On a packet scheduling problem for smart antennas and polyhedra defined by circular-ones matrices

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Abstract

In [1,2] E. Amaldi et al. posed a combinatorial optimization problem that arises when scheduling packets in a smart antenna. The objective is to partition the set of users so as to minimize the number of time slots needed to transmit all the given packets. Here we will present a polynomial time algorithm for solving this packet scheduling problem. More generally, the algorithm solves an integer decomposition problem for polyhedra determined by a circular-ones constraint matrix, which might make it interesting also for other cyclic scheduling problems.

Key words: packet scheduling, cyclic scheduling, polytime algorithm, integer programming

1 Introduction

In recent years, there has been a growing interest in adaptive antenna arrays known as "smart antennas". The combination of an antenna array and digital signal processing capabilities, enables a smart antenna to transmit and receive signals in a spatially sensitive manner. The spatial radiation pattern can be adjusted in real time in response to the signal environment. Exploiting this, signals to different users can be transmitted simultaneously over the same radio channel.

This allows us to view a smart antenna as a collection of co-located directive antennas that each transmit to (or receive from) a narrow beam (approximately 12 degrees). Each of these directive antennas can be independently oriented and can serve one user at a time. However, in order to avoid unwanted interference, there is a restriction on the sets of users that can be served simultaneously: a user that is being served, cannot be in the beam corresponding to a directive antenna that serves another user. This restriction limits the number of users that can be served during the same time slot.

As an example, suppose that the angle of the beams from the directive antennas is 12 degrees and that three users are in a common sector of 12 degrees. If the middle of the three users is served, then the beam corresponding to the antenna that serves it must either contain the clockwise or the anticlockwise neighbour which therefore cannot be served at the same time. This implies that for a set of users that are served simultaneously, the angle between any of these users and it's second clockwise neighbour is more than 12 degrees. Hence the number of users that can be served in a single time slot is less than 60. In fact we will assume that the number of available directive antennas is unlimited and the sets of users that can be served simultaneously are determined exactly by this interference constraint.

2 Modelling the packet scheduling problem

In [2], Amaldi et al. considered the following scheduling problem: given a set of users, we want to serve all of them, minimizing the total number of time slots needed. That is, we want to partition the users into a minimal number of classes, where the members of each class can be served simultaneously by the smart antenna.

Following Amaldi et al., we model the problem in the following manner. Since the exact position of the users is not needed, only their direction as seen from the smart antenna, we model the users by points on the unit circle and let the beams from the directive antennas correspond to arcs of a fixed length α of the unit circle. We will always assume that $0 < \alpha < 2\pi$.

For two points a, b on the unit circle the closed segment running clockwise from a to b is called an *arc* and is denoted by $[a, b]$. Let $\alpha > 0$ be fixed. A finite set S of points on the unit circle will be called *independent* [1] if there exist $|S|$ arcs on the unit circle, each of length α, such that each point in S is in exactly one of these arcs and each of these arcs contains exactly one element of S. Note that any two of the $|S|$ arcs may intersect as long as the intersection does not contain a point in S. The independent sets correspond to the sets of users that can be served simultaneously. The scheduling problem can now be stated as follows.

[1] In [2] arcs are half-open segments, but for the definition, this is equivalent to using closed segments of the same length.

Problem 1 *Given a finite subset V of the unit circle, find a partition of V into a minimal number of independent sets.*

3 Results

A basic observation, which led us to a polynomial time algorithm for the scheduling problem is the following

Observation 1 *A finite set S of points on the unit circle is independent if and only if $|S \cap [s, s']| \leq 2$ for each arc $[s, s']$ of length α starting at a point $s \in S$.*

To see necessity, suppose that some arc of length α contains $u, v, w \in S$ in this order, then any arc of length α containing v also contains u or w and hence S is not independent. For sufficiency, suppose that $|S \cap [s, s']| \leq 2$ for each arc $[s, s']$ of length α with $s \in S$. Let $v \in S$ and let u and w be the anticlockwise and clockwise neighbour in S of v respectively. The length of $[u, w]$ must be larger than α since $|[u, w] \cap S| > 2$, and hence there exists an arc of length α intersecting S only in v. Note that the last argument also shows that given an independent set S, $|S|$ arcs of length α as in the definition of independent set, are easily constructed from S.

This observation allows us to identify the independent sets with the zero-one solutions to a system of linear inequalities $Ax \leq \mathbf{2}$. Here the zero-one constraint matrix A is a *circular-ones matrix*: in each row of A, the ones or the zeros form a contiguous block. The scheduling problem is then to partition the all-one vector into a minimal number of zero-one solutions to this system. By extending the algorithm in [3] for colouring proper circular arc graphs, we obtain an algorithm that given an $m \times n$ circular-ones matrix A, a vector $b \in \mathbb{Z}_+^m$, and a vector $x \in \mathbb{Z}_+^n$, decomposes x into a minimal number of integral solutions to $Ax \leq b$. The running time of the algorithm is $O(nm \log(x^T \mathbf{1}))$. The packet scheduling problem is a special case where b is the all-two vector and x is the all-one vector. Applied to the packet scheduling problem, the algorithm finds in time $O(n^2 \log n)$ an optimal schedule, where n is the number of users.

References

[1] E. Amaldi, A. Capone, F. Malucelli, Circular Arc Models and Algorithms for Packet Scheduling in Smart Antennas, *IV ALIO/EURO Workshop on Applied Combinatorial Optimization*, see: http://www-di.inf.puc-rio.br/~celso/artigos/pucon.ps.

[2] E. Amaldi, A. Capone, F. Malucelli, Discrete models and algorithms for packet scheduling in smart antennas, *2nd Cologne Twente Workshop on Graphs and Combinatorial Optimization.*

[3] J.B. Orlin, M.A. Bonuccelli, D.P. Bovet, An $O(n^2)$ algorithm for colouring proper circular arc graphs, *SIAM J. Algebraic Discrete Methods* 2 (1981), no. 2, 88–93.

[4] A. Perez-Neira, X. Mestre, J.R. Fonollosa, Smart antennas in software radio base stations, *IEEE Communications Magazine*, Vol. 39 (2), Feb. 2001, 166–173.

[5] K. Sheikh, D. Gesbert, D. Gore, A. Paulraj, Smart antennas for broadband wireless access networks, *IEEE Communications Magazine*, Vol. 37 (11), Nov. 1999, 100–105.