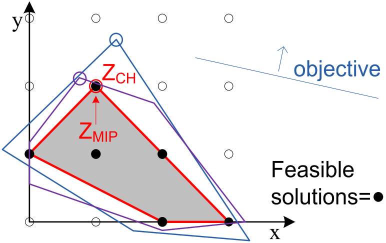
Convex Hull: The Tightest Formulation

Convex Hull (CH)

Smallest convex feasible region containing all feasible integer points



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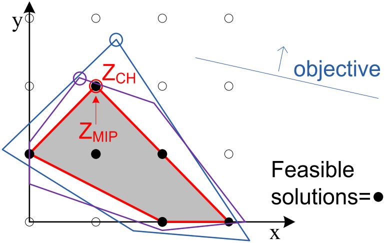

- The *convex hull* problem solves an MIP as an LP
  - Each vertex satisfies the integrality constraints
  - So an LP optimum is also an MIP optimum
- Unfortunately,

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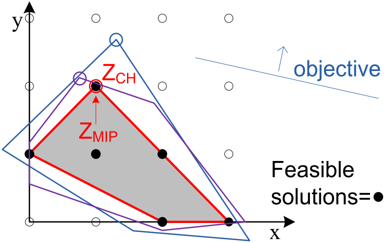

- The *convex hull* problem solves an MIP as an LP
  - Each vertex satisfies the integrality constraints
  - So an LP optimum is also an MIP optimum
- Unfortunately, the *convex hull* is typically too difficult to obtain
  - To solve an MIP is usually easier than trying to find its *convex hull*

Concepts: Tightness and Compactness

- Tightness: defines the search space (relaxed feasible region) that the solver needs to explore to find the solution
- Compactness (problem size): defines the searching speed (data to process) that the solver takes to find the solution

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- Compactness (problem size): defines the searching speed (data to process) that the solver takes to find the solution
- *Convex hull*: The tightest formulation $\Rightarrow$ MIP solved as LP

Tightening an MIP Formulation

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  - Trade-off: Tightness vs. Compactness
- Improving the MIP formulation
  - Provide the *convex hull* for some set of constraints
  - If available, use the *convex hull* for some set of constraints

Tight and Compact (TC) Formulation

- Let's focus on the core of UC formulations:
  - Min/max outputs
  - SU & SD ramps
  - Minimum up/down (TU/TD) times

Tight and Compact (TC) Formulation

- Let's focus on the core of UC formulations:
  - Min/max outputs
  - SU & SD ramps
  - Minimum up/down (TU/TD) times, *convex hull* already available[1]
- The whole formulation can be found in the paper TC-UC[2] and the convex hull proof in gentile *et al.*[3]

[1] D. Rajan and S. Takriti, "Minimum Up/Down Polytopes of the Unit Commitment Problem with Start-Up Costs," IBM, Research Report RC23628, Jun. 2005

[2] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and Compact MILP Formulation for the Thermal Unit Commitment Problem," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4897–4908, Nov. 2013

[3] C. Gentile, G. Morales-España, and A. Ramos, "A tight MIP formulation of the unit commitment problem with start-up and shut-down constraints," en, *EURO Journal on Computational Optimization*, vol. 5, no. 1, pp. 177–201, Apr. 2016

Formulation for a generating unit (I)

- Generation limits taking into account:

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} - \max\left(SD{-}SU, 0\right) v_t \quad \forall t \tag{1}$$

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) v_t - \max\left(SU{-}SD, 0\right) w_{t+1} \quad \forall t \tag{2}$$

Total generation $= \underline{P} \cdot u_t + p_t$.

Variables		Parameters	
p_t	Energy production above $\underline{P}$	$\underline{P}$	Minimum power output
u_t	Commitment status	$\overline{P}$	Maximum power output
v_t	Startup status	SU	Startup ramp
w_t	Shutdown status	SD	Shutdown ramp

Formulation for a generating unit (II)

- Logical relationship:

$$u_t - u_{t-1} = v_t - w_t \quad \forall t \tag{3}$$
$$v_t \le u_t \quad \forall t \tag{4}$$
$$w_t \le 1 - u_t \quad \forall t \tag{5}$$

where (4) and (5) avoid the simultaneous startup and shutdown.

- Variable bounds

$$p_t \ge 0 \quad \forall t \tag{6}$$
$$0 \le u_t, v_t, w_t \le 1 \quad \forall t \tag{7}$$

Tightness of the Formulation

Let's study the polytope (1)-(7) using PORTA[4]:

- PORTA enumerates all vertices of a convex feasible region

[4] T. Christof and A. Löbel, "PORTA: POlyhedron Representation Transformation Algorithm, Version 1.4.1," *Konrad-Zuse-Zentrum für Informationstechnik Berlin, Germany*, 2009

Tightness of the Formulation

Let's study the polytope (1)-(7) using PORTA[4]:

- PORTA enumerates all vertices of a convex feasible region
- Example: 3 periods and $\overline{P} = 200, \underline{P} = SU = SD = 100$ for:
  - Case 1: $TU = TD = 1$
  - Case 2: $TU = TD = 2$

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Case 1: Providing The Convex Hull

Formulation:

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$u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3$:

```
DIM = 10

CONV_SECTION
(  1) 0 0 0 0 0 0 0   0   0   0
(  2) 0 0 1 0 1 0 0   0   0   0
(  3) 1 0 0 0 0 1 0   0   0   0
(  4) 0 1 0 1 0 0 1   0   0   0
(  5) 0 1 1 1 0 0 0   0   0   0
(  6) 0 1 1 1 0 0 0   0   0 100
(  7) 1 1 0 0 0 0 1   0   0   0
(  8) 1 1 0 0 0 0 1 100   0   0
(  9) 1 1 1 0 0 0 0   0   0   0
( 10) 1 1 1 0 0 0 0   0   0 100
( 11) 1 1 1 0 0 0 0   0 100   0
( 12) 1 1 1 0 0 0 0   0 100 100
( 13) 1 1 1 0 0 0 0 100   0   0
( 14) 1 1 1 0 0 0 0 100   0 100
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( 16) 1 1 1 0 0 0 0 100 100 100
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END
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All vertices are integer

$\Downarrow$

Convex Hull

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END
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Case 2: Providing and Using Convex Hulls (I)

Formulation + TU/TD *Convex hull*:

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PORTA results for $(TU = TD = 2)$

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How to remove the fractional vertices?

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DIM = 10

CONV_SECTION

	u_1	u_2	u_3	v_2	v_3	w_2	w_3	p_1	p_2	p_3
(1)	0	0	0	0	0	0	0	0	0	0
(2)	1/2	1	1/2	1/2	0	0	1/2	0	50	0
(3)	1/2	1	1/2	1/2	0	0	1/2	0	50	50
(4)	1/2	1	1/2	1/2	0	0	1/2	50	50	0
(5)	1/2	1	1/2	1/2	0	0	1/2	50	50	50
(6)	0	0	1	0	1	0	0	0	0	0
(7)	1	0	0	0	0	1	0	0	0	0
(8)	0	1	1	1	0	0	0	0	0	0
(9)	0	1	1	1	0	0	0	0	0	100
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(16)	1	1	1	0	0	0	0	100	0	0
(17)	1	1	1	0	0	0	0	100	0	100
(18)	1	1	1	0	0	0	0	100	100	0
(19)	1	1	1	0	0	0	0	100	100	100

END

Case 2: Providing and Using Convex Hulls (II)

Reformulating (1) and (2) for $TU \geq 2$:

$$\sout{p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} - \max\left(SD - SU, 0\right) v_t} \quad (1)$$

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$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) v_t - \left(\overline{P} - SD\right) w_{t+1} \quad (8)$$

$$u_t - u_{t-1} = v_t - w_t \quad (3)$$

$$\sum_{i=t-TU+1}^{t} v_i \leq u_t \quad (4)$$

$$\sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t \quad (5)$$

PORTA results for $(TU = TD = 2)$
$u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3$:

```
DIM = 10

CONV_SECTION
(  1) 0 0 0 0 0 0 0   0   0   0
(  2) 0 0 1 0 1 0 0   0   0   0
(  3) 1 0 0 0 0 1 0   0   0   0
(  4) 0 1 1 1 0 0 0   0   0   0
(  5) 0 1 1 1 0 0 0   0   0 100
(  6) 1 1 0 0 0 0 1   0   0   0
(  7) 1 1 0 0 0 0 1 100   0   0
(  8) 1 1 1 0 0 0 0   0   0   0
(  9) 1 1 1 0 0 0 0   0   0 100
( 10) 1 1 1 0 0 0 0   0 100   0
( 11) 1 1 1 0 0 0 0   0 100 100
( 12) 1 1 1 0 0 0 0 100   0   0
( 13) 1 1 1 0 0 0 0 100   0 100
( 14) 1 1 1 0 0 0 0 100 100   0
( 15) 1 1 1 0 0 0 0 100 100 100
END
```

Case 2: Providing and Using Convex Hulls (II)

Reformulating (1) and (2) for $TU \geq 2$:

~~$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} - \max\left(SD - SU, 0\right) v_t \quad (1)$$~~

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$\Rightarrow$ *Convex Hull*

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$u_1, u_2, u_3, v_2, v_3, w_2, w_3, p_1, p_2, p_3$:

```
DIM = 10

CONV_SECTION
(  1) 0 0 0 0 0 0 0   0   0   0
(  2) 0 0 1 0 1 0 0   0   0   0
(  3) 1 0 0 0 0 1 0   0   0   0
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(  6) 1 1 0 0 0 0 1   0   0   0
(  7) 1 1 0 0 0 0 1 100   0   0
(  8) 1 1 1 0 0 0 0   0   0   0
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( 10) 1 1 1 0 0 0 0   0 100   0
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( 13) 1 1 1 0 0 0 0 100   0 100
( 14) 1 1 1 0 0 0 0 100 100   0
( 15) 1 1 1 0 0 0 0 100 100 100
END
```

Outline

Self-UC case Study

- Case Study: Self-UC for 10-units, for 32-512 days time span
  - Basic constraints: max/min, SU/SD and TU/TD

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  - *1bin*[6] 1-binary variable (u): $2^{10\times512}$ combinations
  - *3binTUTD*[7]: 3-binary variable version (u,v,w) + TU/TD *convex hull:* $2^{3\times10\times512}$ combinations

[5] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and Compact MILP Formulation for the Thermal Unit Commitment Problem," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4897–4908, Nov. 2013

[6] M. Carrion and J. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1371–1378, 2006

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  - *3binTUTD*[7]: 3-binary variable version (u,v,w) + TU/TD *convex hull:* $2^{3\times10\times512}$ combinations
- All results are expressed as percentages of *1bin* results

[5] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and Compact MILP Formulation for the Thermal Unit Commitment Problem," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4897–4908, Nov. 2013

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Case Study: Self-UC (I)

Results presented as percentages of *1bin:*

	3binTUTD (%)	*TC* (%)
Constraints	$<$78	$<$48
Nonzeros	89	72
Real Vars	33.3	33.3
Bin Vars	$=$300	$=$300

$\Downarrow$

TC is more Compact

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Integrality Gap	34	$=0$

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TC is Tighter **and Simultaneously** more Compact

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Constraints	<78	<48
Nonzeros	89	72
Real Vars	33.3	33.3
Bin Vars	$=300$	$=300$
Integrality Gap	34	$=0$
MIP runtime (speedup)	4.9 (20x)	0.107 (995x)

$\Downarrow$

TC is Tighter **and Simultaneously** more Compact

Case Study: Self-UC (II)

Results presented as percentages of *1bin:*

	3binTUTD (%)	*TC* (%)
Constraints	$<$78	$<$48
Nonzeros	89	72
Real Vars	33.3	33.3
Bin Vars	=300	=300
Integrality Gap	34	**=0**
MIP runtime	4.9	0.107
LP runtime	80	**49.8**

$\Downarrow$

The *TC* formulation describe the *convex hull*
then solving MIP (non-convex) as LP (convex)

Outline

Stochastic UC: Case Study

- 10 generating units for a time span of 2 days
- 10 to 200 scenarios in demand

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[8] T. Li and M. Shahidehpour, "Price-based unit commitment: A case of Lagrangian relaxation versus mixed integer programming," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 2015–2025, Nov. 2005

[9] J. Arroyo and A. Conejo, "Optimal response of a thermal unit to an electricity spot market," *Power Systems, IEEE Transactions on*, vol. 15, no. 3, pp. 1098–1104, 2000

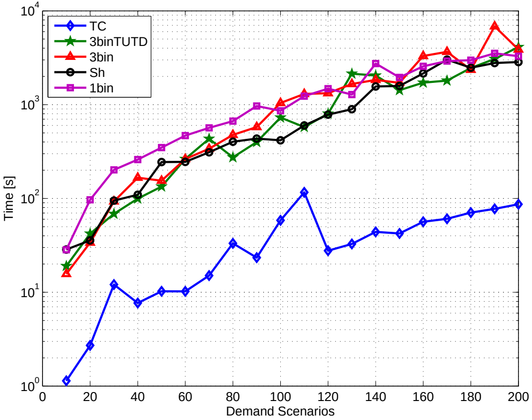
Stochastic UC: Case Study

- 10 generating units for a time span of 2 days
- 10 to 200 scenarios in demand
- Comparing *TC*, *1bin*, *3binTUTD* and Two additional formulations *Sh*[8] and *3bin*[9]
- Different Solvers
  - Cplex 12.6.0
  - Gurobi 5.6.2
  - XPRESS 25.01.07
- Stop criteria:
  - Time limit: 5 hours or
  - Optimality tolerance: 0.1 %

[8] T. Li and M. Shahidehpour, "Price-based unit commitment: A case of Lagrangian relaxation versus mixed integer programming," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 2015–2025, Nov. 2005

[9] J. Arroyo and A. Conejo, "Optimal response of a thermal unit to an electricity spot market," *Power Systems, IEEE Transactions on*, vol. 15, no. 3, pp. 1098–1104, 2000

Stochastic: Cplex



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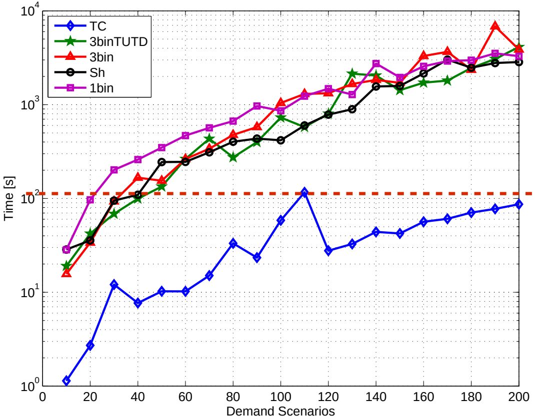
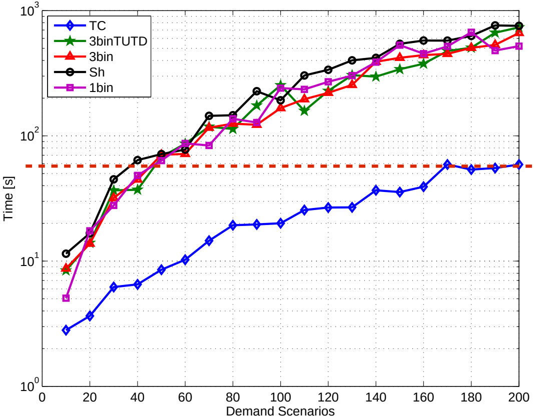


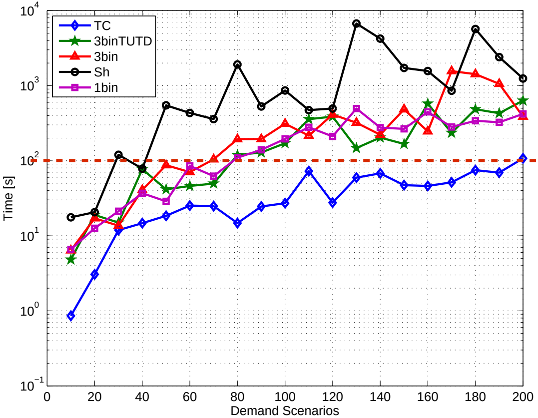
TC deals with 200 scenarios within the time that others deal with 40

Stochastic: Gurobi



TC deals with 200 scenarios within the time that others deal with 50

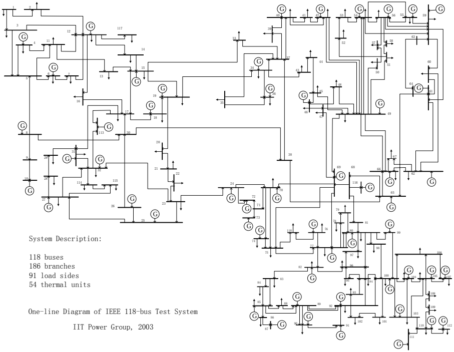
Stochastic: XPRESS



TC deals with 200 scenarios within the time that others deal with 80

Outline

IEEE-118 Bus System



- 54 thermal units; 118 buses; 186 transmission lines; 91 loads
  - 24 hours time span
  - 3 wind farms, 20 wind power scenarios
  - Stop Criteria in Cplex 12.6.0
    - 0.05% opt. tolerance or 24h time limit

UC performance comparisons (I)

	Traditional Energy-Block Scheduling	
	3binTUTD[10]	*TC*
o.f. [k$]	829.04	829.02
opt.tol [%]	0.224	0.023
IntGap [%]	1.27	0.58

- Compared with *3binTUTD*, *TC*:
  - lowered IntGap by 53.3%

[10] FERC, "RTO Unit Commitment Test System," Federal Energy and Regulatory Commission, Washington DC, USA, Tech. Rep., Jul. 2012, p. 55

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MIP runtime [s]	**86400**	**206.5**

- Compared with *3binTUTD*, *TC*:
  - lowered IntGap by 53.3%
  - is more than 420x faster

[10] FERC, "RTO Unit Commitment Test System," Federal Energy and Regulatory Commission, Washington DC, USA, Tech. Rep., Jul. 2012, p. 55

UC performance comparisons (II)

	Traditional Energy-Block Scheduling	
	3binTUTD[11]	*TC*
o.f. [k$]	829.04	829.02
opt.tol [%]	0.224	0.023
IntGap [%]	1.27	0.58
MIP runtime [s]	86400	**206.5**
LP runtime [s]	**246.76**	22.03

- *TC* solved the MIP before *3binTUTD* solved the LP
  - within the required opt. tolerance (0.05%)

[11] FERC, "RTO Unit Commitment Test System," Federal Energy and Regulatory Commission, Washington DC, USA, Tech. Rep., Jul. 2012, p. 55

UC performance comparisons (III)

	Traditional Energy-based UC		Power-Based UC
	3binTUTD	*TC*	*P-TC*
o.f. [k$]	829.04	829.02	**818.13**
opt.tol [%]	0.224	0.023	0.049
IntGap [%]	1.27	0.58	
MIP runtime [s]	**86400**	206.5	
LP runtime [s]	246.76	22.03	

- *P-TC*[12] has a more detailed and accurate UC representation

[12] G. Morales-Espana, A. Ramos, and J. Garcia-Gonzalez, "An MIP Formulation for Joint Market-Clearing of Energy and Reserves Based on Ramp Scheduling," *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 476–488, 2014

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opt.tol [%]	0.224	0.023	0.049
IntGap [%]	1.27	0.58	0.74
MIP runtime [s]	**86400**	206.5	**867.9**
LP runtime [s]	246.76	22.03	38.1

- *P-TC*[12] has a more detailed and accurate UC representation
  - it solved 100x faster than *3binTUTD*
  - its UC core is also a **convex hull**[13]

[12] G. Morales-Espana, A. Ramos, and J. Garcia-Gonzalez, "An MIP Formulation for Joint Market-Clearing of Energy and Reserves Based on Ramp Scheduling," *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 476–488, 2014

[13] G. Morales-España, C. Gentile, and A. Ramos, "Tight MIP formulations of the power-based unit commitment problem," *OR Spectrum*, vol. 37, no. 4, pp. 929–950, May 2015

Outline

Conclusions (I)

- Beware of what matters in good MIP formulations
    - Tightness & Compactness

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Conclusions (I)

- Beware of what matters in good MIP formulations
  - Tightness & Compactness
  - $\uparrow$ Binaries $\Rightarrow \uparrow$ Solving time **False myth**
- Use the *convex hull* of some set of constraints
  - Minimum up/down times[14]
  - Unit operation in Energy-based UC[15]
  - Unit operation in Power-based UC[16]
  - $\Rightarrow \downarrow$ solving time by **simultaneously** T&Cing the final UCs[17,18]

---

[14] D. Rajan and S. Takriti, "Minimum Up/Down Polytopes of the Unit Commitment Problem with Start-Up Costs," IBM, Research Report RC23628, Jun. 2005

[15] C. Gentile, G. Morales-España, and A. Ramos, "A tight MIP formulation of the unit commitment problem with start-up and shut-down constraints," en, *EURO Journal on Computational Optimization*, vol. 5, no. 1, pp. 177–201, Apr. 2016

[16] G. Morales-España, C. Gentile, and A. Ramos, "Tight MIP formulations of the power-based unit commitment problem," *OR Spectrum*, vol. 37, no. 4, pp. 929–950, May 2015

[17] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and Compact MILP Formulation for the Thermal Unit Commitment Problem," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4897–4908, Nov. 2013

[18] G. Morales-Espana, J. M. Latorre, and A. Ramos, "Tight and Compact MILP Formulation of Start-Up and Shut-Down Ramping in Unit Commitment," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1288–1296, 2013

Conclusions

- Better UC core in stochastic UCs $\Rightarrow$
  - Critical solving time reductions

Conclusions

- Better UC core in stochastic UCs ⇒
  - Critical solving time reductions
- If *convex hulls* are not available ⇒
  - Create simultaneously tight and compact models
  - by reformulating the problem, e.g., CCGTs[19]
  - Key hint: start removing all big-M parameters

[19] G. Morales-España, C. M. Correa-Posada, and A. Ramos, "Tight and Compact MIP Formulation of Configuration-Based Combined-Cycle Units," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1350–1359, Mar. 2016

Possible collaboration?

- With PhD Students (papers envisioned)
  - Convex hull for variable SU costs
    - Almost there (just one constraint needs to be added)
  - Stochastic UC dealing with intra-period uncertainty
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  - Long term investment model for combined heat and power systems

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- Master theses with Jedlix
  - Robust smart charging of EVs
  - Aggregated EV models to deal with uncertainty

Questions

Thank you for your attention

Contact Information:
g.a.moralesespama@tudelft.nl

For Further Reading

J. Arroyo and A. Conejo, "Optimal response of a thermal unit to an electricity spot market," *Power Systems, IEEE Transactions on*, vol. 15, no. 3, pp. 1098–1104, 2000.

M. Carrion and J. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1371–1378, 2006.

T. Christof and A. Löbel, "PORTA: POlyhedron Representation Transformation Algorithm, Version 1.4.1," *Konrad-Zuse-Zentrum für Informationstechnik Berlin, Germany*, 2009.

FERC, "RTO Unit Commitment Test System," Federal Energy and Regulatory Commission, Washington DC, USA, Tech. Rep., Jul. 2012, p. 55.

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For Further Reading (cont.)

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