Comment on ‘Optimum depth of the information pit on the data surface of a compact disk’

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In a recent paper [1] Li et al. investigated the question of optimum pit depth in optical disc read-out. Data on such a disc (e.g. CD or DVD) is encoded in a spiral track of pits of varying length and read out by scanning a diffraction limited spot along this spiral track. When the scanning spot is over land all light is reflected and collected by the objective lens. The resulting intensity measured at the photo-detector is near maximum. When the scanning spot is over a pit light is scattered away from the aperture of the objective lens, and as a result the measured intensity at the photo-detector will be lower than for the land case. The question is how deep the pits should be in order to maximize the modulation between the land and pit signal. In reference [1] and in a previous paper of similar character [2] the authors calculated the interference between two axially displaced point sources that are imaged by a thin lens to two points close to the detector plane. This results in an interference term that varies as \[ \cos(k \Delta(1 + M_T^2)) \], with \( M_T \) being the lateral magnification of the lens and \( k = \frac{2\pi}{\lambda} \), where \( \lambda \) is the wavelength. Minimum intensity (optimized pit depth) is obtained when

\[
\Delta = \frac{\lambda}{2} \frac{m}{1 + M_T^2}, \quad (m = 1, 3, 5, \ldots).
\] (1)

This claim stands in stark contrast to the common belief that the optimum pit depth should be \( m\lambda/4 \) with \( m = 1, 3, 5, \ldots \) [3]. It will be shown in this comment that the result of reference [1] is incorrect.

Figure 1 shows the optical model considered in [1] and [2]. The reflected field generated at the pit bottom is approximated by the field of a single point source \( S_1 \), and the field generated at the land area is approximated by the field of a single point source \( S_2 \). These two effective point sources are imaged by a thin lens to points \( S'_1 \) and \( S'_2 \). The fields at the observation points \( Q \) and \( Q' \) in the object and image spaces, which are in the far field of the source and image points, are the sum of two spherical waves

\[
U_{ob}(Q) = A \frac{e^{ikr_1 + \psi_1}}{r_1} + A \frac{e^{ikr_2 + \psi_2}}{r_2}
\] (2a)
The amplitudes $A$ of the two spherical waves have been chosen equal for the sake of convenience. The quantities $\psi_1$, $\psi_2$, $\psi'_1$, and $\psi'_2$ are the phases at the two source points and their corresponding image points. The crucial mistake of Li and co-authors is in the assignment of these phases. The correct assignment can be derived as follows: one of the four phases may be set equal to zero, for example $\psi'_2 = 0$. The distance from the light source (the laser diode) to the pit bottom is an amount $\Delta$ larger than the distance from the light source to the land area. The phase difference between the two effective point emitters is therefore $k\Delta$. This phase difference fixes the other phase at the object side as $\psi_1 = k\Delta$. In [1] and [2] the phase difference between the two image points is set equal to the phase difference between the two source points leading to $\psi'_1 - \psi'_2 = k\Delta$ (e.g. equations (5) in [2]). This setting incorrectly ignores the difference in phase delay between the object and image points $S_1$ and $S'_1$ on the one hand and the object and image points $S_2$ and $S'_2$ on the other hand. The phase delay between the object and image points is given by the optical path length from the object to the image point. Because both points are conjugate all path lengths through the lens are equal. In the thin lens approximation it therefore follows that

$$\psi'_1 - \psi_1 = kR_1 + kR'_1$$

(3a)

$$\psi'_2 - \psi_2 = kR_2 + kR'_2.$$  

(3b)

As a consequence, the phase difference between the two image points is given by

$$\psi'_1 - \psi'_2 = \psi_1 - \psi_2 + k(R'_1 - R'_2) + k(R_1 - R_2) = k\Delta + k\Delta' + k\Delta$$

$$= (2 - M^2)k\Delta$$,  

(4)

where it is to be noted that $\Delta = R_1 - R_2$ and $\Delta' = R'_1 - R'_2$ are of opposite sign in the present sign convention. The interference term in the focal region has a phase $\psi'_1 - \psi'_2 - k\Delta'$. According to the phase setting of [1] and [2] this results in $\psi'_1 - \psi'_2 - k\Delta' = (1 + M^2)k\Delta$, but with the correct phase setting according to

$$U_{im}(Q') = A \frac{e^{i(-kr'_1 + \psi_1)}}{r'_1} + A \frac{e^{i(-kr'_2 + \psi_2)}}{r'_2}. \quad (2b)$$

The field at point $Q'$ in image space is the sum of two spherical waves converging on the image points $S'_1$ and $S'_2$ of the two point sources.
equation (4) this is $\psi_1 - \psi_2 - k\Delta' = 2k\Delta$. Consequently, the interference term varies as $\cos(2k\Delta)$, leading to the $\lambda/4$ condition for optimum pit-land contrast.

This conclusion is supported by a consideration of the light collected at the aperture on the object side of the lens. It turns out that no light is collected at all when the pit depth satisfies the $\lambda/4$ condition. It follows that the intensity at the detector on the image side of the lens will be zero as well, regardless of the optics in between the collecting aperture and the detector. Clearly, the optimum pit depth can in no way depend on a system parameter such as magnification.

The collected fraction may be calculated as follows. The aperture is the part of the sphere with radius $R = (R_1 + R_2)/2$ and polar angles $\theta \leq \arcsin(NA)$, where $NA$ is the numerical aperture. The distances from the point sources $S_1$ and $S_2$ to an observation point $Q$ on the aperture at a polar angle $\alpha$ are

$$r_1 = \sqrt{(R \cos \theta + \Delta/2)^2 + R^2 \sin^2 \theta} \approx R + \frac{1}{2} \Delta \cos \theta$$

$$r_2 = \sqrt{(R \cos \theta - \Delta/2)^2 + R^2 \sin^2 \theta} \approx R - \frac{1}{2} \Delta \cos \theta,$$

which gives the following field as a function of the polar angle $\theta$

$$U_{ob}(\theta) = A \frac{e^{ikR}}{R} \left( e^{ik\Delta \cos \theta/2} + e^{-ik\Delta \cos \theta/2} \right).$$

The flux through the aperture (normalized by the total flux emitted by the two point sources in the forward direction) is found by straightforward integration as

$$I = \varepsilon \frac{1 + [\sin(k\Delta \varepsilon) / k\Delta \varepsilon] \cos(2k\Delta - k\Delta \varepsilon)}{1 + [\sin(2k\Delta) - \sin(k\Delta)] / k\Delta},$$

where $\varepsilon = \left(1 - \sqrt{1 - NA^2}\right)/2$, and is seen to depend on $k\Delta$ and $NA$ alone. In the limit of small $NA$ this may be approximated as

$$I = 2\varepsilon \frac{\cos^2(k\Delta)}{1 + [\sin(2k\Delta) - \sin(k\Delta)] / k\Delta}.$$

Clearly, for a pit depth satisfying the $\lambda/4$ condition the flux through the aperture is zero.

Finally, we would like to stress that Li and co-authors use a rather schematic model of optical disc read-out. Instead of a model with two effective axially shifted point sources, a model in which laterally shifted point sources are present is closer to the physical reality. In an accurate model of optical disc read-out all points of the data surface act as point emitters. Each contribution must be weighted by the field of the illuminating scanning spot, and all these contributions must be added. This integration over the reflecting disc surface is essential for a more reliable model. A second integration, namely over the detector area, is essential as well. In optical disc read-out the detector area is much larger than the spot on the detector. In the nomenclature of [4] optical disc read-out is an example of type I scanning microscopy, as opposed to type II, or confocal scanning microscopy, where the photo-detector area is much smaller than the light spot (see also [5] for an analysis of scanning microscopy). Due to differences between these two types a calculation of the intensity at the focal point is insufficient for an analysis of actual optical disc read-out. For a type I scanning microscope the intensity integrated over the
detector area is equal to the intensity collected at the aperture of the objective lens, which follows directly from flux conservation. This simplifies the analysis considerably. Effects of high $NA$ and polarization also have a significant influence on the diffraction of light by sub-wavelength sized pits. The formalism of vector diffraction theory is the proper framework for describing these effects. For example, the reflection of elongated pits for linear polarizations parallel and perpendicular to the long axis of the pit differ substantially [6]. This has a significant effect on optimum pit depth. A semi-quantitative estimate of the effect of high $NA$ can already be made using the expression for the collected flux at the aperture of the scanning lens [equation (7)]. It turns out that the minima are still reasonably described by the $m\lambda/4$ condition, provided that $m$ is not too large. The minima are not zero anymore, and the deviation from zero increases with $m$. However, for the first minimum $m = 1$ the effect of $NA$ is very small.

In conclusion, we have shown that (a) the correct assignment of the relative phases of the two effective point sources and their image points leads to the well-known $\lambda/4$ condition for the optimum pit depth; (b) the optimum pit depth cannot depend on a system parameter such as magnification; (c) a more accurate model should take into account the lateral extension of the diffracting pits, the lateral extension of the detector and the effects of polarization and high $NA$.

References