1. Find the general solution of the differential equation
\[ y''(x) + 4y'(x) + 5y(x) = 3e^{-2x} - 25x + 8 \cos(x). \]

2. Consider \( x = 0.1 = 0.11111 \ldots \)
   (a) Evaluate \( 10x - 1 \) and use this to show that \( x = \frac{1}{9} \).
   (b) Express \( x \) as a geometric series and find its sum.

3. Find the sum of \( \sum_{n=1}^{\infty} \frac{n+1}{3^n} \) through the following steps:
   (a) Start with the geometric series \( \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \) for \( |x| < 1 \) and show that
   \[ \frac{d}{dx} \left( \frac{x^2}{1-x} \right) = \sum_{n=0}^{\infty} (n+2)x^{n+1} \quad \text{for} \quad |x| < 1. \]
   (b) Use this to find the sum of the series \( \sum_{n=1}^{\infty} \frac{n+1}{3^n} \).

4. (a) Use a binomial series to show that
   \[ \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \left( -1 \right)^n \left( \frac{1}{2} \right)^n x^{2n} \quad \text{for} \quad |x| < 1. \]
   (b) Use this to find the first three nonzero terms of the Taylor series of \( \arccos(x) \).
   Hint: the derivative of \( \arccos(x) \) is \( \frac{-1}{\sqrt{1-x^2}} \).

5. Use the first four nonzero terms of the Taylor series of \( \sin(x^4) \) to find an approximation of \( \int_0^1 \sin(x^4) \, dx \). You don’t need to simplify your answer.

6. Use Taylor series to evaluate \( \lim_{x \to 0} \frac{1 + \frac{1}{2}x^2 - \sqrt{1+x^2} - \frac{1}{2}e^{x^2} - \frac{1}{2}e^{-x^2} \cos(x)}{1 + \frac{1}{2}e^{x^2} - e^{x^2} \cos(x)} \).

7. Find the arc length of the curve given by \( y = \cosh(x) := \frac{e^x + e^{-x}}{2} \) and \(-2 \leq x \leq 2\).
   Hint: You may use that \( \cosh'(x) = \sinh(x) := \frac{e^x - e^{-x}}{2} \) and \( \cosh^2(x) - \sinh^2(x) = 1 \).

8. Use the linearization of \( f(x, y) = xe^{xy} \) at the point \((1, 0)\) to find an approximation of \( f(1.1, -0.1) \).

Grade: \( \frac{\text{earned score} + 4}{4} \) rounded to one decimal