1. Find (if possible) the solution of the boundary-value problem

\[ y''(x) + 4y'(x) + 4y(x) = 0, \quad y(0) = 2, \quad y(1) = 0. \]

2. Find the general solution of the differential equation

\[ y''(x) + y'(x) - 2y(x) = 9e^x - 4e^{2x}. \]

3. Consider the power series \( \sum_{n=0}^{\infty} \frac{3^n(x + 1)^n}{5n+1} \).

   (a) Determine the central point and the radius of convergence of this power series.
   (b) Find the sum of the series for all \( x \) for which the series converges.

   Hint: it is a geometric series.

4. Consider the function \( f(x) = \frac{1}{\sqrt{1 + x^2}} \) for \(-1 < x < 1\).

   (a) Show by using the formula for the binomial series that

   \[ f(x) = 1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 - \frac{5}{16}x^6 + \ldots. \]

   (b) Use the first three terms of this series to estimate

   \[ \int_{0}^{\frac{1}{2}} \frac{dx}{\sqrt{1 + x^2}}. \]

   (c) Evaluate the limit

   \[ \lim_{x \to 0} \frac{1}{\sqrt{1 + x^2}} - 1 + \frac{1}{2}x^2 \left( \frac{1}{1 - \cos(x^2)} \right). \]

5. Determine the (arc) length of the curve given by

\[ \mathbf{r}(t) = \langle t\sqrt{2}, e^t, e^{-t} \rangle, \quad 0 \leq t \leq 1. \]

6. Show that \( \lim_{(x,y) \to (0,0)} \frac{xy}{\sqrt{x^2 + y^2}} = 0. \)

Grade: \( \frac{\text{earned score} + 4}{4} \) rounded to one decimal