Practice worksheet on multi-agent epistemic logic

Scenario 0 Two people, *Amina (A)* (female) and *Bao (B)* (male), enter a large room. On the table, is a remote-control mechanical coin flipper. One presses a button, and the coin spins through the air, landing in a small box on the table. The box closes. The two people are much too far to see the coin. In reality, the coin shows heads.

Scenario 1 After Scenario 0, they both go up and take a look together.

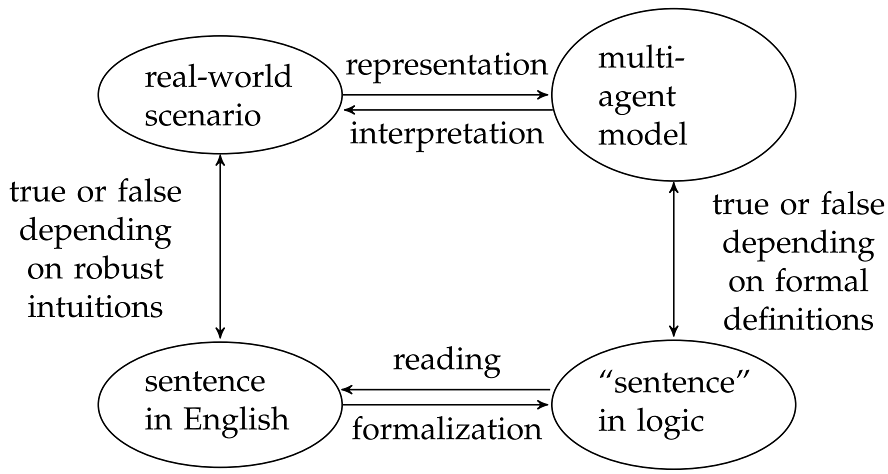
Scenario 2 After Scenario 0, A goes up and looks at the coin while B stands at the door. B watches her open the box, but he doesn't see what she sees.

Scenario 3 After Scenario 0, A mentions that she left something in a different room and asks B to get it. She goes to the table and looks at the coin while B is away. She races back and pretends that nothing happens. B (wrongly) believes that she has been standing there the whole time, so he thinks he is in Scenario 0. A knows all of this about B.

Fill in the table. The second column should be the logic statement corresponding to the first. The last four columns should be your intuitions about whether the English statement is true (√) or false (×) *after* the given Scenario.

English	logic	Sc. 0	Sc. 1	Sc. 2	Sc. 3
The coin shows heads	h	√	√	√	√
The coin shows tails	t	×	×	×	×
The coin is not both heads and tails					
If the coin is not heads, it's tails					
A knows that the coin shows heads	$K_a h$	×	√	√	√
B does not know that the coin shows heads	$\neg K_b h$				
Either A knows that the coin shows heads, or she knows that the coin shows tails					
A knows that Either the coin shows heads or it shows tails					
If the coin shows heads, then A knows that it shows heads					
B "knows" that A does not know that the coin shows heads	$K_b \neg K_a h$				
A knows that B doesn't know that the coin shows heads	$K_a \neg K_b h$				

Here is the big picture. Which part did we work on before, and what are we doing on the next pages?



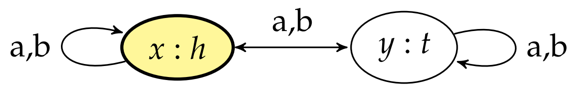
The formal definitions of truth:

$x \models p$	iff	p is written on x (for p atomic)
$x \models \varphi \land \psi$	iff	$x \models \varphi$ and $x \models \psi$
$x \models \varphi \lor \psi$	iff	$x \models \varphi$ or $x \models \psi$
$x \models \neg\varphi$	iff	$x \not\models \varphi$
$x \models \varphi \rightarrow \psi$	iff	if $x \models \varphi$, then $x \models \psi$
$x \models \varphi \leftrightarrow \psi$	iff	$x \models \varphi$ if and only if $x \models \psi$
$x \models K_b\, \varphi$	iff	$y \models \varphi$ for all y such that $x \rightarrow y$ for b

These follows from the definitions, and they are useful facts:

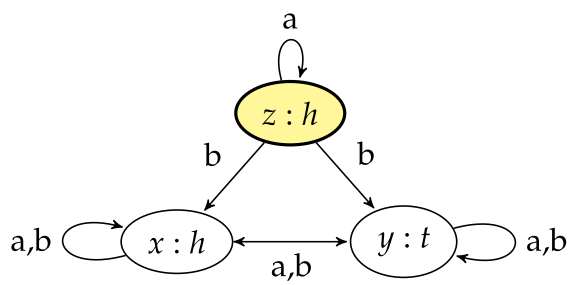
$x \not\models p$	iff	p is not written on x (for p atomic)
$x \not\models \varphi \land \psi$	iff	$x \not\models \varphi$ or $x \not\models \psi$
$x \not\models \varphi \lor \psi$	iff	$x \not\models \varphi$ and $x \not\models \psi$
$x \not\models \neg\varphi$	iff	$x \models \varphi$
$x \not\models \varphi \rightarrow \psi$	iff	$x \models \varphi$, but $x \not\models \psi$
$x \not\models \varphi \leftrightarrow \psi$	iff	($x \models \varphi$, but $x \not\models \psi$) or ($x \models \psi$, but $x \not\models \varphi$)
$x \not\models K_b\, \varphi$	iff	for some y such that $x \rightarrow y$ for b, $y \not\models \varphi$

Now fill in the charts below, using the formal semantics

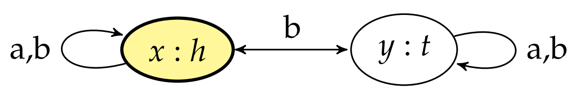


logic sentence	true in x, the real world	true in y
$K_a\, h$	×	×
$K_b\, h$	×	×
$K_b\, K_a\, h$	×	×
$K_b\, \neg K_a\, h$	√	√
$K_a\, K_b\, h$	×	×
$h \rightarrow K_a\, h$	×	√
$K_b(h \rightarrow K_a\, h)$		

logic sentence	true in x, the real world	true in y
$K_a\, h$	√	×
$K_b\, h$	√	×
$K_b\, K_a\, h$		
$K_b\, \neg K_a\, h$		
$K_a\, K_b\, h$		
$h \rightarrow K_a\, h$		
$K_b(h \rightarrow K_a\, h)$		



logic sentence	true in x	true in y	true in z, the real world
$K_a\ h$	×	×	√
$K_b\ h$			
$K_b\ K_a\ h$			
$K_b\ \neg K_a\ h$			
$K_a\ K_b\ h$			
$h \rightarrow K_a\ h$			
$K_b(h \rightarrow K_a\ h)$			



logic sentence	true in x, the real world	true in y
$K_a\ h$		
$K_b\ h$	×	×
$K_b\ K_a\ h$		
$K_b\ \neg K_a\ h$		
$K_a\ K_b\ h$		
$h \rightarrow K_a\ h$		
$K_b(h \rightarrow K_a\ h)$		

Your final task

We started with four scenarios, and now we have seen four models. Figure out which of the original scenarios goes with which of the models. To do this, check your intuitions against what is formally true in the real world of the models.