
The area of the triangle $ABC$ equals half the area of the parallelogram spanned by the vectors $b - a = (-2, 3, -5)$ and $c - a = (-5, 4, -9)$. Hence:

$$\text{area}(ABC) = \frac{1}{2} |(b - a) \times (c - a)| = \frac{1}{2} |(-7, 7, 7)| = \frac{7}{2} |(-1, 1, 1)| = \frac{7}{2} \sqrt{3}.$$

2. G. See: Stewart, § 1.6. This is example 13.

Suppose that $y = \arctan x$, then we have: $\tan y = x$ and $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. For $x > 0$ we have that $0 < y < \frac{\pi}{2}$. Draw a right-angled triangle with an orthogonal side of length $x$ and an other orthogonal side of length 1. Then the hypotenuse has length $\sqrt{1 + x^2}$ (according to the theorem of Pythagoras). Then we have:

$$\cos(\arctan x) = \frac{1}{\sqrt{1 + x^2}}.$$

Using the fact that $\arctan(-x) = -\arctan x = -y$ it is not difficult to see that this holds for $x \leq 0$ as well.


An equation of the tangent line at the point $(-1, 2)$ has the form $y - 2 = r(x + 1)$, where $r$ denotes the slope. Now we have $r = \frac{dy}{dx} \big|_{(-1,2)}$. Using implicit differentiation we obtain:

$$2(x + 2y - 1) \left(1 + 2\frac{dy}{dx}\right) = 4x + 2y\frac{dy}{dx}.$$

For $x = -1$ and $y = 2$ we find that

$$4 + 8\frac{dy}{dx} = -4 + 4\frac{dy}{dx} \quad \Rightarrow \quad r = \frac{dy}{dx} \big|_{(-1,2)} = -2.$$

Finally we have: $y - 2 = -2(x + 1)$ or equivalently $y = -2x$.


Using integration by parts we obtain:

$$\int_1^e \frac{\ln x}{x^2} \, dx = -\int_1^e \ln x \, d\left(\frac{1}{x}\right) = \left[-\frac{\ln x}{x}\right]_1^e + \int_1^e \frac{1}{x} \, dx = -\frac{1}{e} + \left[\frac{1}{x}\right]_1^e = -\frac{1}{e} - \frac{1}{e} + 1 = 1 - \frac{2}{e}.$$
5. B. See: Stewart, § 5.5. Compare with § 7.5, exercise 63.

Using the substitution $t = \sqrt{x}$ we obtain:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = 2 \int e^{\sqrt{x}} \, d\sqrt{x} = 2 \int e^{t} \, dt = 2e^{t} + C = 2e^{\sqrt{x}} + C.$$

6. B. See: Stewart, § 7.8. Compare with example 4, example 8 and exercise 57.

We have:

$$\int_{1}^{0} \ln x \, dx = \lim_{\epsilon \downarrow 0} \int_{\epsilon}^{1} \ln x \, dx = \lim_{\epsilon \downarrow 0} \left( -\epsilon \ln \epsilon - 1 + \epsilon \right) = -1,$$

since

$$\int_{\epsilon}^{1} \ln x \, dx = \left[ x \ln x \right]_{\epsilon}^{1} - \int_{\epsilon}^{1} \frac{1}{x} \, dx = -\epsilon \ln \epsilon - 1 + \epsilon$$

and

$$\int_{1}^{\infty} \frac{1}{x^{2}} \, dx = \lim_{A \to \infty} \left( -\frac{2}{\sqrt{A}} + 2 \right) = 2.$$

7. G. See: Stewart, § 9.3.

The differential equation is separable:

$$\frac{dy}{1 + y^{2}} = (1 + x^{2}) \, dx \implies \arctan y = x + \frac{1}{3}x^{3} + C \implies y(x) = \tan \left( x + \frac{1}{3}x^{3} + C \right).$$

With $y(0) = 1$ we have: $C = \frac{\pi}{4}$.


The differential equation is linear. Divide by $x$ for the standard form:

$$y'(x) + \frac{2}{x} y(x) = \frac{e^{-x}}{x}.$$ 

Now we look for an integrating factor $I(x)$ such that:

$$I(x)y'(x) + \frac{2}{x} I(x)y(x) = I(x) \frac{e^{-x}}{x} \quad \text{and} \quad I'(x) = \frac{2}{x} I(x).$$

This implies that $I(x) = e^{\ln x} = x^{2}$ (for instance). Then we have:

$$\frac{d}{dx} \left[ x^{2} y(x) \right] = x e^{-x} \implies x^{2} y(x) = \int x e^{-x} \, dx = -\int x \, e^{-x} \, dx = -x e^{-x} + \int e^{-x} \, dx = -xe^{-x} - e^{-x} + C$$

and therefore: $y(x) = \frac{-xe^{-x} - e^{-x} + C}{x^{2}}$. With $y(-1) = 1$ we have: $C = 1$.

9. D. See: Stewart, Appendix H.

In the polar form is: $1 - i = \sqrt{2} e^{-\frac{1}{4} \pi i}$ and $-1 + i = \sqrt{2} e^{\frac{3}{4} \pi i}$. Then we have:

$$\frac{(1 - i)^{11}}{(-1 + i)^{4}} = \frac{\left( \sqrt{2} e^{-\frac{1}{4} \pi i} \right)^{11}}{\left( \sqrt{2} e^{\frac{3}{4} \pi i} \right)^{4}} = 2^{\frac{11}{2}} e^{-\frac{22}{4} \pi i} = 8\sqrt{2} e^{\frac{1}{4} \pi i} = 8(1 + i).$$