1. **B.** See: Stewart, § 12.4. Compare with example 3 and the exercises 29-32.

Take two vectors in the plane through $A$, $B$ and $C$, for instance: $\mathbf{b} - \mathbf{a} = \langle 1, 1, -2 \rangle$ and $\mathbf{c} - \mathbf{a} = \langle -1, 0, 1 \rangle$. Then we have

$$(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a}) = \langle 1, 1, -2 \rangle \times \langle -1, 0, 1 \rangle = \langle 1, 1, 1 \rangle$$

is a vector perpendicular to the plane.

2. **H.** See: Stewart, § 1.6. Compare with example 13 and the exercises 69-72.

Suppose that $y = \arccos x$, then we have: $\cos y = x$ and $y \in [0, \pi]$. For $0 < x < 1$ we have that $0 < y < \frac{\pi}{2}$. Draw a right angled triangle with an orthogonal side of length $x$ and the hypotenuse of length 1. Then the other orthogonal side has length $\sqrt{1 - x^2}$ (according to the theorem of Pythagoras). Then we have:

$$\tan(\arccos x) = \frac{\sqrt{1 - x^2}}{x}.$$ 

3. **H.** See: Stewart, § 3.5. Compare with exercise 29.

An equation of the tangent line at the point $(2, 1)$ has the form $y - 1 = r(x - 2)$, where $r$ denotes the slope. Now we have $r = \frac{dy}{dx} \bigg|_{(2,1)}$. Using implicit differentiation we obtain:

$$18x + 18y \frac{dy}{dx} = 10(x^2 + y^2 - x) \left(2x + 2y \frac{dy}{dx} - 1\right).$$

For $x = 2$ and $y = 1$ we find that

$$36 + 18 \frac{dy}{dx} = 120 + 60 \frac{dy}{dx} - 30 \quad \Rightarrow \quad r = \frac{dy}{dx} \bigg|_{(2,1)} = -\frac{54}{42} = -\frac{9}{7}.$$ 

Finally we have: $7y - 7 = -9x + 18$ or equivalently $9x + 7y = 25$.


Using the substitution $y = \ln x$ we obtain:

$$\int_1^e \frac{(\ln x)^2}{x} \, dx = \int_1^e (\ln x)^2 \, d\ln x = \int_0^1 y^2 \, dy = \left[ \frac{1}{3}y^3 \right]_0^1 = \frac{1}{3}.$$
5. **H.** See: Stewart, § 5.5. This is exercise 43 of § 7.5.

Using the substitution \( t = x\sqrt{x} \) we obtain:

\[
\int \frac{\sqrt{x}}{1 + x^3} \, dx = \frac{2}{3} \int \frac{d(x\sqrt{x})}{1 + x^3} = \frac{2}{3} \int \frac{dt}{1 + t^2} = \frac{2}{3} \arctan t + C = \frac{2}{3} \arctan (x\sqrt{x}) + C.
\]


We have:

\[
\int_0^\infty \frac{x}{1 + x^3} \, dx = \int_0^1 \frac{x}{1 + x^3} \, dx + \int_1^\infty \frac{x}{1 + x^3} \, dx \quad \text{and} \quad \frac{x}{1 + x^3} < \frac{x}{x^3} = \frac{1}{x^2} \quad \text{for} \quad x > 0.
\]

And since \( \int_1^\infty \frac{dx}{x^2} = 1 \) this implies that \( \int_0^\infty \frac{x}{1 + x^3} \, dx \) is convergent.

For \( x > 0 \) we have that \( 0 < \frac{\arctan x}{2 + e^x} < \frac{\pi}{2} \frac{2 + e^x}{e^x} < \frac{\pi}{2} e^{-x} \). Since \( \int_0^\infty e^{-x} \, dx = 1 \) this implies that \( \int_0^\infty \frac{\arctan x}{2 + e^x} \, dx \) is convergent.

Note that \( 0 \leq \frac{\cos^2 x}{\sqrt{x}} \leq \frac{1}{\sqrt{x}} \) and that \( \int_0^1 \frac{dx}{\sqrt{x}} = 2 \). This implies that \( \int_0^1 \frac{\cos^2 x}{\sqrt{x}} \, dx \) is convergent too.

7. **C.** See: Stewart, § 9.3. Compare with example 3.

The differential equation is separable. Note that \( y = 0 \) is a solution. For \( y \neq 0 \) we have:

\[
\frac{dy}{y} = x^2 \, dx \quad \Rightarrow \quad \ln |y| = \frac{1}{3} x^3 + K \quad \Rightarrow \quad y(x) = \pm e^{K \cdot e^{\frac{1}{3} x^3}}.
\]

Hence: \( y(x) = Ce^{\frac{1}{3} x^3} \). With \( y(0) = 2 \) we have: \( C = 2 \). Hence: \( y(x) = 2e^{\frac{1}{3} x^3} \).

8. **D.** See: Stewart, § 9.5. This is exercise 19.

The differential equation is linear. Divide by \( x \) for the standard form:

\[
y'(x) - \frac{1}{x} y(x) = x \sin x.
\]

Now we look for an integrating factor \( I(x) \) such that:

\[
I(x)y'(x) - \frac{1}{x} I(x)y(x) = xI(x) \sin x \quad \text{and} \quad I'(x) = -\frac{1}{x} I(x).
\]

This implies that \( I(x) = \frac{1}{x} \) (for instance). Then we have:

\[
\frac{d}{dx} \left[ \frac{1}{x} y(x) \right] = \sin x \quad \Rightarrow \quad \frac{1}{x} y(x) = \int \sin x \, dx = -\cos x + C
\]

and therefore: \( y(x) = -x \cos x + Cx \). With \( y(\pi) = 0 \) we have: \( 0 = \pi + C\pi \) or equivalently \( C = -1 \). Hence: \( y(x) = -x \cos x - x \).

9. **D.** See: Stewart, Appendix H.

In the polar form is: \( -1 + i\sqrt{3} = 2e^{\frac{2\pi i}{3}} \) and \( \sqrt{3} + i = 2e^{\frac{\pi i}{6}} \). Then we have:

\[
\left( \frac{-1 + i\sqrt{3}}{\sqrt{3} + i} \right)^4 = \left( \frac{2e^{\frac{2\pi i}{3}}}{2e^{\frac{\pi i}{6}}} \right)^4 = 2^4 e^{\frac{14\pi i}{6}} = 16 e^{\frac{3\pi i}{2}} = 16 \left(-\frac{1}{2} + \frac{1}{2}i\sqrt{3} \right) = -8 + 8i\sqrt{3}.
\]