1. Find the general solution of the differential equation
\[ y''(x) + y(x) = \sin(x) + \sin(2x). \]

2. Find the sum of \( \sum_{n=2}^{\infty} \frac{n(n-1)}{2^n} \) through the following steps:

(a) Start with the geometric series \( \sum_{n=0}^{\infty} x^n \) and deduce from this the sum of the series \( \sum_{n=1}^{\infty} nx^{n-1} \) and the sum of the series \( \sum_{n=2}^{\infty} n(n-1)x^{n-2} \) for \( |x| < 1 \).

(b) Now find the sum of \( \sum_{n=2}^{\infty} \frac{n(n-1)}{2^n} \).

3. Consider the function \( f(x) = \sqrt{1 + x^3} \) for \(-1 < x < 1\).

(a) Use the formula for the binomial series to show that
\[ f(x) = 1 + \frac{1}{2} x^3 - \frac{1}{8} x^6 + \frac{1}{16} x^9 + \ldots. \]

(b) Evaluate \( \lim_{x \to 0} \frac{1 - \sqrt{1 + x^3}}{x^3} \).

4. Consider the curve \( C \) given by \( \mathbf{r}(t) = 4t \mathbf{i} + 3t \mathbf{j} + 2t^2 \sqrt{t} \mathbf{k} \) with \( 0 \leq t \leq \frac{1}{2} \).

(a) Show that the (arc) length of \( C \) is equal to \( 5 \int_{0}^{\frac{1}{2}} \sqrt{1 + t^2} \, dt \).

(b) Use the first three terms of the series in part (a) of the preceding problem to find an approximation of this (arc) length of \( C \). Give your answer in the form of a (simplified) fraction.

5. Find an equation of the tangent plane to the surface \( z = 2xe^{xy} \) at the point \( (2, 0, 4) \).